THE FIRM: DEMAND AND SUPPLY

MICROECONOMICS
Principles and Analysis
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Moving on from the optimum…

- We derive the firm's reactions to changes in its environment
- These are the *response functions*
  - We will examine three types of them
  - Responses to different types of market events
- In effect we treat the firm as a black box
The firm as a “black box”

- Behaviour can be predicted by necessary and sufficient conditions for optimum
- The FOC can be solved to yield behavioural response functions
- Their properties derive from the solution function
  - we need the solution function’s properties
  - apply them again and again
Overview

Firm: Comparative Statics

- Conditional Input Demand
  - Output Supply
  - Ordinary Input Demand
  - Short-run problem

Response function for stage 1 optimisation
The first response function

- Review the cost-minimisation problem and its solution

- Choose \( z \) to minimise
  \[
  \sum_{i=1}^{m} w_i z_i \text{ subject to } q \leq \phi(z), \ z \geq 0
  \]

- The firm’s cost function:
  \[
  C(w, q) := \min \sum w_i z_i \quad \{ \phi(z) \geq q \}
  \]

- Cost-minimising value for each input:
  \[
  z^*_i = H^i(w, q), \ i=1,2,\ldots,m
  \]

- The “stage 1” problem

- The solution function

- \( H^i : \text{conditional input demand function} \)

- Demand for input \( i \), conditional on given output level \( q \)

- A graphical approach
Mapping into \((z_1, w_1)\)-space

- **Left-hand panel:** conventional case of \(Z\)
- **the slope of the tangent:** value of \(w_1\)
- **Repeat for a lower value of** \(w_1\)
- **…and again to get…**
- **Green curve:** conditional demand curve

- **Constraint set is convex, with smooth boundary**
- **Response function is a continuous map:**

\[
H^1(w, q)
\]

Now try a different case
Another map into \((z_1, w_1)\)-space

- **Left-hand panel:** nonconvex \(Z\)
- **Start with a high value of** \(w_1\)
- **Repeat for a very low value of** \(w_1\)
- **Points “nearby” work the same way**
- **But what happens in between?**
- **A demand correspondence**

- Constraint set is nonconvex
- Response is **discontinuous:** jumps in \(z^*\)
- Map multivalued at discontinuity

Multiple inputs at this price

no price yields a solution here
Conditional input demand function

- Assume that single-valued input-demand functions exist
- How are they related to the cost function $C$?
- What are their properties?
- How are their properties related to those of $C$?
  - tip if you’re not sure about the cost function:
  - check the presentation “Firm Optimisation”
  - revise the five main properties of the function $C$
Use the cost function

- Recall this relationship?
  \[ C_i(w, q) = z_i^* \]
  The slope: 
  \[ \frac{\partial C(w, q)}{\partial w_i} \]
  Optimal demand for input \( i \)

- So we have:
  \[ C_i(w, q) = H^i(w, q) \]
  conditional input demand function

- Differentiate this with respect to \( w_j \)
  \[ C_{ij}(w, q) = H^i_j(w, q) \]
  Second derivative

- Yes, it's Shephard's lemma

- Link between conditional input demand and cost functions

- Slope of input conditional demand function: effect of \( \Delta w_j \) on \( z_i^* \) for given \( q \)

April 2018

Frank Cowell: Firm- Demand & Supply
Simple result 1

- Use a standard property
  \[ \frac{\partial^2(\cdot)}{\partial w_i \partial w_j} = \frac{\partial^2(\cdot)}{\partial w_j \partial w_i} \]

- So in this case
  \[ C_{ij}(w, q) = C_{ji}(w, q) \]

- Therefore we have:
  \[ H_{ji}(w, q) = H_{ij}(w, q) \]

- second derivatives of a function “commute”

- The order of differentiation is irrelevant

- The effect of the price of input $i$ on conditional demand for input $j$ equals the effect of the price of input $j$ on conditional demand for input $i$
Simple result 2

- Use the standard relationship:
  \[ C_{ij}(w, q) = H_j^i(w, q) \]

- We can get the special case:
  \[ C_{ii}(w, q) = H_i^i(w, q) \]

- Because cost function is concave:
  \[ C_{ii}(w, q) \leq 0 \]

- Therefore:
  \[ H_i^i(w, q) \leq 0 \]

- \textbf{Slope of conditional input demand function derived from second derivative of cost function}
  \[ H_i^i(w, q) \leq 0 \]

- We've just put \( j = i \)

- \textbf{A general property}

- \textbf{The relationship of conditional demand for an input with its own price cannot be positive}

and so…
Conditional input demand curve

Consider the demand for input 1

"Downward-sloping" conditional demand

In some cases it is possible that \( H_i^j = 0 \)

