WELFARE: EFFICIENCY

MICROECONOMICS
Principles and Analysis
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Welfare Principles

- Try to find general principles for running the economy
  - may be more fruitful than the constitution approach
- We have already slipped in one notion of “desirability”
  - “Technical Efficiency”
  - …applied to the firm
- What about a similar criterion for the whole economy?
  - a generalised version of efficiency
  - subsumes technical efficiency?
- And what about other desirable principles?
Overview

Welfare: efficiency

Pareto efficiency

CE and PE

Extending efficiency

Fundamental criterion for judging economic systems
A short agenda

- Describe states of the economy in welfare terms...
- Use this analysis to define efficiency
- Apply efficiency analysis to an economy
  - Use standard multi-agent model
  - Same as analysed in earlier presentations
- Apply efficiency concept to uncertainty
  - distinguish ex-ante and ex-post cases
The essential concepts

- **Social state**
  - describes economy completely
  - for example, an allocation

- **Pareto superiority**
  1. at least as much utility for all
  2. strictly greater utility for some

- **Pareto efficiency**
  - uses concept of Pareto superiority
  - also needs a definition of feasibility…
A definition of efficiency

- The basis for evaluating social states: \( v^h(\theta) \)

- A social state \( \theta \) is *Pareto superior* to state \( \theta' \) if:
  1. For all \( h \): \( v^h(\theta) \geq v^h(\theta') \)
  2. For some \( h \): \( v^h(\theta) > v^h(\theta') \)

- A social state \( \theta \) is *Pareto efficient* if:
  1. It is feasible
  2. No other feasible state is Pareto superior to \( \theta \)

The utility level enjoyed by person \( h \) under social state \( \theta \)

Note the similarity with the concept of blocking by a coalition

“Feasibility” could be determined in terms of the usual economic criteria
Derive the utility possibility set

- From the attainable set...
- ...take an allocation
- Evaluate utility for each agent
- Repeat to get utility possibility set

\[ U_a = U^a(x_1^a, x_2^a) \]
\[ U_b = U^b(x_1^b, x_2^b) \]
Utility possibility set – detail

- Utility levels of each person
- $U$-possibility derived from $A$
- Fix all but one at some utility level then max $U$ of that person
- Repeat for other persons and $U$-levels to get PE points

- Efficient points on boundary of $U$
- Not all boundary points are efficient

Find the corresponding efficient allocation
Finding an efficient allocation (1)

- Use essentially the same method
- Do this for the case where
  - all goods are purely private
  - households 2, ..., $n_h$ are on fixed utility levels $\psi^h$
- Then problem is to maximise $U^1(x^1)$ subject to:
  - $U^h(x^h) \geq \psi^h$, $h = 2, \ldots, n_h$
  - $\Phi^f(q^f) \leq 0$, $f = 1, \ldots, n_f$
  - $x_i \leq q_i + R_i$, $i = 1, \ldots, n$
- Use all this to form a Lagrangian in the usual way...
Finding an efficient allocation (2)

\[
\max \mathcal{L}( [x], [q], \lambda, \mu, \kappa) := \\
U^1(x^1) + \sum_h \lambda_h [U^h(x^h) - \upsilon^h] \\
- \sum_f \mu_f \Phi^f(q^f) \\
+ \sum_i \kappa_i [q_i + R_i - x_i]
\]

where

\[x_i = \sum_h x_i^h\]
\[q_i = \sum_f q_i^f\]
FOCs

- Differentiate Lagrangian w.r.t. $x_i^h$. If $x_i^h$ is positive at the optimum then:
  \[ \lambda_h U_i^h(x^h) = \kappa_i \]
- Likewise for good $j$:
  \[ \lambda_h U_j^h(x^h) = \kappa_j \]
- Differentiate Lagrangian w.r.t. $q_i^f$. If $q_i^f$ is nonzero at the optimum then:
  \[ \mu_f \Phi_i^f(q^f) = \kappa_i \]
- Likewise for good $j$:
  \[ \mu_f \Phi_j^f(q^f) = \kappa_j \]
Interpreting the FOC

