ADVERSE SELECTION

MICROECONOMICS

Principles and Analysis

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The adverse selection problem

- A key aspect of hidden information
- Information relates to personal characteristics
  - for hidden information about actions see *Moral Hazard*
- Focus on the heterogeneity of agents on one side of the market
- Elementary model of a single seller with multiple buyers
Overview

Adverse selection

Principles

Monopoly problem

Insurance

*background and outline*
Key concepts (1)

- **Contract:**
  - agreement to provide specified good or service
  - in exchange for specified payment
  - type of contract will depend on information available

- **Fee schedule:**
  - set-up involving a menu of contracts
  - one party draws up the menu
  - allows some selection by other party
  - again the type of fee schedule will depend on information available

- **Types:**
  - assume that hidden information is well structured
  - general shape of (e.g.) agents’ preferences is common knowledge
  - but there is heterogeneity as to (e.g.) intensity of preference
  - correspond to different types of agents
**Key concepts (2)**

- **Adverse selection:**
  - individuals faced with a contract
  - can choose to accept or reject
  - multiple contracts aimed at different types?
  - then some individuals may choose the “wrong” one

- **Screening:**
  - knowing this, other side of market seeks to respond
  - draw up contracts so that the various groups self-select the “right” ones

- “Adverse selection” and “Screening” are effectively equivalent

- Based on concept of Bayesian-Nash equilibrium

- Follow through a simplified version of the game
Screening: extensive-form game

- "Nature" chooses a person's type
- Probabilities are common knowledge
- Firm may offer a contract, not knowing the type
- Consumer chooses whether to accept contract

Diagram:
- 0
  - π
  - 1−π
  - [LOW] f
  - [HIGH] f
  - [NO] a
  - [OFFER] b
  - [reject] [accept] [reject] [accept]
Outline of the approach

- Begin with monopolist serving a market
  - heterogeneous customers
  - differ in terms of taste for the product
  - other differences (income?) are unimportant

- Easy to see what is going on
  - main points can be established from case of just two customer types

- Lessons from this are easily transferred to other contexts
  - examine these later
Overview

A fee schedule to maximise profits from consumers with known tastes

- Principles
  - Monopoly problem
    - Exploitation: full information
    - Effect of hidden information
    - “Second-best” solution

- Insurance
Model structure

- Monopoly produces good 1 using good 2 as input
  - constant marginal cost
  - zero fixed cost
- Good 1 cannot be resold
- The monopoly sells to heterogeneous customers
- The firm wants to set up a system of payment
  - a fee schedule
- Some customer information might be concealed
  - imagine this information in the form of a parameter
  - knowledge of each customer's parameter value would help the firm exploit the customer
**Alternative fee schedules**

- **A straight unit price**
- **Two-part tariff**
- **Multi-part tariff**

1. \( F(x_1) = p x_1 \)
2. \( F(x_1) = F_0 + px_1 \)
3. \( F(x_1) = F_0 + p' x_1, x_1 \leq x \)
   \[ = F_0 + p' x_1 + p'' [x_1 - x], x_1 > x \]
Single customer type

- Suppose there is just one type of customer
  - income is $y$

- Utility is given by
  - $U(x_1, x_2) = x_2 + \psi(x_1)$ where $\psi(0) = 0$
  - zero income effect for good 1
  - “quasilinear” form

- Welfare can be measured by consumer surplus
  - reservation utility level is $\underline{\nu} = U(y, 0) = y$

- Firm maximises profits subject to reservation constraint

- Monopoly position means firm can appropriate the surplus

- Can do this by imposing two-part tariff:
  - fixed charge $F_0$
  - price $p = c$
A two-part tariff

\[ F(x_1) \]
Exploitative contract

- Income
- Preferences
- Reservation utility
- Budget set, exploitative contract
- Fixed charge
- Optimal consumption

- By not participating consumer can get utility level $\nu = U(0, y)$
- Reservation indifference curve is given by $U(x_1, x_2) = \nu$
Heterogeneous consumers

- Two groups: $a$-types and $b$-types
- Groups differ in their incomes and in their tastes
- Each group is internally homogeneous
- Introduce the single-crossing condition:
  - imposes regularity on indifference curves;
  - makes it possible to compare the groups
  - easy to introduce “tailor-made” fee schedules
Two sets of preferences

- $U^a(x_1^a, x_2^a) = x_2^a + \tau^a \psi(x_1^a)$
- $U^b(x_1^b, x_2^b) = x_2^b + \tau^b \psi(x_1^b)$
- Single-crossing condition is satisfied

- $h = a$
- $h = b$

- $a$-type indifference curves
- $b$-type indifference curves
Full-information contracts

