DUOPOLY

MICROECONOMICS

Principles and Analysis

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Overview

How the basic elements of the firm and of game theory are used
Basic ingredients

- **Two firms:**
  - issue of entry is not considered
  - but monopoly could be a special limiting case

- **Profit maximisation**

- **Quantities or prices?**
  - there’s nothing within the model to determine which “weapon” is used
  - it’s determined *a priori*
  - highlights artificiality of the approach

- **Simple market situation:**
  - there is a known demand curve
  - single, homogeneous product
Reaction

- We deal with “competition amongst the few”
- Each actor has to take into account what others do
- A simple way to do this: *the reaction function*
- Based on the idea of “best response”
  - we can extend this idea
  - in the case where more than one possible reaction to a particular action
  - it is then known as a reaction *correspondence*
- We will see how this works:
  - where reaction is in terms of prices
  - where reaction is in terms of quantities
Overview

Introduction to a simple simultaneous move price-setting problem
Competing by price

▪ Simplest version of model:
  • there is a market for a single, homogeneous good
  • firms announce prices
  • each firm does not know the other’s announcement when making its own

▪ Total output is determined by demand
  • determinate market demand curve
  • known to the firms

▪ Division of output amongst the firms determined by market “rules”

▪ Take a specific case with a clear-cut solution
Bertrand – basic set-up

- Two firms can potentially supply the market
  - each firm: zero fixed cost, constant marginal cost $c$
  - if one firm alone supplies the market it charges monopoly price $p_M > c$
  - if both firms are present they announce prices

- The outcome of these announcements:
  - if $p_1 < p_2$ firm 1 captures the whole market
  - if $p_1 > p_2$ firm 2 captures the whole market
  - if $p_1 = p_2$ the firms supply equal amounts to the market

- What will be the equilibrium price?
Bertrand – best response?

- Consider firm 1’s response to firm 2
  - If firm 2 foolishly sets a price $p^2$ above $p_M$ then it sells zero output
    - firm 1 can safely set monopoly price $p_M$
  - If firm 2 sets $p^2$ above $c$ but less than or equal to $p_M$ then:
    - firm 1 can “undercut” and capture the market
    - firm 1 sets $p^1 = p^2 - \delta$, where $\delta > 0$
    - firm 1’s profit always increases if $\delta$ is made smaller
    - but to capture the market the discount $\delta$ must be positive!
    - so strictly speaking there’s no best response for firm 1
  - If firm 2 sets price equal to $c$ then firm 1 cannot undercut
    - firm 1 also sets price equal to $c$
  - If firm 2 sets a price below $c$ it would make a loss
    - firm 1 would be crazy to match this price
    - if firm 1 sets $p^1 = c$ at least it won’t make a loss
- Let’s look at the diagram
Bertrand model – equilibrium

- Marginal cost for each firm
- Monopoly price level
- Firm 1’s reaction function
- Firm 2’s reaction function
- Bertrand equilibrium
Bertrand – assessment

- Using “natural tools” – prices
- Yields a remarkable conclusion
  - mimics the outcome of perfect competition
  - price = MC
- But it is based on a special case
  - neglects some important practical features
  - fixed costs
  - product diversity
  - capacity constraints
- Outcome of price-competition models usually sensitive to these
Overview

The link with monopoly and an introduction to two simple “competitive” paradigms

Duopoly

Background

Price competition

Quantity competition

Assessment

• Collusion
• The Cournot model
• Leader-Follower
Quantity models

- Now take *output quantity* as the firms’ choice variable
- Price is determined by the market once total quantity is known:
  - an auctioneer?
- Three important possibilities:

1. Collusion:
   - competition is an illusion
   - monopoly by another name
   - but a useful reference point for other cases

2. Simultaneous-move competing in quantities:
   - complementary approach to the Bertrand-price model

3. Leader-follower (sequential) competing in quantities
Collusion – basic set-up

- Two firms agree to maximise joint profits
  - what they can make by acting as though they were a single firm
  - essentially a monopoly with two plants
- They also agree on a rule for dividing the profits
  - could be (but need not be) equal shares
- In principle these two issues are separate
The profit frontier

- To show what is possible for the firms
  - draw the profit frontier

- Show the possible combination of profits for the two firms
  - given demand conditions
  - given cost function

- Distinguish two cases
  1. where cash transfers between the firms are not possible
  2. where cash transfers are possible
Frontier – non-transferable profits

- Take case of identical firms
- Constant returns to scale
- DRTS (1): MC always rising
- DRTS (2): capacity constraints
- IRTS (fixed cost and constant MC)
Frontier – transferable profits

