Measuring Mobility

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Approaches to mobility

- Model of mobility often depends on application:
  - income or wealth mobility
  - wage mobility
  - educational mobility
  - mobility in terms of social status
- Measurement addressed from different standpoints
  - in relation to a specific dynamic model
  - as an abstract distributional concept
- Focus on the mobility measures in the abstract
  - covers income or wealth mobility
  - covers also "rank" mobility where the underlying data are categorical
  - separates out the fundamental components of the mobility-measurement problem

Mobility modelling

- Basic information is the temporal pair $z_i = (x_{i,t-1}, x_{i,t})$
- Bivariate distribution
  - distribution function $F(x_{t-1}, x_t)$
  - marginal distributions F_{t-1} and F_t give income distribution in each period
- Time-aggregated income
  - derived from $(x_{i,t-1}, x_{i,t})$ using weights w_{t-1}, w_t
  - $\bar{x}_i := w_{t-1}x_{i,t-1} + w_t x_{i,t}$
  - Distribution $F_{\mathbf{w}}$ derived from F

Mobility measures in practice

- Stability indices: $1 - \frac{I(F_{\mathbf{w}})}{w_{t-1}I(F_{t-1})+w_t I(F_t)}$
- Hart (1976): $1 - r(\log x_{t-1}, \log x_t)$
  - where r is the correlation coefficient
- King (1983): $1 - \left[\frac{\int\int \left(x_t e^{\gamma r(F,(x_{t-1},x_t))} \right)^k dF(x_{t-1},x_t)}{\mu_k(F_t)} \right]^{\frac{1}{k}}$
  - $k \leq 1,\ k \neq 0,\ \gamma \geq 0$
  - where $r(F;(x_{t-1},x_t))$ is a rank indicator: $\mu_1 (F_t)^{-1} |x_t - Q(F_t;F_{t-1}(x_{t-1}))|$
  - $Q(G;q) := \inf\{x : G(x) \geq q\}$
- Fields-Ok (1999): $c \int\int |\log x_{t-1} - \log x_t| \, dF(x_{t-1}, x_t)$

Fundamentals

- How to characterise mobility
  - in terms of individual income?
  - in terms of social position?
- Ingredients for a theory of mobility measurement:
  1. time frame of two or more periods;
  2. measure of individual status within society
  3. aggregation of changes in status over the time frame.

Ingredients of the problem: classes

- "Income" as a generic term
  - any cardinally measurable, comparable quantity
  - cardinality turns out not to be crucial for our approach
- Ordered set of K income classes
  - class k is associated with income level x_k where $x_k < x_{k+1}, k = 1, 2, ..., K-1$
  - $p_k \in \mathbb{R}_+$ is the size of class k, $k = 1, 2, ..., K$ and
  - $\sum_{k=1}^{K} p_k = n$, the size of the population
- $k_0(i)$ and $k_1(i)$: income class occupied by person i in periods 0 and 1 respectively
- mobility characterised by $\left(x_{k_0(1)}, ..., x_{k_0(n)}\right)$ and $\left(x_{k_1(1)}, ..., x_{k_1(n)}\right)$

Ingredients of the problem: valuation

- Don't have to use simple aggregation of the x_k to compute mobility index
- Could carry out a relabelling of the income classes
- For example use $n_0(x_k) := \sum_{h=1}^{k} p_h,\ k = 1, ..., K$
  - number of persons in, or below, each income class
  - according to the distribution in period 0
- Suppose that the class sizes $(p_1, ..., p_K)$ in period 0 change to $(q_1, ..., q_K)$ in period 1
- Relabelling the income classes:$n_1(x_k) := \sum_{h=1}^{k} q_h,\ k = 1, ..., K$

