INEQUALITY MEASUREMENT

SO478
http://darp.lse.ac.uk/so478

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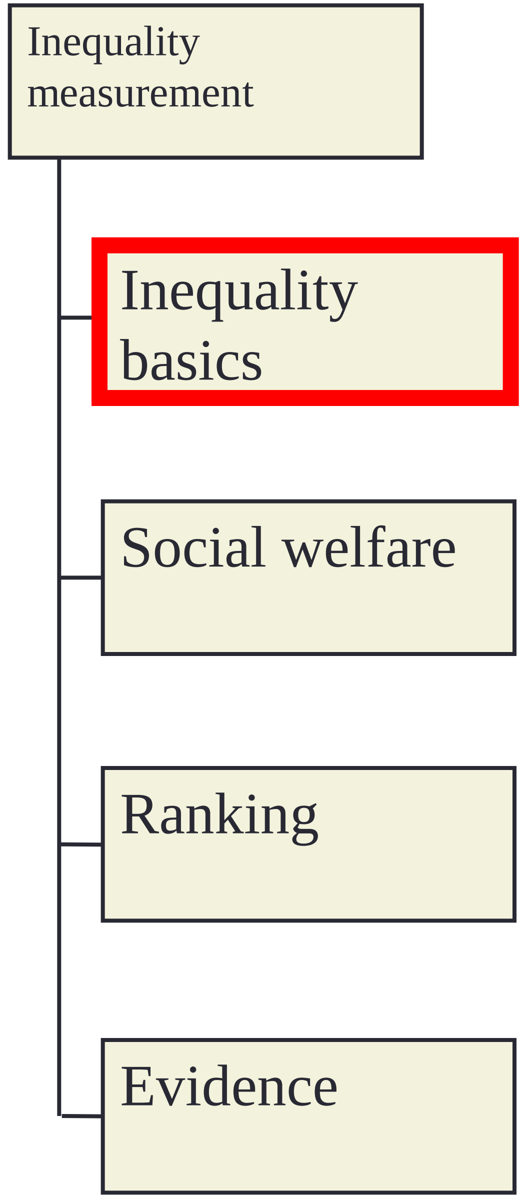
Frank Cowell

Overview...



Intuition, practicalities and principles

Agenda

- Ingredients of the problem
  - what concepts?
  - what kind of data?
- Principles of inequality measurement
  - can we move beyond intuition or guesswork?
  - where do the principles come from?
- Point of the exercise
  - empirical assessment
  - policy guidance

- For overviews: Cowell (2000, 2008, 2011, 2016), Sen and Foster (1997)

Ingredients of the problem

- The parties
  - individuals
  - households
  - larger groups?
- The equalisand "*x*"
  - "income"
  - other monetary quantities
  - other cardinal quantities
  - ordinal/categorical
- The assessment method
  - a distributional ordering
  - a distributional ranking
  - a cardinal index

“x”