- Increasing returns to scale (without transfers)
- Now suppose firms can make “side-payments”
- Profits if everything were produced by firm 1
- Profits if everything were produced by firm 2
- The profit frontier if transfers are possible
- Joint-profit maximisation with equal shares

- Side payments mean profits can be transferred between firms
- Cash transfers “convexify” the set of attainable profits
Collusion – simple model

- Take the special case of the “linear” model where marginal costs are identical:
  \[ c^1 = c^2 = c \]

- Will both firms produce a positive output?
  1. if unlimited output is possible then only one firm needs to incur the fixed cost
     - in other words a true monopoly
  2. but if there are capacity constraints then both firms may need to produce
     - both firms incur fixed costs

- We examine both cases – capacity constraints first
Collusion: capacity constraints

- If both firms are active total profit is
  \[ [a - bq] q - [C_0^1 + C_0^2 + cq] \]
- Maximising this, we get the FOC:
  \[ a - 2bq - c = 0 \]
- Which gives equilibrium quantity and price:
  \[ q = \frac{a - c}{2b}; \quad p = \frac{a + c}{2} \]
- So maximised profits are:
  \[ \Pi_M = \frac{[a - c]^2}{4b} - [C_0^1 + C_0^2] \]
- Now assume the firms are identical: \( C_0^1 = C_0^2 = C_0 \)
- Given equal division of profits each firm’s payoff is
  \[ \Pi_J = \frac{[a - c]^2}{8b} - C_0 \]
Collusion: no capacity constraints

- With no capacity limits and constant marginal costs
  - seems to be no reason for both firms to be active
- Only need to incur *one* lot of fixed costs $C_0$
  - $C_0$ is the smaller of the two firms’ fixed costs
  - previous analysis only needs slight tweaking
  - modify formula for $P_J$ by replacing $C_0$ with $\frac{1}{2}C_0$
- But is the division of the profits still implementable?
Overview

Simultaneous move “competition” in quantities

Duopoly

Background

Price competition

Quantity competition
- Collusion
- The Cournot model
- Leader-Follower

Assessment
Cournot – basic set-up

- Two firms
  - assumed to be profit-maximisers
  - each is fully described by its cost function

- Price of output determined by demand
  - determinate market demand curve
  - known to both firms

- Each chooses the quantity of output
  - single homogeneous output
  - neither firm knows the other’s decision when making its own

- Each firm makes an assumption about the other’s decision
  - firm 1 assumes firm 2’s output to be given number
  - likewise for firm 2

- How do we find an equilibrium?
Cournot – model setup

- Two firms labelled $f = 1, 2$
- Firm $f$ produces output $q^f$
- So total output is:
  - $q = q^1 + q^2$
- Market price is given by:
  - $p = p(q)$
- Firm $f$ has cost function $C^f(\cdot)$
- So profit for firm $f$ is:
  - $p(q) q^f - C^f(q^f)$
- Each firm’s profit depends on the other firm’s output
  - (because $p$ depends on total $q$)
Cournot – firm’s maximisation

- Firm 1’s problem is to choose $q^1$ so as to maximise
  \[ \Pi^1(q^1; q^2) := p (q^1 + q^2) q^1 - C^1(q^1) \]
- Differentiate $\Pi^1$ to find FOC:
  \[
  \frac{\partial \Pi^1(q^1; q^2)}{\partial q^1} = p_q(q^1 + q^2) q^1 + p(q^1 + q^2) - C_q^1(q^1)
  \]
  - for an interior solution this is zero
- Solving, we find $q^1$ as a function of $q^2$
- This gives us 1’s reaction function, $\chi^1$:
  \[ q^1 = \chi^1(q^2) \]
- Let’s look at it graphically
Cournot – the reaction function

- Firm 1’s Iso-profit curves
- Assuming 2’s output constant at $q_0$
- Firm 1 maximises profit
- If 2’s output were constant at a higher level
- 2’s output at a yet higher level
- The reaction function

Firm 1’s choice given that 2 chooses output $q_0$
Cournot – solving the model

- $\chi^1(\cdot)$ encapsulates profit-maximisation by firm 1
- Gives firm’s reaction 1 to fixed output level of competitor:
  - $q^1 = \chi^1(q^2)$
- Of course firm 2’s problem is solved in the same way
- We get $q^2$ as a function of $q^1$:
  - $q^2 = \chi^2(q^1)$
- Treat the above as a pair of simultaneous equations
- Solution is a pair of numbers ($q^{C1}$, $q^{C2}$)
  - So we have $q^{C1} = \chi^1(\chi^2(q^{C1}))$ for firm 1
  - and $q^{C2} = \chi^2(\chi^1(q^{C2}))$ for firm 2
- This gives the Cournot-Nash equilibrium outputs
Cournot-Nash equilibrium (1)