Ingredients of the problem: status

- u_i, v_i denote individual i's status in the 0-distribution, the 1-distribution
- personal history: $z_i := (u_i, v_i)$
- *Distribution-independent, static (1).* $z_i = \left(x_{k_0(i)}, x_{k_1(i)} \right)$
- *Distribution-independent, static (2).*
  $z_i = \left(\varphi \left(x_{k_0(i)} \right), \varphi \left(x_{k_1(i)} \right) \right)$
  - φ essentially arbitrary (utility of x?)
  - mobility independent of φ?
- *Distribution-dependent, static.*
  $z_i = \left(n_0 \left(x_{k_0(i)} \right), n_0 \left(x_{k_1(i)} \right) \right)$
  - cumulative numbers in class "value" the class
- *Distribution-dependent, dynamic.*
  $z_i = \left(n_0 \left(x_{k_0(i)} \right), n_1 \left(x_{k_1(i)} \right) \right).$

Example

- Consider the following example:

	period 0	period 1
x_1	A	_
x_2	B	A
x_3	C	B
x_4	_	C
x_5	_	_

- Different definitions of status will produce different evaluations of such a change.

Basic axioms

- Let m be individual mobility, increasing in $|u_i - v_i|$
- **Continuity** $\succeq$ is continuous on Z^n
- **Monotonicity.** If $\mathbf{z}, \mathbf{z}' \in Z^n$ differ only in their ith component then $m(u_i, v_i) > m(u'_i, v'_i) \iff \mathbf{z} \succ \mathbf{z}'$.
- **Independence.** For $\mathbf{z}, \mathbf{z}' \in Z^n$ such that: $\mathbf{z} \sim \mathbf{z}'$ and $z_i = z'_i$ for some i then $\mathbf{z}(\zeta, i) \sim \mathbf{z}'(\zeta, i)$ for all $\zeta \in [z_{i-1}, z_{i+1}] \cap [z'_{i-1}, z'_{i+1}]$.
- **Local immobility.** Let $\mathbf{z}, \mathbf{z}' \in Z^n$ be such that, for some i and j, $u_i = v_i$, $u_j = v_j$, $u'_i = u_i + \delta$, $v'_i = v_i + \delta$, $u'_j = u_j - \delta$, $v'_j = v_j - \delta$ and, for all $h \neq i, j$, $u'_h = u_h$, $v'_h = v_h$. Then $\mathbf{z} \sim \mathbf{z}'$.

Representation results (1)

- **Theorem.** Given basic axioms:
  - $\succeq$ is representable by the continuous function given by $\sum_{i=1}^{n} \phi_i (z_i), \forall \mathbf{z} \in Z^n$
  - $\phi_i : Z \to \mathbb{R}$ is a continuous function that is strictly decreasing in $|u_i - v_i|$
  - $\phi_i (u, u) = a_i + b_i u$.
- **Corollary.** $\succeq$ is also representable by
  - $\phi \left(\sum_{i=1}^{n} \phi_i (z_i) \right)$
  - $\phi : \mathbb{R} \to \mathbb{R}$ is continuous and strictly monotonic increasing.

Representation results (2)

- **Status scale irrelevance.** For any $\mathbf{z}, \mathbf{z}' \in Z^n$ such that $\mathbf{z} \sim \mathbf{z}'$, $t\mathbf{z} \sim t\mathbf{z}'$for all $t > 0$.:
- **Theorem.** Given Basic axioms and scale irrelevance:
  - $\succeq$ is representable by $\phi\left(\sum_{i=1}^{n} u_i H_i\left(\frac{u_i}{v_i}\right)\right)$
  - where H_i is a real-valued function.

Representation results (3)

- This suggests we can compare the (u, v) vectors in different parts of the distribution in terms of proportional differences
  - $m(z_i) = \max\left(\frac{u_i}{v_i}, \frac{v_i}{u_i}\right)$
- **Mobility scale irrelevance.** Suppose there are $\mathbf{z}_0, \mathbf{z}'_0 \in Z^n$ such that $\mathbf{z}_0 \sim \mathbf{z}'_0$. Then for all $t > 0$ and $\mathbf{z}, \mathbf{z}'$ such that $m(\mathbf{z}) = tm(\mathbf{z}_0)$ and $m(\mathbf{z}') = tm(\mathbf{z}'_0)$: $\mathbf{z} \sim \mathbf{z}'$.
- **Theorem.** Given our axioms $\succeq$ is representable by $\Phi(\mathbf{z}) = \phi\left(\sum_{i=1}^n u_i^\alpha v_i^{1-\alpha}\right)$
  - where $\alpha \neq 1$ is a constant.