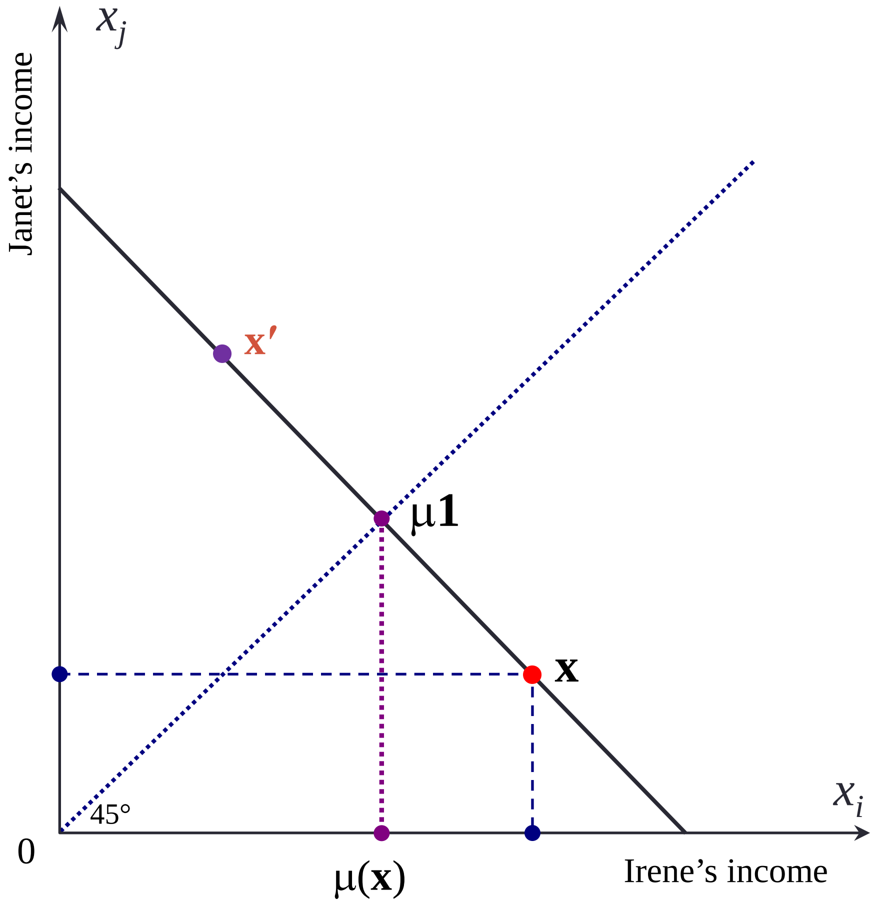
- Cardinal quantity
  - not like health?
  - not like education?
- Comparable between parties
  - needs
  - utility
  - other non-observables
- Non-negative
  - cake
  - food

Intuition

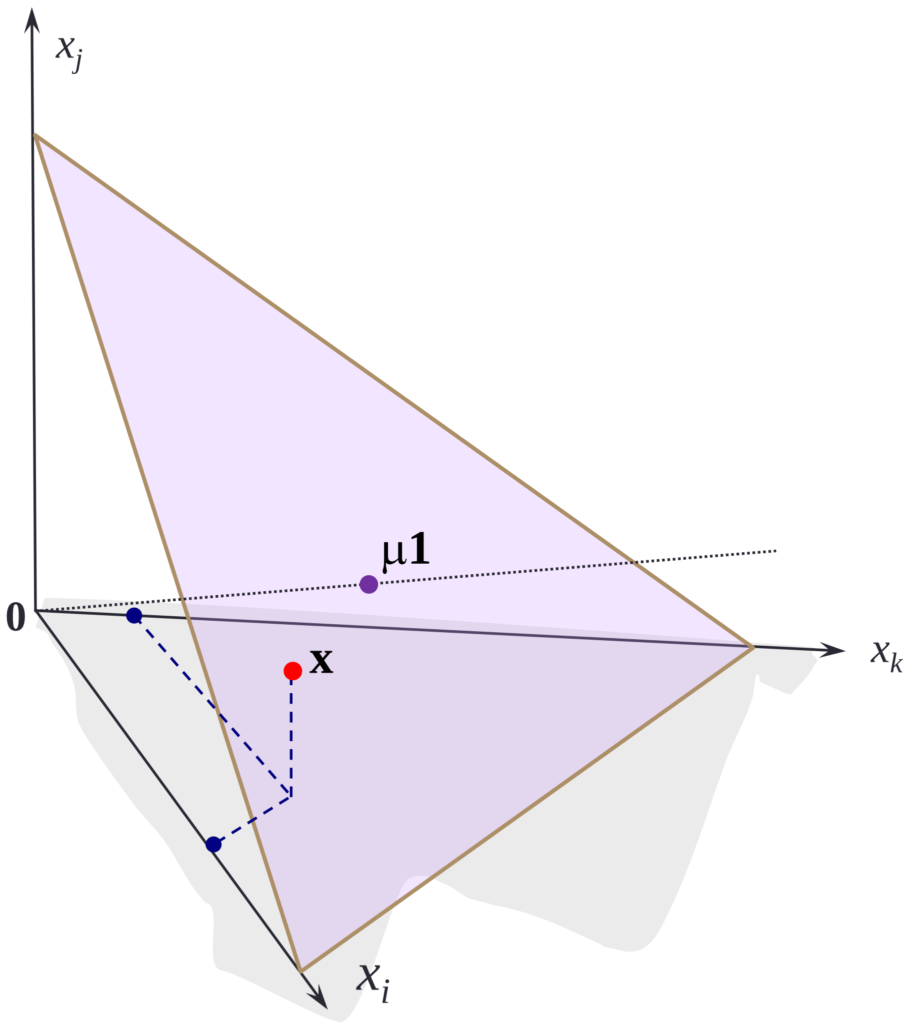
- Numbers
  - P and R
  - P, Q and R
- History
  - inequality as a snapshot?
  - righting past wrongs?
  - inequality as a process?
- Desert and merit
  - should inequality take into account info other than “x”