- Assume there is full information about both types of consumer
- The firm knows
  - preferences of both Alf and Bill
  - incomes of both Alf and Bill
  - therefore can predict Alf, Bill’s behaviour
- Uses this information to design two-part tariffs
  - tailor-made for each type of consumer
  - forces each down to the reservation utility levels $\psi^a$ and $\psi^b$
- Outcome can be illustrated as follows
Exploitation of two groups

- $y^a, y^b$: incomes of Alf and Bill
- Preferences of Alf and Bill
- Budget sets, exploitative contracts
- Fixed charges
- Optimal consumptions

Exploitation of two groups

Alf

Bill

$y^a, y^b$: incomes of Alf and Bill
Preferences of Alf and Bill
Budget sets, exploitative contracts
Fixed charges
Optimal consumptions
Overview

Key issues of the adverse-selection problem

Adverse selection

Principles

Monopoly problem

• Exploitation: full information
• Effect of hidden information
• “Second-best” solution

Insurance
Concealed information

- Now suppose personal information is private
  - can’t observe a customer’s taste characteristic
  - can’t implement a fee-schedule conditioned on taste
- Possibility of severe loss of profit to the firm
- The reason is that some individuals may masquerade:
  - high-valuation consumers can claim the contract appropriate to low-valuation consumers
  - imitate the behaviour of low-valuation consumers
  - enjoy a surplus by “hiding” amongst the others
- Will this lead to an inefficient outcome?
Bill’s incentive to masquerade

- **Alf’s income and preferences**
- **Purple: Contract intended for a-type**
- **Green: Contract intended for b-type**
- **Alf would be better off with a b-type contract**
- **Now try cutting the fixed charge on an a-contract**

- To masquerade as a b-type Alf mimics Bill’s consumption
- Alf finds the new a-contract at least as good as a b-contract
Insight from the masquerade

- A “pooling contract” is not optimal
- By cutting $F_0$ for $a$-types sufficiently:
  - the $a$-types are at least as well off as if they had masqueraded as $b$-types
  - the firm makes higher profits
  - there is allocative efficiency
- But this new situation is not a solution:
  - shows that masquerading is suboptimal
  - illustrates how to introduce incentive-compatibility
  - but we have not examined profit maximisation
- Requires separate modelling
Overview

First look at a design problem

Adverse selection

Principles

Monopoly problem

Insurance

- Exploitation: full information
- Effect of hidden information
- “Second-best” solution
More on masquerading

- Now consider profit maximisation
- Build in informational constraints
  - will act as additional side constraint on the optimisation problem
  - because of this usually called a “second-best” approach
- Another way of seeing that a pooling contract not optimal
- Let us examine the elements of an approach
Elements of the approach

- Choose $F(\cdot)$ to maximise profits
- Subject to
  1. participation constraint
  2. incentive-compatibility constraint
- The *participation constraint* is just as before
  - can opt out and not consume at all
- *Incentive compatibility* encapsulates the information problem
  - individuals know that tastes cannot be observed
  - so they can select a contract “meant for” someone else
  - they will do so if it results in a utility gain
- Run through logic of solution
Logic of the solution: (1)

- **In principle** we have a profit-maximisation problem subject to *four* constraints:
  - high valuation *a*-types’ participation constraint
  - low valuation *b*-types’ participation constraint
  - *a*-types' incentive compatibility: must get as much utility as they would from a *b*-type contract
  - *b*-types' incentive compatibility: must get as much utility as they would from an *a*-type contract

- But *a*-types cannot be forced on to reservation utility level $\upsilon^a$
  - we’ve seen what happens: they would grab a *b*-type contract
  - so *a*-types’ participation constraint is redundant

- Also *b*-types have no incentive to masquerade
  - they would *lose* from an *a*-type contract
  - so *b*-type incentive-compatibility constraint is redundant
  - can show this formally

- So the problem can be simplified
Logic of the solution: (2)

- In practice we have a profit-maximisation problem subject to two constraints:
  - $b$-types’ participation constraint
  - $a$-types’ incentive compatibility: must get as much utility as they would from an $a$-type contract
- $b$-types can be kept on reservation utility level $\underline{v}_b$
  - there is an infinity of fee schedules that will do this
- $a$-types must be prevented from masquerading
  - do this by distorting upwards the unit price for the $b$-types
  - force the $a$-types down to the indifference curve they could attain with a $b$-type contract
  - maximise profit from them by charging price = MC
- Check this in a diagram
Second-best contracts

- Income and preferences
- Budget sets for full-information case
- The b-type contract
- a’s utility with a b-type contract
- The a-type contract
Second-best contracts (2)

- Income and preferences for the two types
- The b-type contract
- a’s utility with a b-type contract
- The a-type contract
- The attainable set constructed by the firm

- Implementing the contract with a multipart tariff
- A b-type is forced down on to the reservation indifference curve
- An a-type gets the utility possible by masquerading as a b-type
- Multipart tariff has kink at \( x_1 \)
Second-best contracts (3)