Generating an aggregate mobility index

- Consider a subset of Z :
  - $Z(\bar{u}, \bar{v}) := \{\mathbf{z} \in Z |\ \sum_{i=1}^{n} z_i = (\bar{u}, \bar{v})\}$.
- From theorem 3, that the mobility index must take the form:
  - $\Phi(\mathbf{z}) = \bar{\phi}\left(\sum_{i=1}^{n} u_i^{\alpha} v_i^{1-\alpha};\ \bar{u},\ \bar{v}\right)$.
- $\Phi(\mathbf{z})$ should be zero when there is no mobility
  - using the standard interpretation of mobility
  - $\bar{\phi}\left(\sum_{i=1}^{n} u_i;\ \bar{u},\ \bar{u}\right) = 0$,
  - i.e. $\bar{\phi}(\bar{u};\ \bar{u},\ \bar{u}) = 0$.

Using a broader interpretation of zero mobility

- Scaling up everyone's status should not matter
  - $v_i = \lambda u_i, i = 1, .., n$ (where $\lambda = \bar{v}/\bar{u}$)
  - $\bar{\phi}\left(\lambda^{1-\alpha} \sum_{i=1}^{n} u_i;\ \bar{u},\ \bar{v}\right) = 0$
  - $\bar{\phi}\left(\bar{u}^{\alpha}\bar{v}^{1-\alpha};\ \bar{u},\ \bar{v}\right) = 0.$
- This requires ϕ and $\bar{\phi}$ are equivalent to:
  - $\psi\left(\sum_{i=1}^{n} \left[\frac{u_i}{\mu_u}\right]^{\alpha} \left[\frac{v_i}{\mu_v}\right]^{1-\alpha}\right).$
- A suitable cardinalisation of $\psi(.)$ gives M.
  - $M_\alpha := \frac{1}{\alpha[\alpha-1]n} \sum_{i=1}^{n} \left[\left[\frac{u_i}{\mu_u}\right]^{\alpha} \left[\frac{v_i}{\mu_v}\right]^{1-\alpha} - 1\right].$

Limiting cases

- Two limiting cases
- $\alpha = 0$:
  - $M_0 = -\frac{1}{n}\sum_{i=1}^{n} \frac{v_i}{\mu_v} \log\left(\frac{u_i}{\mu_u} \Big/ \frac{v_i}{\mu_v} \right)$,
- $\alpha = 1$
  - $M_1 = \frac{1}{n}\sum_{i=1}^{n} \frac{u_i}{\mu_u} \log\left(\frac{u_i}{\mu_u} \Big/ \frac{v_i}{\mu_v} \right)$.
- We have a *class* of aggregate mobility measures
  - high $\alpha > 0$: M sensitive to downward movements
  - $\alpha < 0$: M sensitive to upward movements

Discussion 1

- Concerned with *ranks* not *income levels*? Then make status an ordinal concept (Chakravarty 1984)
- Variety of ways to define status ordinally: mobility tables or transition matrices.
- However, these approaches are sensitive to the adjustment of class boundaries:
  - Consider the case where in the original set of classes $p_k = 0$ and $p_{k+1} > 0$;
  - if the mobility index is sensitive to small values of p and the income boundary between classes k and $k+1$ is adjusted there could be a big jump in the mobility index.
  - This will not happen if the index is defined in terms of u_i and v_i

Discussion 2

- The derivation is value free. Can we introduce a social valuation of mobility?
- Could construct an explicit welfare approach
  - something analagous to Atkinson inequality? (Gottschalk-Spolaore 2002)
  - but you must go beyond simplistic welfare models (Markandya 1982, 1983)
- Can also introduce normative elements in the above framework
  - definition of status
  - value range of α