Income distributions $n = 2$

$$\mu(\mathbf{x}) := \frac{1}{n}\sum_{i=1}^{n} x_i$$



Income distributions $n = 3$



- *A representation with 3 incomes*
- *Income distributions with given total*
- *Equal income distributions*
- *income distribution* $\mathbf{x}$

A fundamental question

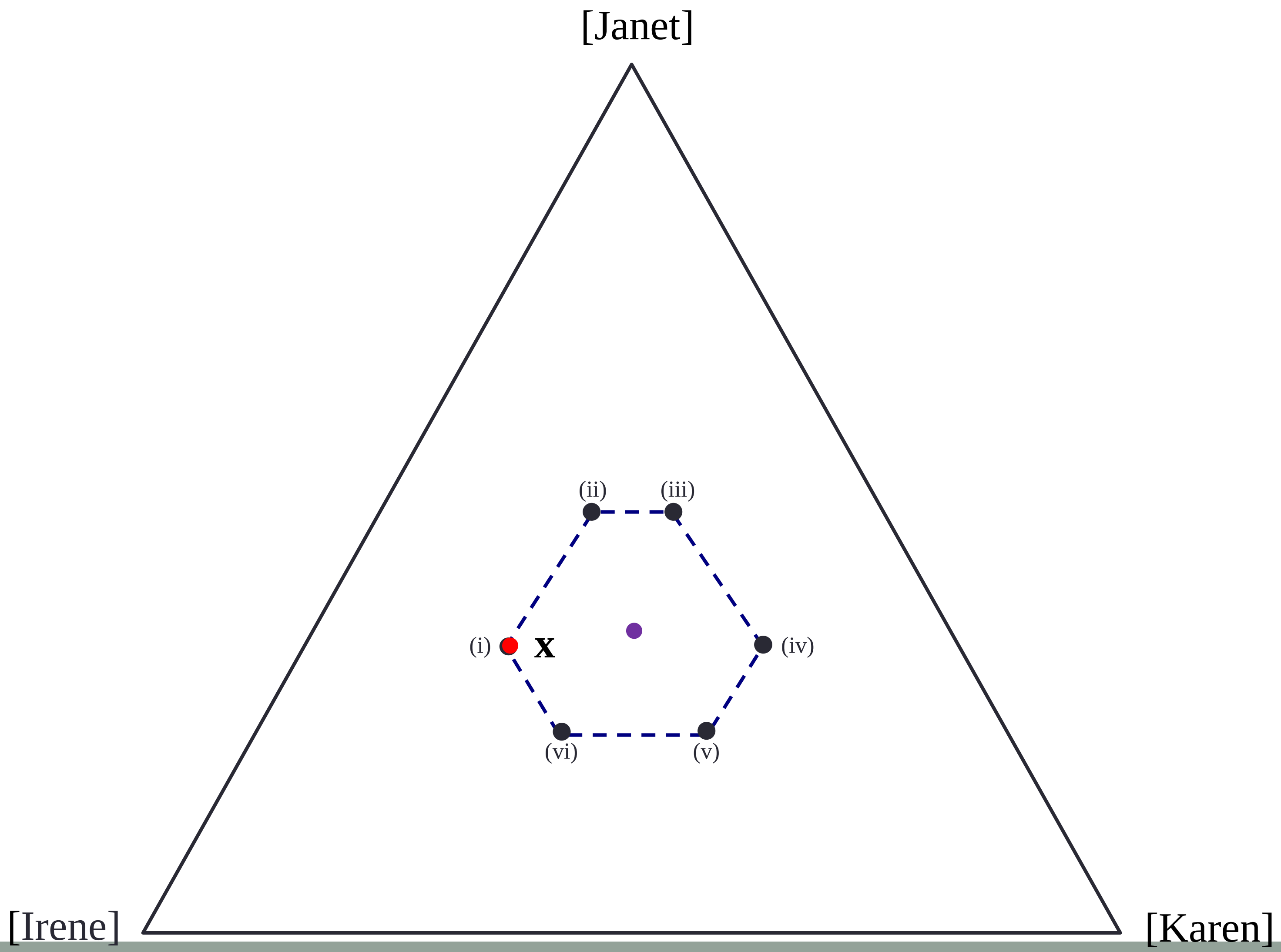
- What makes a “good” set of principles?
- There is no such thing as a “right” or “wrong” axiom
- However axioms could be appropriate or inappropriate
  - Need some standard of “reasonableness”
  - For example, how do people view income distribution comparisons?
- Use a simple framework to list some of the basic axioms
  - Assume a fixed population of size n
  - Assume that individual well-beingcan be measured by x
  - Wellbeing = income normalised by equivalence scales
- Follow the approach of Amiel-Cowell (1999) Appendix A