Corresponds to case where isoquant is kinked: multiple \( w \) values consistent with same \( z^* \)
Conditional demand function: summary

- Nonconvex $Z$ yields discontinuous $H$
- Cross-price effects are symmetric
- Own-price demand slopes downward
- (exceptional case: own-price demand could be constant)
Overview

Firm: Comparative Statics

- Conditional Input Demand
- Output Supply
- Ordinary Input Demand
- Short-run problem

Response function for stage 2 optimisation
The second response function

- Review the profit-maximisation problem and its solution
- Choose \( q \) to maximise:
  \[ pq - C(w, q) \]

- From the FOC:
  \[ p = C_q(w, q^*), \text{ if } q^* > 0 \]
  \[ pq^* \geq C(w, q^*) \]

- The “stage 2” problem
- “Price equals marginal cost”
- “Price covers average cost”

- Profit-maximising value for output:
  \[ q^* = S(w, p) \]

- \( S \) is the supply function
- (again it may be a correspondence)
Supply of output and output price

- Use the FOC:
  \[ C_q(w, q^*) = p \]
  \[ "marginal cost equals price" \]

- Use the supply function for \( q \):
  \[ C_q(w, S(w, p)) = p \]
  \[ Gives an equation in w and p \]

- Differentiate with respect to \( p \)
  \[ C_{qq}(w, S(w, p)) S_p(w, p) = 1 \]
  \[ Use the "function of a function" rule \]

- Rearrange:
  \[ S_p(w, p) = \frac{1}{C_{qq}(w, q^*)} \]
  \[ Gives slope of supply function \]
  \[ Positive if MC is increasing \]
The firm’s supply curve

- AC (green) and MC (red) curves
- For given $p$ read off optimal $q^*$
- Continues down to $p$
- Check what happens below $p$

Supply response given by $q = S(w, p)$

Case illustrated is for $\phi$ with first decreasing AC, then increasing AC, Response is a discontinuous map: jumps in $q^*$

Multivalued at the discontinuity

Multiple $q^*$ at this price

no price yields a solution here
Supply of output and price of input $j$

- Use the FOC:
  \[ C_q(w, S(w, p)) = p \]

- Differentiate with respect to $w_j$
  \[ C_{qj}(w, q^*) + C_{qq}(w, q^*) S_j(w, p) = 0 \]

- Rearrange:
  \[ S_j(w, p) = -\frac{C_{qj}(w, q^*)}{C_{qq}(w, q^*)} \]

  - Same as before: “price equals marginal cost”
  - Use the “function of a function” rule again
  - Supply of output must fall with $w_j$ if MC increases with $w_j$
Supply function: summary

- Supply curve slopes upward
- Supply decreases with the price of an input, if MC increases with the price of that input
- Nonconcave $\phi$ yields discontinuous $S$
- IRTS means $\phi$ is nonconcave and so $S$ is discontinuous
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Response function for combined optimisation problem
The third response function

- Recall the first two response functions:
  \[ z_i^* = H_i(w, q) \]
  \[ q^* = S(w, p) \]

- Now substitute for \( q^* \):
  \[ z_i^* = H_i(w, S(w, p)) \]

- Use this to define a new function:
  \[ D_i(w, p) := H_i(w, S(w, p)) \]

- **Demand for input \( i \), conditional on output \( q \)**

- **Supply of output**

- **Stages 1 & 2 combined…**

- **Demand for input \( i \) (unconditional)**

- **Use this relationship to analyse firm’s response to price changes**
Demand for \( i \) and the price of output

- Take the relationship
  \[ D_i(w, p) = H_i(w, S(w, p)) \]

- Differentiate with respect to \( p \):
  \[ D_p i(w, p) = H_q i(w, q^*) S_p(w, p) \]

  \( D_i \) increases with \( p \) iff \( H_i \) increases with \( q \). Reason? Supply increases with price ( \( S_p > 0 \) )

- But we also have, for any \( q \):
  \[ H_i(w, q) = C_i(w, q) \]
  \[ H_q i(w, q) = C_{iq}(w, q) \]

- Shephard’s Lemma again

- Substitute in the above:
  \[ D_p i(w, p) = C_{qi}(w, q^*) S_p(w, p) \]

  Demand for input \( i \) (\( D_i \)) increases with \( p \) iff marginal cost (\( C_q \)) increases with \( w_i \)
Demand for \(i\) and the price of \(j\)

- Again take the relationship \(D_i(w, p) = H_i(w, S(w, p))\)

- Differentiate with respect to \(w_j\):
  \[
  D_j^i(w, p) = H_j^i(w, q^*) + H_q^i(w, q^*)S_j(w, p)
  \]

- Use Shephard’s Lemma again: \(H_q^i(w, q) = C_{iq}(w, q)\)