Income mobility

- Simplest case: status *before* and *after*
  - i.e. distribution-independent, static
  - Movements of incomes: $u_i = x_{0i}$ and $v_i = x_{1i}$.
- Define moment $\mu_{g(u,v)} = n^{-1} \sum_{i=1}^{n} g(u_i, v_i)$
  - $g(.)$ is a specific function
  - consider three cases: M_α, M_0 and M_1

General case

- Rewrite the index as $M_\alpha = \frac{1}{\alpha(1-\alpha)}\left[\frac{n^{-1}\sum u_i^\alpha v_i^{1-\alpha}}{\mu_u^\alpha\,\mu_v^{1-\alpha}} - 1\right]$.
- In terms of moments: $M_\alpha = \frac{1}{\alpha(\alpha-1)}\left[\frac{\mu_{u^\alpha v^{1-\alpha}}}{\mu_u^\alpha \mu_v^{1-\alpha}} - 1\right]$.
- Central Limit Theorem implies asymptotic normality under standard regularity conditions.
- $\widehat{\mathrm{Var}}(M_\alpha) = D\hat{\Sigma}D^\top$ with $D = \left[\frac{\partial M_\alpha}{\partial \mu_u}\,;\quad \frac{\partial M_\alpha}{\partial \mu_v}\,;\quad \frac{\partial M_\alpha}{\partial \mu_{u^\alpha v^{1-\alpha}}}\right]$
- D in terms of sample moments:
  $D = \left[\frac{-\mu_{u^\alpha v^{1-\alpha}}\,\mu_u^{-\alpha-1}\mu_v^{\alpha-1}}{(\alpha-1)}\,;\ \frac{\mu_{u^\alpha v^{1-\alpha}}\,\mu_u^{-\alpha}\mu_v^{\alpha-2}}{\alpha}\,;\ \frac{\mu_u^{-\alpha}\mu_v^{\alpha-1}}{\alpha(\alpha-1)}\right]$.

Covariance matrix

- $\hat{\Sigma}$ is the estimator of the covariance matrix of μ_u, μ_v and $\mu_{u^\alpha v^{1-\alpha}}$
- We have:

$$\hat{\Sigma} = \frac{1}{n}\begin{bmatrix} \mu_{u^2} - (\mu_u)^2 & \mu_{uv} - \mu_u\mu_v & \mu_{u^{1+\alpha}v^{1-\alpha}} - \mu_u\mu_{u^\alpha v^{1-\alpha}} \\ \mu_{uv} - \mu_u\mu_v & \mu_{v^2} - (\mu_v)^2 & \mu_{u^\alpha v^{2-\alpha}} - \mu_v\mu_{u^\alpha v^{1-\alpha}} \\ \mu_{u^{1+\alpha}v^{1-\alpha}} - \mu_u\mu_{u^\alpha v^{1-\alpha}} & \mu_{u^\alpha v^{2-\alpha}} - \mu_v\mu_{u^\alpha v^{1-\alpha}} & \mu_{u^{2\alpha}v^{2-2\alpha}} - (\mu_{u^\alpha v^{1-\alpha}})^2 \end{bmatrix}$$

Limiting case (1)

- M_0 as a function of four moments:
  $M_0 = \frac{\mu_{v \log v} - \mu_{v \log u}}{\mu_v} + \log\left(\frac{\mu_u}{\mu_v}\right).$
- $\widehat{\mathrm{Var}}(M_0) = D_0 \hat{\Sigma}_0 D_0^\top$
- $D_0 = \left[\frac{\partial M_0}{\partial \mu_u}\ ;\quad \frac{\partial M_0}{\partial \mu_v}\ ;\quad \frac{\partial M_0}{\partial \mu_{v \log v}}\ ;\quad \frac{\partial M_0}{\partial \mu_{v \log u}}\right]$
- $\hat{\Sigma}_0$:

$$\frac{1}{n}\begin{bmatrix} \mu_{u^2} - (\mu_u)^2 & \mu_{uv} - \mu_u\mu_v & \mu_{uv \log v} - \mu_u\mu_{v \log v} & \mu_{uv \log u} - \mu_u\mu_{v \log u} \\ \mu_{uv} - \mu_u\mu_v & \mu_{v^2} - (\mu_v)^2 & \mu_{v^2 \log v} - \mu_v\mu_{v \log v} & \mu_{v^2 \log u} - \mu_v\mu_{v \log u} \\ \mu_{uv \log v} - \mu_u\mu_{v \log v} & \mu_{v^2 \log v} - \mu_v\mu_{v \log v} & \mu_{(v \log v)^2} - (\mu_{v \log v})^2 & \mu_{v^2 \log u \log v} - \mu_{v \log u}\mu_{v \log v} \\ \mu_{uv \log u} - \mu_u\mu_{v \log u} & \mu_{v^2 \log u} - \mu_v\mu_{v \log u} & \mu_{v^2 \log u \log v} - \mu_{v \log u}\mu_{v \log v} & \mu_{(v \log u)^2} - (\mu_{v \log u})^2 \end{bmatrix}$$

Limiting case (2)

- M_1 as a function of four moments:
  $M_1 = \frac{\mu_{u \log u} - \mu_{u \log v}}{\mu_u} + \log\left(\frac{\mu_v}{\mu_u}\right)$
- $\widehat{\text{Var}}(M_1) = D_1 \hat{\Sigma}_1 D_1^\top$ with
  $D_1 = \left[\frac{\partial M_1}{\partial \mu_u}\,;\quad \frac{\partial M_1}{\partial \mu_v}\,;\quad \frac{\partial M_1}{\partial \mu_{u \log u}}\,;\quad \frac{\partial M_1}{\partial \mu_{u \log v}}\right]$
- $D_1 = \left[\frac{-\mu_{u \log u} + \mu_{u \log v} - \mu_u}{\mu_u^2}\,;\ \frac{1}{\mu_v}\,;\ \frac{1}{\mu_u}\,;\ -\frac{1}{\mu_u}\right]$
- $\hat{\Sigma}_1$:

$$\frac{1}{n}\begin{bmatrix} \mu_{u^2} - (\mu_u)^2 & \mu_{uv} - \mu_u\mu_v & \mu_{u^2 \log u} - \mu_u\mu_{u \log u} & \mu_{u^2 \log v} - \mu_u\mu_{u \log v} \\ \mu_{uv} - \mu_u\mu_v & \mu_{v^2} - (\mu_v)^2 & \mu_{uv \log u} - \mu_v\mu_{u \log u} & \mu_{uv \log v} - \mu_v\mu_{u \log v} \\ \mu_{u^2 \log u} - \mu_u\mu_{u \log u} & \mu_{uv \log u} - \mu_v\mu_{u \log u} & \mu_{(u \log u)^2} - (\mu_{u \log u})^2 & \mu_{u^2 \log u \log v} - \mu_{u \log u}\mu_{u \log v} \\ \mu_{u^2 \log v} - \mu_u\mu_{u \log v} & \mu_{uv \log v} - \mu_v\mu_{u \log v} & \mu_{u^2 \log u \log v} - \mu_{u \log u}\mu_{u \log v} & \mu_{(u \log v)^2} - (\mu_{u \log v})^2 \end{bmatrix}$$

Rank mobility

- we now consider the *distribution-dependent, dynamic* status.
- Scale independence means we can define status using proportions
  - $u_i = \hat{F}_0(x_{0i})$ and $v_i = \hat{F}_1(x_{1i})$
- $\hat{F}_0(.)$ and $\hat{F}_1(.)$ are the empirical distribution functions
  - $\hat{F}_k(x) = \frac{1}{n} \sum_{j=1}^{n} I(x_{kj} \leq x)$
- Method of moments does not apply as the values in u and v are non i.i.d.