Inequality axioms (1)

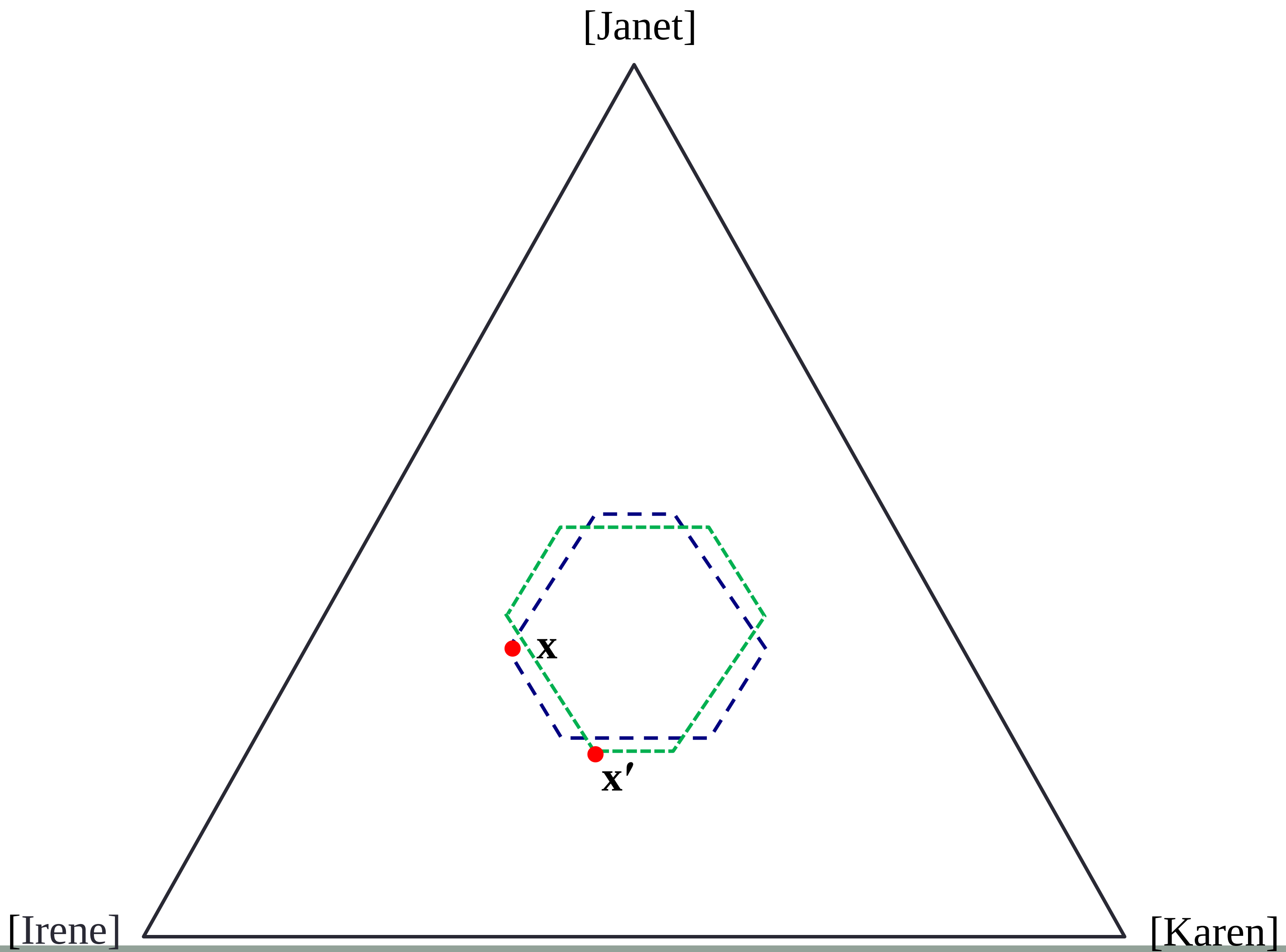
- **1 Anonymity.** Suppose $\mathbf{x}'$ is a permutation of $\mathbf{x}$. Then:
  $I(\mathbf{x}') = I(\mathbf{x})$
- **2 Population principle.**
  $I(\mathbf{x}) \geq I(\mathbf{y}) \Rightarrow I(\mathbf{x},\mathbf{x},\ldots,\mathbf{x}) \geq I(\mathbf{y},\mathbf{y},\ldots,\mathbf{y})$
- **3 Transfer principle.** (Dalton 1920) Suppose $x_i < x_j$ then, for small δ**:**