- Use this and the previous result on \(S_j(w, p)\) to give a decomposition into a “substitution effect” and an “output effect”:

\[
D_j^i(w, p) = H_j^i(w, q^*) - \frac{C_{jq}(w, q^*)}{C_{qq}(w, q^*)} C_{iq}(w, q^*)
\]

- **Substitution effect** is just slope of conditional input demand curve

- **Output effect** is \([\text{effect of } \Delta w_j \text{ on } q] \times [\text{effect of } \Delta q \text{ on demand for } i]\)
Results from decomposition formula

- Take the general relationship:

\[ D_j^i(w, p) = H_j^i(w, q^*) - \frac{C_{iq}(w, q^*)C_{jq}(w, q^*)}{C_{qq}(w, q^*)}. \]

We know this is symmetric in \( i \) and \( j \).

- The effect \( w_i \) on demand for input \( j \) equals the effect of \( w_j \) on demand for input \( i \).

- Now take the special case where \( j = i \):

\[ D_i^i(w, p) = H_i^i(w, q^*) - \frac{C_{iq}(w, q^*)^2}{C_{qq}(w, q^*)}. \]

We know this is negative or zero.

\( D_i^i(w, p) \) cannot be positive.

- If \( w_i \) increases, the demand for input \( i \) cannot rise.

Symmetric in \( i \) and \( j \).
Input-price fall: substitution effect

\[ H^1(w,q) \]

- \( z_1^* \): initial equilibrium
- grey arrow: fall in \( w_1 \)
- shaded area: value of price fall

Change in cost

Notional increase in factor input if output target is held constant
Input-price fall: total effect

- $z_1^*$ : initial equilibrium
- green line: substitution effect
- $z_1^{**}$ : new equilibrium

Conditional demand at original output
Conditional demand at new output

Initial price level

Ordinary demand curve

$W_1$ vs. $z_1$ graph
### Ordinary demand function: summary

- Nonconvex $Z$ may yield a discontinuous $D$
- Cross-price effects are symmetric
- Own-price demand slopes downward
- Same basic properties as for $H$ function
Overview

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- Short-run problem

Optimisation subject to side-constraint
The short run: concept

- This is not a moment in time
- It is defined by additional constraints within the model
- Counterparts in other economic applications where one may need to introduce side constraints
The short-run problem

- We build on the firm’s standard optimisation problem
- Choose $q$ and $z$ to maximise
- \[ \Pi := pq - \sum_{i=1}^{m} w_i z_i \]
- subject to the standard constraints:
  - $q \leq \phi (z)$
  - $q \geq 0, \ z \geq 0$
- But we add a side condition to this problem:
  - $z_m = \bar{z}_m$
- Let $\bar{q}$ be the value of $q$ for which $z_m = \bar{z}_m$ would have been freely chosen in the unrestricted cost-min problem…
The short-run cost function

- \( \tilde{C}(w, q, \bar{z}_m) := \min \sum w_i z_i \) for \( \{z_m = \bar{z}_m\} \)

- Short-run demand for input \( i \):
  \( \tilde{H}^i(w, q, \bar{z}_m) = \tilde{C}_i(w, q, \bar{z}_m) \)

- Compare with the ordinary cost function
  \( C(w, q) \leq \tilde{C}(w, q, \bar{z}_m) \)

- So, dividing by \( q \):
  \( \frac{C(w, q)}{q} \leq \frac{\tilde{C}(w, q, \bar{z}_m)}{q} \)

The solution function with the side constraint

- Follows from Shephard’s Lemma

- By definition of the cost function. We have “=” if \( q = \bar{q} \)

- Short-run AC \( \geq \) long-run AC.
  \( SRAC = LRAC \) at \( q = \bar{q} \)

Supply curves
MC, AC and supply in the short and long run

- green curve: AC if all inputs variable
- red curve: MC if all inputs variable
- $\bar{q}$: given output level
- black curve: AC if input m kept fixed
- brown curve: MC if input m kept fixed
- LR supply curve follows LRMC
- SR supply curve follows SRMC

- SRAC touches LRAC at given output
- SRMC cuts LRMC at given output
- Supply curve steeper in the short run
Conditional input demand

- **Brown curve:** Demand for input 1
- **Purple curve:** Demand for input 1 in problem with the side constraint

- “Downward-sloping” conditional demand
- Conditional demand curve is steeper in the short run
Key concepts

- Basic functional relations
- Price signals → firm → input/output responses

- $H^i(w,q)$ demand for input $i$, conditional on output
- $S(w,p)$ supply of output
- $D^i(w,p)$ demand for input $i$ (unconditional)

And they all hook together like this:

- $H^i(w, S(w,p)) = D^i(w,p)$
What next?

- Analyse the firm under a variety of market conditions
- Apply the analysis to the consumer’s optimisation problem