- Multipart tariff: firm’s viewpoint
- An a-type choice
- A b-type choice
- The cost function
- Profit on each a-type
- Profit on each b-type

\[
\begin{align*}
F(x_1) \\
C(x_1)
\end{align*}
\]

\[
F(\cdot) \\
C(\cdot)
\]

\[
\begin{align*}
\Pi^a \\
\Pi^b
\end{align*}
\]
Second best: principles

- \(a\)-types have to be made as well off as they could get by masquerading
  - so they have to keep some surplus
- Full surplus can be extracted from \(b\)-types
- High valuation \(a\)-type contract involves price = MC
  - "No Distortion at the Top"
- Low-valuation \(b\)-types face higher price, lower fixed charge than under full information
  - they consume less than under full information
  - this acts to dissuade \(a\)-types
Overview

Heterogeneous risk types in an insurance market
Adverse selection: competition

- So far we have assumed an extreme form of market organisation
  - power in the hands of a monopolist
  - can draw up menu of contracts
  - limited only by possibility of non-participation or masquerading
- Suppose the monopoly can be broken
  - free entry into the market
  - numbers determined by zero-profit condition
- What type of equilibrium will emerge?
- Will there be an equilibrium?
The insurance problem

- Apply the standard model of risk-taking
- The individual enjoys a random endowment
  - has given wealth \( y \)
  - but faces a potential loss \( L \)
  - consider this as a prospect \( P_0 \) with payoffs \((y, y - L)\)
- Individual’s preferences satisfy von-Neumann-Morgenstern axioms
  - can use concept of expected utility
  - if \( \pi \) is probability of loss
  - slope of indifference curve where it crosses the 45° line is \(-[1 - \pi]/\pi\)
- Competitive market means actuarially fair insurance
  - slope of budget line given by \(-[1 - \pi]/\pi\)
A single risk type

- Indifference map
- Income and possible loss
- Actuarially expected income
- Actuarially fair insurance, premium $\kappa$
- Attainable set

- Slope is same on $45^\circ$ line (here $\pi$ is probability of loss, state BLUE)
- Gets “flatter” as $\pi$ increases
- Endowment point $P_0$ has coordinates $(y, y - L)$
- Full insurance guarantees expected income
The insurance problem: types

- An information problem can arise if there is heterogeneity of the insured persons
- Assume that heterogeneity concerns probability of loss
  - \( a \)-types: high risk, high demand for insurance
  - \( b \)-types: low risk, low demand for insurance
  - types associated with risk rather than pure preference
- Each individual is endowed with the prospect \( P_0 \)
- Begin with full-information case
Efficient risk allocation

- $P_0$: Endowment point
- Purple: $a$-type indifference curves
- Brown: $b$-type indifference curves
- Attainable set and equilibrium, $a$-types
- Attainable set and equilibrium, $b$-types

- $\pi^a > \pi^b$

- An $a$-type would prefer to get a $b$-type contract if it were possible.

* Detail on slide can only be seen if you run the slideshow
Possibility of adverse selection

- Indifference curves
- Endowment
- b-type (low-risk) insurance contract
- a-type (high-risk) insurance contract
- If Alf insures fully with a b-type contract
- If over-insurance were possible

* detail on slide can only be seen if you run the slideshow
Pooling

- Suppose the firm “pools” all customers
- Same price offered for insurance to all
- Assume that $a$-types and $b$-types are in the proportions $(\gamma, 1 - \gamma)$
- Pooled probability of loss is therefore
  \[ \pi := \gamma \pi^a + [1 - \gamma] \pi^b \]
  \[ \pi^a > \pi > \pi^b \]
- Can this be an equilibrium?
Pooling equilibrium?

- Endowment & indiff curves
- Pure a-type, b-type contracts
- Pooling contract, high $\gamma$
- Pooling contract, low $\gamma$
- Pooling contract, intermediate $\gamma$
- $b$-type’s choice with pooling
- $a$-type’s unrestricted choice
- $a$-type mimics a $b$-type
- A profitable contract preferred by $b$-types but not by $a$-types

Proposed pooling contract always dominated by a separating contract.
Separating equilibrium?

- Endowment & indiff curves
- Pure a-type, b-type contracts
- a-type prefers a pure b-type contract
- Restrict b-types in their coverage
- Then a-types take efficient contract
- a-type’s and b-type’s preferred prospects to \((P^*a, P^b)\)
- A pooled contract preferred by both a-types and b-type

- Proposed separating contract might be dominated by pooling contract
- Could happen if \(\gamma\) small enough
What next?

- Consider other fundamental models of information
- Signalling models
  - also hidden personal characteristics
  - but where the informed party moves first
- Moral hazard
  - hidden information about individual actions