- Firm 2’s Iso-profit curves
- If 1’s output is $q_0$ ...
- …firm 2 maximises profit
- Repeat at higher levels of 1’s output
- Firm 2’s reaction function
- Combine with firm’s reaction function
- “Consistent conjectures”

Firm 2’s choice given that 1 chooses output $q_0$

$\Pi^2(q^2; q^1) = \text{const}$

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Cournot-Nash equilibrium (2)

- Firm 1’s Iso-profit curves
- Firm 2’s Iso-profit curves
- Firm 1’s reaction function
- Firm 2’s reaction function
- Cournot-Nash equilibrium
- Outputs with higher profits for both firms
- Joint profit-maximising solution
The Cournot-Nash equilibrium

- Why “Cournot-Nash”?
- It is the general form of Cournot’s (1838) solution
- It also is the Nash equilibrium of a simple quantity game:
  - players are the two firms
  - moves are simultaneous
  - strategies are actions – the choice of output levels
  - functions give the best-response of each firm to the other’s strategy (action)
- To see more, take a simplified example
Cournot – a “linear” example

- Take the case where the inverse demand function is:
  \[ p = \beta_0 - \beta q \]

- And the cost function for \( f \) is given by:
  \[ C_f(q_f) = C_0^f + c_f q_f \]

- So profits for firm \( f \) are:
  \[ \left[ \beta_0 - \beta q \right] q_f - \left[ C_0^f + c_f q_f \right] \]

- Suppose firm 1’s profits are \( \Pi \)

- Then, rearranging, the iso-profit curve for firm 1 is:
  \[ q^2 = \frac{\beta_0 - c^1}{\beta} - q^1 - \frac{C_0^1 + \Pi}{\beta q^1} \]
Cournot – solving the linear example

- Firm 1’s profits are given by:
  \[ \Pi^1(q^1; q^2) = [\beta_0 - \beta q] q^1 - [C_0^1 + c^1 q^1] \]
- So, choose \( q^1 \) so as to maximise this.
- Differentiating we get:
  \[
  \frac{\partial \Pi^1(q^1; q^2)}{\partial q^1} = -2\beta q^1 + \beta_0 - \beta q^2 - c^1
  \]
- FOC for an interior solution (\( q^1 > 0 \)) sets this equal to zero.
- Doing this and rearranging, we get the reaction function:
  \[
  q^1 = \max \left\{ \frac{\beta_0 - c^1}{2\beta} - \frac{1}{2} q^2, 0 \right\}
  \]
The reaction function again

- Firm 1’s Iso-profit curves
- Firm 1 maximises profit, given $q^2$
- The reaction function

\[ \Pi^1(q^1; q^2) = \text{const} \]
Finding Cournot-Nash equilibrium

- Assume output of both firm 1 and firm 2 is positive
- Reaction functions of the firms, $\chi^1(\cdot)$, $\chi^2(\cdot)$ are given by:
  
  \[ q^1 = \frac{a - c^1}{2b} - \frac{1}{2}q^2 ; \quad q^2 = \frac{a - c^2}{2b} - \frac{1}{2}q^1 \]

- Substitute from $\chi^2$ into $\chi^1$:
  
  \[ q^1_c = \frac{a - c^1}{2b} - \frac{1}{2} \left[ \frac{a - c^2}{2b} - \frac{1}{2}q^1_c \right] \]

- Solving this we get the Cournot-Nash output for firm 1:
  
  \[ q^1_c = \frac{a + c^2 - 2c^1}{3b} \]

- By symmetry get the Cournot-Nash output for firm 2:
  
  \[ q^2_c = \frac{a + c^1 - 2c^2}{3b} \]
Cournot – identical firms

- Take the case where the firms are *identical*
  - useful but very special
- Use the previous formula for the Cournot-Nash outputs
  \[ q_C^1 = \frac{a + c^2 - 2c^1}{3b} \; ; \; q_C^2 = \frac{a + c^1 - 2c^2}{3b} \]
- Put \( c^1 = c^2 = c \). Then we find \( q_C^1 = q_C^2 = q_C \) where
  \[ q_C = \frac{a - c}{3b} \]
- From the demand curve the price in this case is \( \frac{1}{3}[a+2c] \)
- Profits are
  \[ \Pi_C = \frac{[a - c]^2}{9b} - C_0 \]
Symmetric Cournot