$$I(x_1,x_2\ldots, x_i+ \delta,\ldots, x_j-\delta,\ldots, x_n) < I(x_1,x_2,\ldots, x_i,\ldots, x_n)$$

Income distributions $n = 3$ (close-up)



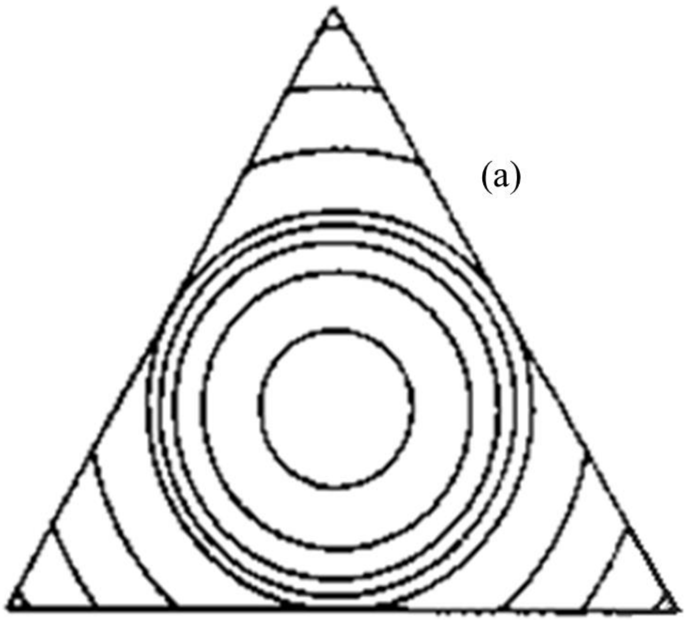
A little difficulty with ranking



Two contour maps

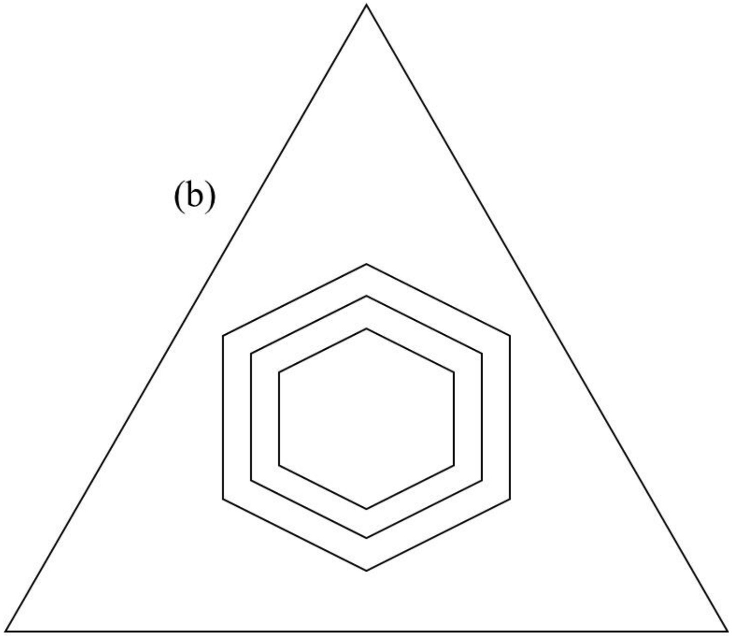
$$I_{\text{CV}}(\mathbf{x}) := \frac{\sqrt{\text{var}(\mathbf{x})}}{\mu(\mathbf{x})}.$$

$$I_{\text{Gini}}(\mathbf{x}) := \frac{1}{2n^2\mu(\mathbf{x})}\sum_{i=1}^{n}\sum_{j=1}^{n}|x_i - x_j|.$$

