Almost essential
Monopoly

Useful, but optional
Game Theory: Strategy and Equilibrium

DUOPOLY

MICROECONOMICS
Principles and Analysis
Frank Cowell
Overview

How the basic elements of the firm and of game theory are used

Duopoly

Background

Price competition

Quantity competition

Assessment
Basic ingredients

- **Two firms:**
  - issue of entry is not considered
  - but monopoly could be a special limiting case

- **Profit maximisation**

- **Quantities or prices?**
  - there’s nothing within the model to determine which “weapon” is used
  - it’s determined *a priori*
  - highlights artificiality of the approach

- **Simple market situation:**
  - there is a known demand curve
  - single, homogeneous product
Reaction

- We deal with “competition amongst the few”
- Each actor has to take into account what others do
- A simple way to do this: the reaction function
- Based on the idea of “best response”
  - we can extend this idea
  - in the case where more than one possible reaction to a particular action
  - it is then known as a reaction correspondence
- We will see how this works:
  - where reaction is in terms of prices
  - where reaction is in terms of quantities
Overview

Introduction to a simple simultaneous move price-setting problem
Competing by price

- Simplest version of model:
  - there is a market for a single, homogeneous good
  - firms announce prices
  - each firm does not know the other’s announcement when making its own

- Total output is determined by demand
  - determinate market demand curve
  - known to the firms

- Division of output amongst the firms determined by market “rules”

- Take a specific case with a clear-cut solution
Bertrand – basic set-up

- Two firms can potentially supply the market
  - each firm: zero fixed cost, constant marginal cost $c$
  - if one firm alone supplies the market it charges monopoly price $p_M > c$
  - if both firms are present they announce prices

- The outcome of these announcements:
  - if $p^1 < p^2$ firm 1 captures the whole market
  - if $p^1 > p^2$ firm 2 captures the whole market
  - if $p^1 = p^2$ the firms supply equal amounts to the market

- What will be the equilibrium price?
Bertrand – best response?

- Consider firm 1’s response to firm 2
- If firm 2 foolishly sets a price $p^2$ above $p_M$ then it sells zero output
  - firm 1 can safely set monopoly price $p_M$
- If firm 2 sets $p^2$ above $c$ but less than or equal to $p_M$ then:
  - firm 1 can “undercut” and capture the market
  - firm 1 sets $p^1 = p^2 - \delta$, where $\delta > 0$
  - firm 1’s profit always increases if $\delta$ is made smaller
  - but to capture the market the discount $\delta$ must be positive!
  - so strictly speaking there’s no best response for firm 1
- If firm 2 sets price equal to $c$ then firm 1 cannot undercut
  - firm 1 also sets price equal to $c$
- If firm 2 sets a price below $c$ it would make a loss
  - firm 1 would be crazy to match this price
  - if firm 1 sets $p^1 = c$ at least it won’t make a loss
- Let’s look at the diagram
Bertrand model – equilibrium

- Marginal cost for each firm
- Monopoly price level
- Firm 1’s reaction function
- Firm 2’s reaction function
- Bertrand equilibrium
Bertrand – assessment

- Using “natural tools” – prices
- Yields a remarkable conclusion
  - mimics the outcome of perfect competition
  - price = MC
- But it is based on a special case
  - neglects some important practical features
  - fixed costs
  - product diversity
  - capacity constraints
- Outcome of price-competition models usually sensitive to these
Overview

The link with monopoly and an introduction to two simple “competitive” paradigms

- Price competition
- Quantity competition
  - Collusion
  - The Cournot model
  - Leader-Follower

Duopoly

Background

Assessment
Quantity models

- Now take *output quantity* as the firms’ choice variable
- Price is determined by the market once total quantity is known:
  - an auctioneer?
- Three important possibilities:
  1. Collusion:
     - competition is an illusion
     - monopoly by another name
     - but a useful reference point for other cases
  2. Simultaneous-move competing in quantities:
     - complementary approach to the Bertrand-price model
  3. Leader-follower (sequential) competing in quantities
Collusion – basic set-up

- Two firms agree to maximise joint profits
  - what they can make by acting as though they were a single firm
  - essentially a monopoly with two plants
- They also agree on a rule for dividing the profits
  - could be (but need not be) equal shares
- In principle these two issues are separate
The profit frontier

- To show what is possible for the firms
  - draw the profit frontier

- Show the possible combination of profits for the two firms
  - given demand conditions
  - given cost function

- Distinguish two cases
  1. where cash transfers between the firms are not possible
  2. where cash transfers are possible
Frontier – non-transferable profits