Establishing the asymptotic distribution

- Ruymgaart and van Zuijlen (1978): asymptotic normality for the multivariate rank statistic $T_n = \frac{1}{n}\sum_{i=1}^{n} c_{in}\phi_1(u_i)\phi_2(v_i)$.
  - c_{in} are given real constants, ϕ_1 and ϕ_2 are (scores) functions defined on (0,1),
  - these are allowed to tend to infinity near 0 and 1 but not too quickly.
  - need to assume the existence of K_1, a_1 and a_2, s.t. $\phi_1(t) \leq \frac{K_1}{[t(1-t)]^{a_1}}$ and $\phi_2(t) \leq \frac{K_1}{[t(1-t)]^{a_2}}$ with $a_1 + a_2 < \frac{1}{2}$ for $t \in (0,1)$.
  - Then, $\phi_1(t)$ and $\phi_2(t)$ tend to infinity near 0 at a rate slower than the functions t^{-a_1} and t^{-a_2}.
  - Variance of T_n is finite, even if not analytically tractable.

Applying the results - general case

- M_α can be written as a function of T_n
  - Note that $\mu_u = \mu_v = \frac{1}{n}\sum_{i=1}^{n}\frac{i}{n} = \frac{n+1}{2n}$.
- M_α as a function of one moment:
  $M_\alpha = \frac{1}{\alpha(\alpha-1)}\left[\frac{2n}{n+1}\mu_{u^\alpha v^{1-\alpha}} - 1\right]$.
  - Hence, $M_\alpha = \frac{1}{\alpha(\alpha-1)}\left[T_n - 1\right]$.

Applying the results - limiting cases

- M_0 as a function of T_n
  - $M_0 = \frac{2n}{n+1}(k - \mu_{v \log u}) = l - T_n.$
- M_1 as a function of T_n
  - $M_1 = \frac{2n}{n+1}(k - \mu_{u \log v}) = l - T_n$
- It can be shown that the relevant conditions are met for $-0.5 < \alpha < 1.5$.
  - M_α is asymptotically normal
  - asymptotic justification for the bootstrap

Income mobility

- coverage error rate of a confidence interval
  - probability that CI does not include the true value of a parameter
  - should be close to the nominal rate
  - e.g. 5% for a 95% CI
- use Monte-Carlo simulations to approximate coverage error rates for different methods:
  - asymptotic
  - percentile bootstrap
  - and studentized bootstrap

Asymptotic confidence intervals

- $CI_{asym} =$
  $[M_\alpha - c_{0.975}\widehat{\text{Var}}(M_\alpha)^{1/2}\,;\; M_\alpha + c_{0.975}\widehat{\text{Var}}(M_\alpha)^{1/2}]$
- $c_{0.975}$ is a critical value from the Student distribution $T(n-1)$.
- finite sample performance often poor
- bootstrap confidence intervals can be expected to perform better

Percentile bootstrap method

- does not require the (asymptotic) standard error
- Method:
  - generate B bootstrap samples by resampling in the original data
  - for each resample, we compute the mobility index.
  - obtain B bootstrap statistics, M_α^b, $b = 1, \ldots, B$.
- The percentile bootstrap confidence interval is equal to $CI_{perc} = [c^b_{0.025}\,;\; c^b_{0.975}]$
  - $c^b_{0.025}$ and $c^b_{0.975}$ are the 2.5 and 97.5 percentiles of the EDF of the bootstrap statistics.

Studentized bootstrap method

- uses the asymptotic standard error
- Method:
  - generate B bootstrap samples by resampling in the original data
  - for each resample, compute a t-statistic.
  - obtain B bootstrap t-statistics $t_\alpha^b = (M_\alpha^b - M_\alpha)/\widehat{\mathrm{Var}}(M_\alpha^b)^{1/2}, b = 1, \ldots, B,$
  - where M_α is the original mobility index
- $CI_{stud} = [M_\alpha - c^*_{0.975}\widehat{\mathrm{Var}}(M_\alpha)^{1/2}\,;\ M_\alpha - c^*_{0.025}\widehat{\mathrm{Var}}(M_\alpha)^{1/2}]$
  - $c^*_{0.025}$ and $c^*_{0.975}$ are percentiles of the EDF of the bootstrap t-statistics.