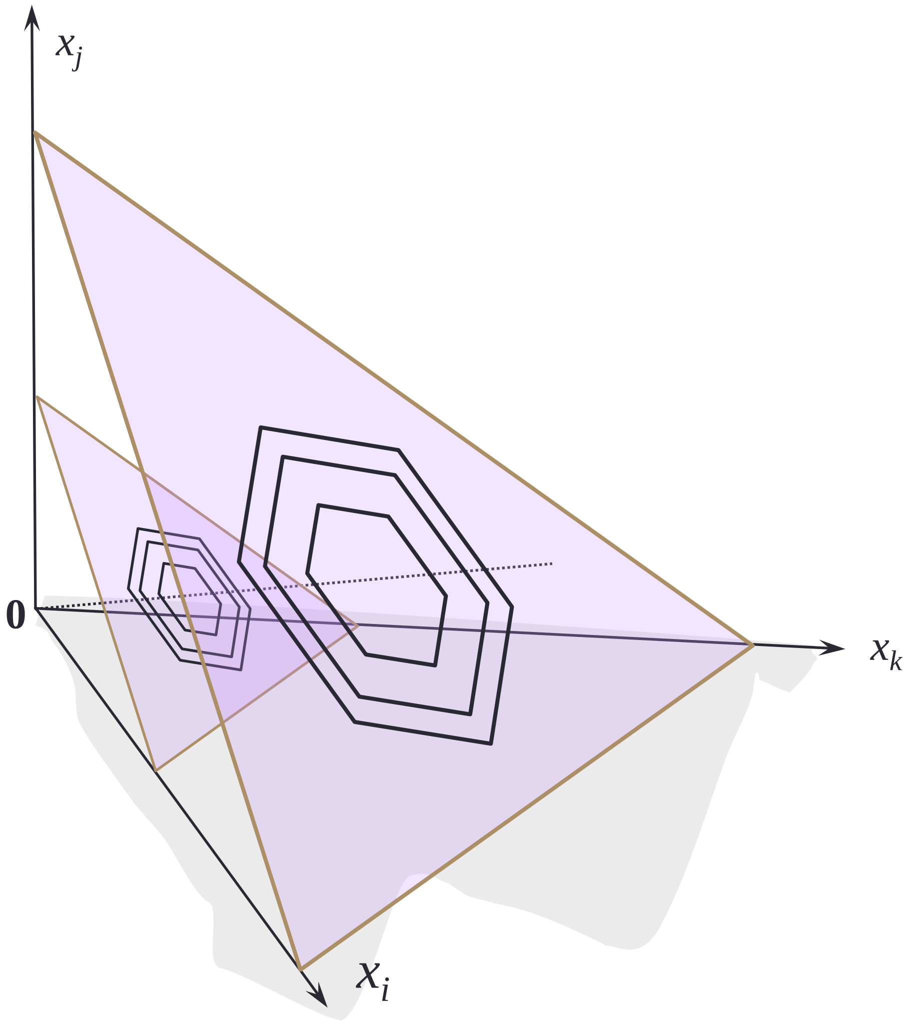


(a)

(b)



Scale invariance



Inequality axioms (2)

- **4 Decomposability**. Suppose **x**' is formed by joining **x** with **z** and **y**' is formed by joining **y** with **z**. Then **:**

  $I(\mathbf{x}) \geq I(\mathbf{y}) \Rightarrow I(\mathbf{x}') \geq I(\mathbf{y}')$
- **5 Scale invariance.** For $\lambda > 0$: $I(\mathbf{x}) \geq I(\mathbf{y}) \Rightarrow I(\lambda\mathbf{x}) \geq I(\lambda\mathbf{y})$
- **6 Translation invariance.** $I(\mathbf{x}) \geq I(\mathbf{y}) \Rightarrow I(\mathbf{x}+\mathbf{1}\delta) \geq I(\mathbf{y}+\mathbf{1}\delta)$