- Take case of identical firms
- Constant returns to scale
- DRTS (1): MC always rising
- DRTS (2): capacity constraints
- IRTS (fixed cost and constant MC)
Increasing returns to scale (without transfers)
Now suppose firms can make “side-payments”
Profits if everything were produced by firm 1
Profits if everything were produced by firm 2
The profit frontier if transfers are possible
Joint-profit maximisation with equal shares

Side payments mean profits can be transferred between firms
Cash transfers “convexify” the set of attainable profits
Collusion – simple model

- Take the special case of the “linear” model where marginal costs are identical:
  \[ c^1 = c^2 = c \]

- Will both firms produce a positive output?
  1. if unlimited output is possible then only one firm needs to incur the fixed cost
     - in other words a true monopoly
  2. but if there are capacity constraints then both firms may need to produce
     - both firms incur fixed costs

- We examine both cases – capacity constraints first
Collusion: capacity constraints

- If both firms are active total profit is
  \[ (a - bq) q - (C_0^1 + C_0^2 + cq) \]
- Maximising this, we get the FOC:
  \[ a - 2bq - c = 0 \]
- Which gives equilibrium quantity and price:
  \[ q = \frac{a - c}{2b}; \quad p = \frac{a + c}{2} \]
- So maximised profits are:
  \[ \Pi_M = \frac{(a - c)^2}{4b} - [C_0^1 + C_0^2] \]
- Now assume the firms are identical: \( C_0^1 = C_0^2 = C_0 \)
- Given equal division of profits each firm’s payoff is
  \[ \Pi_J = \frac{(a - c)^2}{8b} - C_0 \]
Collusion: no capacity constraints

- With no capacity limits and constant marginal costs
  - seems to be no reason for both firms to be active
- Only need to incur one lot of fixed costs $C_0$
  - $C_0$ is the smaller of the two firms’ fixed costs
  - previous analysis only needs slight tweaking
  - modify formula for $\Pi_j$ by replacing $C_0$ with $\frac{1}{2}C_0$
- But is the division of the profits still implementable?
Overview

Simultaneous move “competition” in quantities

- Background
- Price competition
  - Collusion
  - The Cournot model
  - Leader-Follower
- Quantity competition
- Assessment
Cournot – basic set-up

- Two firms
  - assumed to be profit-maximisers
  - each is fully described by its cost function

- Price of output determined by demand
  - determinate market demand curve
  - known to both firms

- Each chooses the quantity of output
  - single homogeneous output
  - neither firm knows the other’s decision when making its own

- Each firm makes an assumption about the other’s decision
  - firm 1 assumes firm 2’s output to be given number
  - likewise for firm 2

- How do we find an equilibrium?
Cournot – model setup

- Two firms labelled $f = 1, 2$
- Firm $f$ produces output $q^f$
- So total output is:
  - $q = q^1 + q^2$
- Market price is given by:
  - $p = p(q)$
- Firm $f$ has cost function $C^f(\cdot)$
- So profit for firm $f$ is:
  - $p(q) q^f - C^f(q^f)$
- Each firm’s profit depends on the other firm’s output
  - (because $p$ depends on total $q$)
Cournot – firm’s maximisation

- Firm 1’s problem is to choose $q^1$ so as to maximise
  \[\Pi^1(q^1; q^2) := p(q^1 + q^2) q^1 - C^1(q^1)\]
- Differentiate $\Pi^1$ to find FOC:
  \[
  \frac{\partial \Pi^1(q^1; q^2)}{\partial q^1} = p_q(q^1 + q^2) q^1 + p(q^1 + q^2) - C_q^1(q^1)
  \]
  - for an interior solution this is zero
- Solving, we find $q^1$ as a function of $q^2$
- This gives us 1’s reaction function, $\chi^1$:
  \[q^1 = \chi^1(q^2)\]
- Let’s look at it graphically
Cournot – the reaction function

- Firm 1’s Iso-profit curves
- Assuming 2’s output constant at $q_0$
- Firm 1 maximises profit
- If 2’s output were constant at a higher level
- 2’s output at a yet higher level
- The reaction function

Firm 1’s choice given that 2 chooses output $q_0$
Cournot – solving the model

- $\chi^1(\cdot)$ encapsulates profit-maximisation by firm 1
- Gives firm’s reaction 1 to fixed output level of competitor:
  - $q^1 = \chi^l(q^2)$
- Of course firm 2’s problem is solved in the same way
- We get $q^2$ as a function of $q^1$:
  - $q^2 = \chi^2(q^1)$
- Treat the above as a pair of simultaneous equations
- Solution is a pair of numbers $(q_C^1, q_C^2)$
  - So we have $q_C^1 = \chi^1(\chi^2(q_C^1))$ for firm 1
  - and $q_C^2 = \chi^2(\chi^1(q_C^2))$ for firm 2
- This gives the Cournot-Nash equilibrium outputs
Cournot-Nash equilibrium (1)