Comparison

- studentized bootstrap based on asymptotically pivotal statistic
- t-statistics follow (asymptotically) a known distribution
  - superior performance of the bootstrap over asymptotic confidence intervals
- both bootstrap intervals asymmetric, asymptotic confidence interval is symmetric
  - bootstrap CIs more accurate if the underlying distribution is asymmetric.

Experiments

- Bivariate Lognormal distribution: $(x_0, x_1) \sim LN(\mu, \Sigma)$ with $\mu = (0, 0)$ and $\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$
- mobility increases as ρ decreases
- we try different mobility indices, different sample sizes and different mobility levels
- for each combination:
    - draw 10,000 samples (from the bivariate lognormal distribution)
    - compute M_α
    - compute confidence interval at 95%
    - How often does the interval does not include the true parameter?

Asymptotic confidence intervals

α	-1	0	0.5	1	2
$n = 100, \rho = 0$	0.3686	0.1329	0.1092	0.1357	0.3730
$n = 100, \rho = 0.2$	0.3160	0.1334	0.1136	0.1325	0.3194
$n = 100, \rho = 0.4$	0.2664	0.1353	0.1221	0.1351	0.2889
$n = 100, \rho = 0.6$	0.2175	0.1346	0.1275	0.1361	0.2263
$n = 100, \rho = 0.8$	0.1718	0.1349	0.1304	0.1345	0.1753
$n = 100, \rho = 0.9$	0.1528	0.1321	0.1308	0.1329	0.1531
$n = 100, \rho = 0.99$	0.1355	0.1340	0.1331	0.1324	0.1333
$n = 200, \rho = 0$	0.3351	0.1077	0.0923	0.1107	0.3153
$n = 500, \rho = 0$	0.2594	0.0830	0.0696	0.0818	0.2631
$n = 1000, \rho = 0$	0.2164	0.0703	0.0609	0.0726	0.2181
$n = 5000, \rho = 0$	0.1713	0.0554	0.0469	0.0522	0.2066
$n = 10000, \rho = 0$	0.1115	0.0532	0.0527	0.0534	0.1151

Table: Coverage error rate of asymptotic confidence intervals at 95% of income mobility measures. The nominal error rate is 0.05, 10.000 replications

Results

- Distribution-independent, static.
- Recall: coverage error rate should be close to 5%
  - asymptotic intervals perform poorly for $\alpha = -1, 2$
  - coverage error rate is stable as ρ varies (for $\alpha = 0$, 0.5, 1 and $n = 100$)
  - coverage error rate decreases as n increases
  - coverage error rate close to 0.05 for $n \geq 5.000$ and $\alpha = 0$, 0.5, 1.
- asymptotic confidence intervals perform well in very large sample, with $\alpha \in [0, 1]$.
- dismal performance of asymptotic confidence intervals for small and moderate samples
  - poorest results for $\rho = 0.8$
  - try other two methods for this value of ρ

Other methods

α	-1	0	0.5	1	2
$n = 100, \rho = 0.8$					
Asymptotic	0.1718	0.1349	0.1304	0.1345	0.1753
Boot-perc	0.1591	0.1294	0.1215	0.1266	0.1552
Boot-stud	0.0931	0.0751	0.0732	0.076	0.0952
$n = 200, \rho = 0.8$					
Asymptotic	0.1315	0.0973	0.0927	0.0973	0.1276
Boot-perc	0.1222	0.0943	0.0900	0.0950	0.1176
Boot-stud	0.0794	0.0666	0.0660	0.0688	0.0791
$n = 500, \rho = 0.8$					
Asymptotic	0.1127	0.0847	0.0828	0.0857	0.1124
Boot-perc	0.1054	0.0814	0.0813	0.0843	0.1036
Boot-stud	0.0765	0.0641	0.0629	0.0630	0.0779
$n = 1.000, \rho = 0.8$					
Asymptotic	0.0880	0.0678	0.0659	0.0672	0.0864
Boot-perc	0.0862	0.0672	0.0661	0.0689	0.0851
Boot-stud	0.0680	0.0585	0.0589	0.0596	0.0693