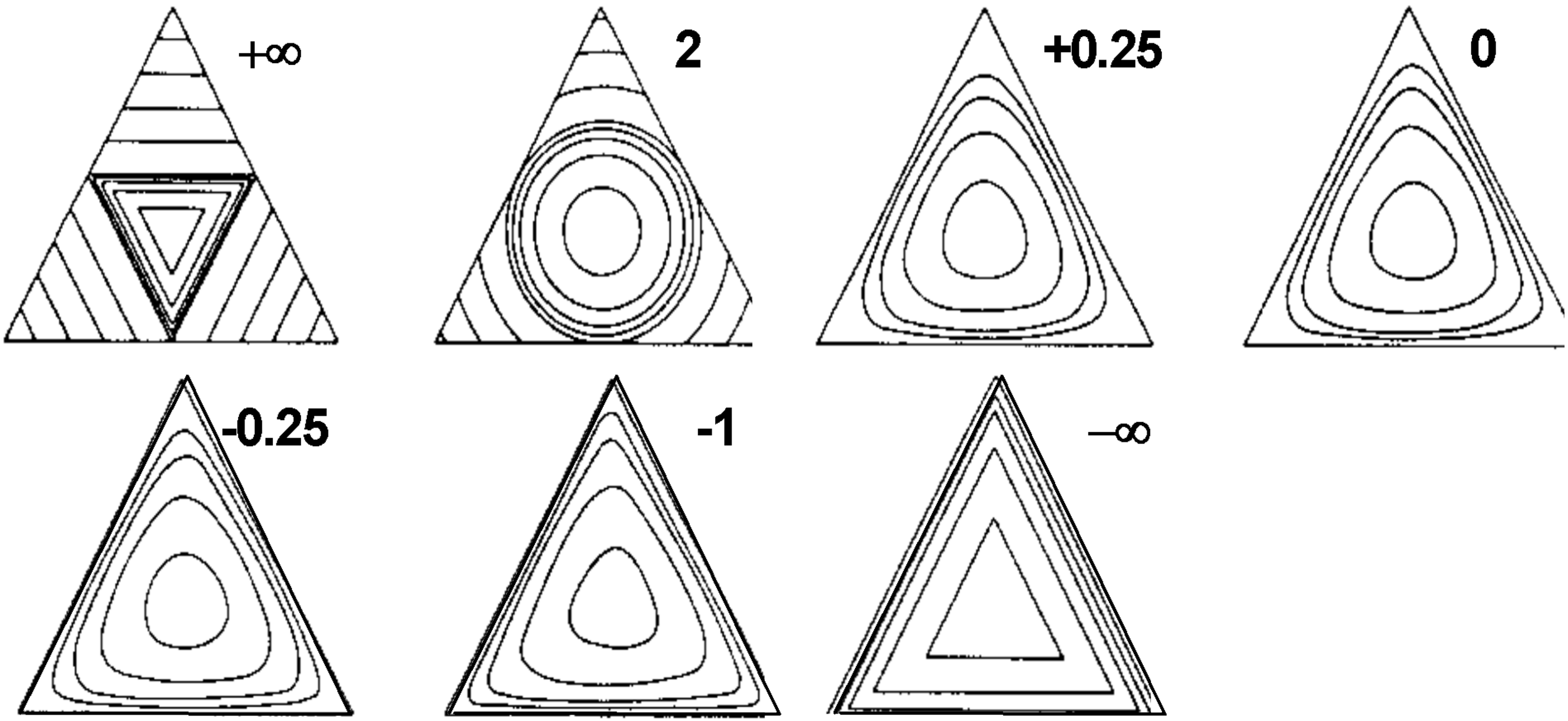
- Axioms 1-5 yield the (relative) Generalised Entropy indices

$$I_{\mathrm{GE}}^{\alpha}(\mathbf{x}) = \frac{1}{\alpha^2 - \alpha}\left[\frac{1}{n}\sum_{i=1}^{n}\left[\frac{x_i}{\mu(\mathbf{x})}\right]^{\alpha} - 1\right]$$