- From the FOCs for any household \( h \) and goods \( i \) and \( j \):

\[
\frac{U_i^h(x^h)}{U_j^h(x^h)} = \kappa_i \kappa_j
\]

for every household: MRS = shadow price ratio

- From the FOCs for any firm \( f \) and goods \( i \) and \( j \):

\[
\frac{\Phi_i^f(q^f)}{\Phi_j^f(q^f)} = \kappa_i \kappa_j
\]

for every firm: MRT = shadow price ratio
Efficiency in an Exchange Economy

- Alf’s indifference curves
- Bill’s indifference curves
- The contract curve

Set of efficient allocations is the contract curve
Includes cases where Alf or Bill is very poor

Allocations where $MRS_{12}^a = MRS_{12}^b$
Efficiency with production

- Household $h$'s indifference curves
- $h$’s consumption in the efficient allocation
- $MRS$ is the tangent
- Firm $f$’s technology set
- $f$’s net output in the efficient allocation

$MRS = MRT$ at efficient point
Overview

Relationship between competitive equilibrium and efficient allocations

- Pareto efficiency
- CE and PE
- Extending efficiency
Efficiency and equilibrium

- The Edgeworth Box diagrams are suggestive
- Points on the contract curve:
  - are efficient
  - could be CE allocations
- Examine connection of market with efficiency:
  - will the equilibrium be efficient?
  - can we use the market to implement any efficient outcome?
  - or are there cases where the market “fails”? 
- Focus on two important theorems
Welfare theorem 1

- Assume a competitive equilibrium
- What is its efficiency property?

THEOREM: if all consumers are greedy and there are no externalities then a competitive equilibrium is efficient

Explanation:
- If they are not greedy, there may be no incentive to trade
- If there are externalities the market takes no account of spillovers
Welfare theorem 2

- Pick any Pareto-efficient allocation
- Can we find a property distribution $d$ so that this allocation is a CE for $d$?

**THEOREM**: if, in addition to conditions for theorem 1, there are no non-convexities then an arbitrary PE allocation be supported by a competitive equilibrium

**Explanation**:
- If “lump sum” transfers are possible then we can arbitrarily change the initial property distribution
- If there are non-convexities the equilibrium price signals could take us away from the efficient allocation
Supporting a PE allocation

- The contract curve
- An efficient allocation
- Supporting price ratio = MRS
- The property distribution
- A lump-sum transfer

Allocations where \( MRS_{12}^a = MRS_{12}^b \)

Support allocation by a CE
- This needs adjustment of the initial endowment
- Lump-sum transfers may be tricky to implement
Individual household behaviour

- Household $h$'s indifference curves
- $h$'s consumption in the efficient allocation
- Supporting price ratio = MRS

- $h$'s consumption in the allocation is utility-maximising for $h$
- $h$'s consumption in the allocation is cost-minimising for $h$
Supporting a PE allocation (production)

- Firm $f$’s technology set
- $f$’s net output in the efficient allocation
- Supporting price ratio = MRT

- $f$’s net output in the allocation is profit-maximising for $f$

what if preferences and production possibilities were different?
Household \( h \) makes "wrong" choice

- Nonconvexity leads to "market failure".

- Suppose we want to allocate this consumption bundle to \( h \).

- Introduce prices 

- \( h \)'s choice given this budget.

- Household \( h \)'s utility function violates second theorem.
Firm $f$ makes “wrong” choice

- Firm $f$’s production function violates second theorem
- Suppose we want to allocate this net output to $f$
- Introduce prices
- $f$’s choice at those prices

Example of “market failure”
Firm $f$ makes “wrong” choice (2)

- Firm $f$’s production function violates second theorem
- Suppose we want to allocate this net output to $f$
- Introduce prices
- $f$'s choice at those prices

- A twist on the previous example
- Big fixed-cost component to producing good 1
- “market failure” once again
PE allocations – two issues

- Same production function
- Is PE at first red dot…?
- or at second red dot?
- Implicit prices for MRS=MRT
- Competitive outcome

- Issue 1 – what characterises the PE?
- Issue 2 – how to implement the PE
Competitive Failure and Efficiency

- The market “failures” raise two classes of problem that lead to two distinct types of analysis:
  - The characterisation problem
    - description of modified efficiency conditions
    - when conditions for the welfare theorems are not met
  - The implementation problem
    - design of a mechanism to achieve the allocation
- These two types of problem pervade modern micro economics
- Important to keep them distinct
Overview

Attempt to generalise the concept of Pareto superiority

Welfare: efficiency

Pareto efficiency

CE and PE

Extending efficiency
Why extend the efficiency concept?