- Firm 2’s Iso-profit curves
- If 1’s output is $q_0$ ...
- …firm 2 maximises profit
- Repeat at higher levels of 1’s output
- Firm 2’s reaction function
- Combine with firm’s reaction function
- “Consistent conjectures”
Cournot-Nash equilibrium (2)

- Firm 1’s Iso-profit curves
- Firm 2’s Iso-profit curves
- Firm 1’s reaction function
- Firm 2’s reaction function
- Cournot-Nash equilibrium
- Outputs with higher profits for both firms
- Joint profit-maximising solution
The Cournot-Nash equilibrium

- Why “Cournot-Nash”? 
- It is the general form of Cournot’s (1838) solution 
- It also is the Nash equilibrium of a simple quantity game:
  - players are the two firms
  - moves are simultaneous
  - strategies are actions – the choice of output levels
  - functions give the best-response of each firm to the other’s strategy (action)
- To see more, take a simplified example
Cournot – a “linear” example

- Take the case where the inverse demand function is:
  \[ p = \beta_0 - \beta q \]
- And the cost function for \( f \) is given by:
  \[ C_f(q^f) = C_0^f + c^f q^f \]
- So profits for firm \( f \) are:
  \[ [\beta_0 - \beta q^f] q^f - [C_0^f + c^f q^f] \]
- Suppose firm 1’s profits are \( \Pi \)
- Then, rearranging, the iso-profit curve for firm 1 is:
  \[ q^2 = \frac{\beta_0 - c^1}{\beta} - \frac{C_0^1 + \Pi}{\beta q^1} \]
Cournot – solving the linear example

- Firm 1’s profits are given by
  \[ \Pi^1(q^1; q^2) = [\beta_0 - \beta q^1] q^1 - [C_0^1 + c^1 q^1] \]
- So, choose \( q^1 \) so as to maximise this
- Differentiating we get:
  \[ \frac{\partial \Pi^1(q^1; q^2)}{\partial q^1} = -2\beta q^1 + \beta_0 - \beta q^2 - c^1 \]
- FOC for an interior solution (\( q^1 > 0 \)) sets this equal to zero
- Doing this and rearranging, we get the reaction function:
  \[ q^1 = \max \left\{ \frac{\beta_0 - c^1}{2\beta} - \frac{1}{2} q^2 , 0 \right\} \]
The reaction function again

- Firm 1’s Iso-profit curves
- Firm 1 maximises profit, given $q^2$
- The reaction function

\[ \Pi^1(q^1; q^2) = \text{const} \]
Finding Cournot-Nash equilibrium

- Assume output of both firm 1 and firm 2 is positive
- Reaction functions of the firms, $\chi^1(\cdot), \chi^2(\cdot)$ are given by:
  \[ q^1 = \frac{a - c^1}{2b} - \frac{1}{2}q^2 ; \quad q^2 = \frac{a - c^2}{2b} - \frac{1}{2}q^1 \]
- Substitute from $\chi^2$ into $\chi^1$:
  \[ q^1_C = \frac{a - c^1}{2b} - \frac{1}{2} \left[ \frac{a - c^2}{2b} - \frac{1}{2}q^1_C \right] \]
- Solving this we get the Cournot-Nash output for firm 1:
  \[ q^1_C = \frac{a + c^2 - 2c^1}{3b} \]
- By symmetry get the Cournot-Nash output for firm 2:
  \[ q^2_C = \frac{a + c^1 - 2c^2}{3b} \]
Cournot – identical firms

- Take the case where the firms are *identical*
  - useful but very special
- Use the previous formula for the Cournot-Nash outputs
  \[ q^1_C = \frac{a + c^2 - 2c^1}{3b}; \quad q^2_C = \frac{a + c^1 - 2c^2}{3b} \]
- Put \( c^1 = c^2 = c \). Then we find \( q^1_C = q^2_C = q_C \) where
  \[ q_C = \frac{a - c}{3b} \]
- From the demand curve the price in this case is \( \frac{1}{3}[a+2c] \)
- Profits are
  \[ \Pi_C = \frac{[a - c]^2}{9b} - C_0 \]
Symmetric Cournot

- A case with identical firms
- Firm 1's reaction to firm 2
- Firm 2's reaction to firm 1
- The Cournot-Nash equilibrium
Cournot – assessment