Table: Coverage error rate of asymptotic and bootstrap confidence intervals at 95% of income mobility measures. 10 000 replications, 199 bootstraps

Results

- percentile bootstrap and asymptotic confidence intervals perform similarly
- studentized bootstrap confidence intervals outperform other methods
  - siginificant improvement over asymptotic confidence intervals

Rank mobility

- *distribution-dependent*, *dynamic* status
- variance of M_α is not analytically tractable
  - cannot use asymptotic and studentized bootstrap confidence intervals
  - use the percentile bootstrap method

Percentile bootstrap method (n=100)

α	-0.5	0	0.5	1	1.5
$\rho = 0$	0.5592	0.1575	0.1088	0.1583	0.5282
$\rho = 0.2$	0.3176	0.1122	0.0884	0.1135	0.3231
$\rho = 0.4$	0.1883	0.0931	0.0755	0.0913	0.1876
$\rho = 0.6$	0.1122	0.0767	0.0651	0.0741	0.1118
$\rho = 0.8$	0.0671	0.0593	0.0555	0.0590	0.0652
$\rho = 0.9$	0.0432	0.0430	0.0431	0.0441	0.0446
$\rho = 0.99$	0.0983	0.0985	0.0981	0.0984	0.0992

Table: Coverage error rate of percentile bootstrap confidence intervals at 95% of rank-mobility measures. 10 000 replications, 199 bootstraps and 100 observations.

Results

- the coverage error rate can be very different for different values of ρ and α,
- it decreases as ρ increases, except for the case of "nearly" zero mobility ($\rho = 0.99$).
- the coverage error rate is close to 0.05 for $\rho = 0.8, 0.9$ and $\alpha = 0, 0.5, 1$.
- What happens as the sample size increases?

Percentile bootstrap method (variable n)

α	-0.5	0	0.5	1	1.5
$n = 100, \quad \rho = 0$	0.5592	0.1575	0.1088	0.1583	0.5282
$n = 200$	0.4613	0.1143	0.0833	0.1180	0.4723
$n = 500$	0.3548	0.0868	0.0645	0.0814	0.3644
$n = 1000$	0.3135	0.0672	0.0556	0.0735	0.3170
$n = 100, \quad \rho = 0.9$	0.0432	0.0430	0.0431	0.0441	0.0446
$n = 200$	0.0454	0.0441	0.0456	0.0454	0.0459
$n = 500$	0.0500	0.0499	0.0485	0.0480	0.0483
$n = 1000$	0.0511	0.0509	0.0539	0.0538	0.0538
$n = 100, \quad \rho = 0.99$	0.0983	0.0985	0.0981	0.0984	0.0992
$n = 200$	0.0981	0.0971	0.0970	0.0974	0.0977
$n = 500$	0.0855	0.0838	0.0833	0.0822	0.083
$n = 1000$	0.0788	0.0777	0.0762	0.0767	0.0771

Table: Coverage error rate of percentile bootstrap confidence intervals at 95% of rank-mobility measures. 10 000 replications, 199 bootstraps.

Results

- the coverage error rate gets closer to 0.05 as the sample size increases,
- the coverage error rate is smaller when $\alpha = 0, 0.5, 1$.
- better statistical properties as the sample size increases

Conclusion

- Key step involves a logical separation of fundamental concepts
  - measure of individual status
  - aggregation of changes in status
- Status concept derived directly from the information in the marginals
- Apply standard principles to movements in status
  - get a superclass of mobility measures
  - generally applicable to wide variety of status concepts
  - parameter α that determines type of mobility measure
- Principal status types yield statistically tractable mobility indices