- A case with identical firms
- Firm 1's reaction to firm 2
- Firm 2's reaction to firm 1
- The Cournot-Nash equilibrium
Cournot — assessment

- Cournot-Nash outcome straightforward
  - usually have continuous reaction functions

- Apparently “suboptimal” from the selfish point of view of the firms
  - could get higher profits for all firms by collusion

- Unsatisfactory aspect is that price emerges as a “by-product”
  - contrast with Bertrand model

- Absence of time in the model may be unsatisfactory
Overview

Duopoly

- Background
- Price competition
  - Collusion
  - The Cournot model
  - Leader-Follower
- Quantity competition
- Assessment

Sequential “competition” in quantities
Leader-Follower – basic set-up

- Two firms choose the quantity of output
  - single homogeneous output
- Both firms know the market demand curve
- But firm 1 is able to choose first
  - It announces an output level
- Firm 2 then moves, knowing the announced output of firm 1
- Firm 1 knows the reaction function of firm 2
- So it can use firm 2’s reaction as a “menu” for choosing its own output
Firm 1 (the leader) knows firm 2’s reaction
  - if firm 1 produces $q^1$ then firm 2 produces $c^2(q^1)$
Firm 1 uses $\chi^2$ as a feasibility constraint for its own action
Building in this constraint, firm 1’s profits are given by

$$p(q^1 + \chi^2(q^1)) q^1 - C^1(q^1)$$

In the “linear” case firm 2’s reaction function is

$$q^2 = \frac{a - c^2}{2b} - \frac{1}{2}q^1$$

So firm 1’s profits are

$$\left[ a - b \left[ q^1 + \frac{a - c^2}{2b} - \frac{1}{2}q^1 \right] \right] q^1 - \left[ C^1_0 + c^1 q^1 \right]$$
Solving the leader-follower model

- Simplifying the expression for firm 1’s profits we have:
  \[ \frac{1}{2} [a + c^2 - bq^1] q^1 - [C_0^1 + c^1 q^1] \]
- The FOC for maximising this is:
  \[ \frac{1}{2} [a + c^2] - bq^1 - c^1 = 0 \]
- Solving for \( q^1 \) we get:
  \[ q_{S1} = \frac{a + c^2 - 2c^1}{2b} \]
- Using 2’s reaction function to find \( q^2 \) we get:
  \[ q_{S2} = \frac{a + 2c^1 - 3c^2}{4b} \]
Leader-follower – identical firms

- Again assume that the firms have the same cost function
- Take the previous expressions for the Leader-Follower outputs:
  \[ q^1_s = \frac{a + c^2 - 2c^1}{2b} ; \quad q^2_s = \frac{a + 2c^1 - 3c^2}{4b} \]
- Put \( c^1 = c^2 = c \); then we get the following outputs:
  \[ q^1_s = \frac{a - c}{2b} ; \quad q^2_s = \frac{a - c}{4b} \]
- Using the demand curve, market price is \( \frac{1}{4} [a + 3c] \)
- So profits are:
  \[ \Pi^1_s = \frac{[a - c]^2}{8b} - C_0 ; \quad \Pi_s = \frac{2[a - c]^2}{16b} - C_0 \]
Leader-Follower

- Firm 1’s Iso-profit curves
- Firm 2’s reaction to firm 1
- Firm 1 takes this as an opportunity set
- and maximises profit here
- Firm 2 follows suit

- Leader has higher output (and follower less) than in Cournot-Nash
- “S” stands for von Stackelberg
Overview

How the simple price- and quantity-models compare
Comparing the models

- The price-competition model may seem more “natural”
- But the outcome ($p = MC$) is surely at variance with everyday experience
- To evaluate the quantity-based models we need to:
  - compare the quantity outcomes of the three versions
  - compare the profits attained in each case
Output under different regimes

- Reaction curves for the two firms
- Joint-profit maximisation with equal outputs
- Cournot-Nash equilibrium
- Leader-follower (Stackelberg) equilibrium
Profits under different regimes

- Attainable set with transferable profits
- Joint-profit maximisation with equal shares
- Profits at Cournot-Nash equilibrium
- Profits in leader-follower (Stackelberg) equilibrium

Cournot and leader-follower models yield profit levels inside the frontier.
What next?

- Introduce the possibility of entry
- General models of oligopoly
- Dynamic versions of Cournot competition