- Cournot-Nash outcome straightforward
  - usually have continuous reaction functions

- Apparently “suboptimal” from the selfish point of view of the firms
  - could get higher profits for all firms by collusion

- Unsatisfactory aspect is that price emerges as a “by-product”
  - contrast with Bertrand model

- Absence of time in the model may be unsatisfactory
Overview

Sequential “competition” in quantities

Duopoly

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Price competition

Quantity competition

Assessment

- Collusion
- The Cournot model
- Leader-Follower
Leader-Follower – basic set-up

- Two firms choose the quantity of output
  - single homogeneous output
- Both firms know the market demand curve
- But firm 1 is able to choose first
  - it announces an output level
- Firm 2 then moves, knowing the announced output of firm 1
- Firm 1 knows the reaction function of firm 2
- So it can use firm 2’s reaction as a “menu” for choosing its own output
Leader-follower – model

- Firm 1 (the leader) knows firm 2’s reaction
  - if firm 1 produces $q^1$ then firm 2 produces $c^2(q^1)$
- Firm 1 uses $\chi^2$ as a feasibility constraint for its own action
- Building in this constraint, firm 1’s profits are given by
  \[ p(q^1 + \chi^2(q^1)) q^1 - C^1(q^1) \]
- In the “linear” case firm 2’s reaction function is
  \[ q^2 = \frac{a - c^2}{2b} - \frac{1}{2}q^1 \]
- So firm 1’s profits are
  \[ [a - b [q^1 + [a - c^2]2b - \frac{1}{2}q^1]]q^1 - [C^1_0 + c^1q^1] \]
Solving the leader-follower model

- Simplifying the expression for firm 1’s profits we have:
  \[ \frac{1}{2} \left[ a + c^2 - bq^1 \right] q^1 - \left[ C_0^1 + c^1 q^1 \right] \]

- The FOC for maximising this is:
  \[ \frac{1}{2} [a + c^2] - bq^1 - c^1 = 0 \]

- Solving for \( q^1 \) we get:
  \[ q^1_s = \frac{a + c^2 - 2c^1}{2b} \]

- Using 2’s reaction function to find \( q^2 \) we get:
  \[ q^2_s = \frac{a + 2c^1 - 3c^2}{4b} \]
Leader-follower – identical firms

- Again assume that the firms have the same cost function
- Take the previous expressions for the Leader-Follower outputs:
  \[ q_S^1 = \frac{a + c^2 - 2c^1}{2b} ; \quad q_S^2 = \frac{a + 2c^1 - 3c^2}{4b} \]
- Put \( c^1 = c^2 = c \); then we get the following outputs:
  \[ q_S^1 = \frac{a - c}{2b} ; \quad q_S^2 = \frac{a - c}{4b} \]
- Using the demand curve, market price is \( \frac{1}{4} [a + 3c] \)
- So profits are:
  \[ \Pi_S^1 = \frac{(a-c)^2}{8b} - C_0 ; \quad \Pi_S = \frac{2(a-c)^2}{16b} - C_0 \]

Reminder
Of course they still differ in terms of their strategic position – firm 1 moves first
Leader-Follower

- Firm 1’s Iso-profit curves
- Firm 2’s reaction to firm 1
- Firm 1 takes this as an opportunity set
- and maximises profit here
- Firm 2 follows suit

- Leader has higher output (and follower less) than in Cournot-Nash
- “S” stands for von Stackelberg

Firm 1's iso-profit curves
Firm 2's reaction to firm 1
Firm 1 takes this as an opportunity set
and maximises profit here
Firm 2 follows suit

Leader has higher output (and follower less) than in Cournot-Nash
“S” stands for von Stackelberg
Overview

How the simple price- and quantity-models compare
Comparing the models

- The price-competition model may seem more “natural”
- But the outcome \( p = MC \) is surely at variance with everyday experience
- To evaluate the quantity-based models we need to:
  - compare the quantity outcomes of the three versions
  - compare the profits attained in each case
Output under different regimes

- Reaction curves for the two firms
- Joint-profit maximisation with equal outputs
- Cournot-Nash equilibrium
- Leader-follower (Stackelberg) equilibrium
Profits under different regimes

- Attainable set with transferable profits
- Joint-profit maximisation with equal shares
- Profits at Cournot-Nash equilibrium
- Profits in leader-follower (Stackelberg) equilibrium

Cournot and leader-follower models yield profit levels inside the frontier
What next?

- Introduce the possibility of entry
- General models of oligopoly
- Dynamic versions of Cournot competition