- Pareto efficiency is indecisive
  - What about the infinity of PE allocations along the contract curve?
- Pareto improvements may be elusive
  - Beware the politician who insists that everybody can be made better off
- Other concepts may command support
  - “potential” efficiency
  - fairness
  - equity
Indecisiveness of PE

- Construct utility-possibility set as before
- Two efficient points
- Points superior to $\theta^°$
- Points superior to $\theta'$

- Boundary points cannot be compared on efficiency grounds
- $\theta^°$ and $\theta'$ cannot be compared on efficiency grounds

A way forward?
“Potential” Pareto superiority

- Define $\theta$ to be potentially superior to $\theta'$ if:
  - there is a $\theta^*$ which is actually Pareto superior to $\theta'$
  - $\theta^*$ is “accessible” from $\theta$

- To make use of this concept we need to define accessibility

- This can be done using a tool that we already have from the theory of the consumer
The idea of accessibility

- Usually “θ* accessible from θ” means that income total in θ* is no greater than in θ
  - “if society can afford θ then it can certainly afford θ*”
- This can be interpreted as a “compensation rule”
  - the gainers in the move from θ' to θ…
  - …get enough to be able to compensate the losers of θ' → θ
- Can be expressed in terms of standard individual welfare criteria
  - the CV for each person
A result on potential superiority

- Use the terminology of individual welfare
- \( CV^h(\theta' \rightarrow \theta) \) is the monetary value the welfare change
  - for person \( h \)
  - given a change from state \( \theta' \) to state \( \theta \)
  - valued in terms of the prices at \( \theta \)
- \( CV^h > 0 \) means a welfare gain; \( CV^h < 0 \) a welfare loss

- **THEOREM**: a necessary and sufficient condition for \( \theta \) to be potentially superior to \( \theta' \) is
  \[
  \sum_h CV^h(\theta' \rightarrow \theta) > 0
  \]
- Can we really base welfare economics on the compensating variation?
Applying potential superiority

\[ \theta^\circ \text{ is not superior to } \theta' \text{ and } \theta' \text{ is not superior to } \theta^\circ \]

\[ \theta^* \text{ is superior to } \theta^\circ \]

\[ \text{Points accessible from } \theta' \]

\[ \text{There could be lots of points accessible by lump-sum transfers} \]

\[ \theta' \text{ is potentially superior to } \theta^\circ \]
Problems with accessibility

- What prices should be used in the evaluation
  - those in $\theta^\circ$?
  - those in $\theta'$?

- We speak only of potential income gains
  - compensation is not actually paid
  - does this matter?

- If no income transfer takes place:
  - so that there are no consumer responses to it
  - can we accurately evaluate gains and losses?

- But this isn’t the biggest problem…
Re-examine potential superiority

- The process from $\theta^\circ$ to $\theta'$, as before
- The process in reverse from $\theta'$ to $\theta^\circ$
- Combine the two

- $\theta'$ is potentially superior to $\theta^\circ$ and …
- $\theta^\circ$ is potentially superior to $\theta'$!
Extending efficiency: assessment

- The above is the basis of Cost-Benefit Analysis
- It relies on notion of “accessibility”
  - a curious concept involving notional transfers?
  - more than one possible definition
- “Compensation” is not actually paid
- If equilibrium prices differ substantially, the rule may produce contradictions
What next?

- Second approach to the question “how should the economy be run?”
  - The first was “the constitution”

- The approach covers widely used general principles

- Efficiency
  - neat and simple
  - but perhaps limited

- Potential efficiency
  - Persuasive but perhaps dangerous economics/politics

- A natural way forward:
  - Examine other general principles
  - Consider problems with applying the efficiency concept
  - Go to the third approach: a full welfare function