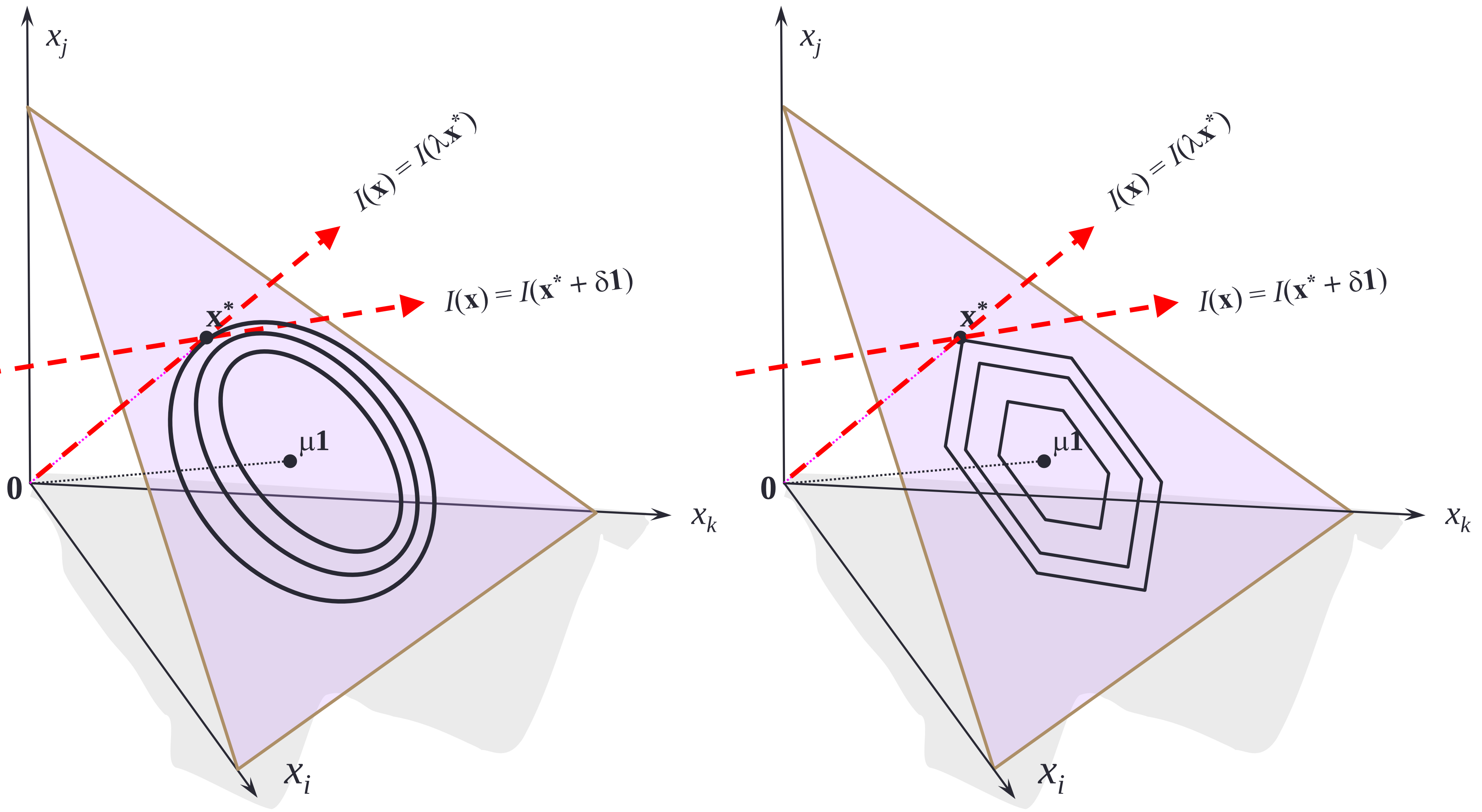
- Axioms 1-4 + 6 yield the (absolute) Kolm indices+ variance

$$I_{\mathrm{K}}^{\beta}(\mathbf{x}) := \frac{1}{\beta}\log\left(\frac{1}{n}\sum_{i=1}^{n} e^{\beta[x_i - \mu(\mathbf{x})]}\right)$$

Generalised Entropy



Scale or translation independence?



Generalised Entropy measures

- Defines a *class* of inequality measures, given parameter α :

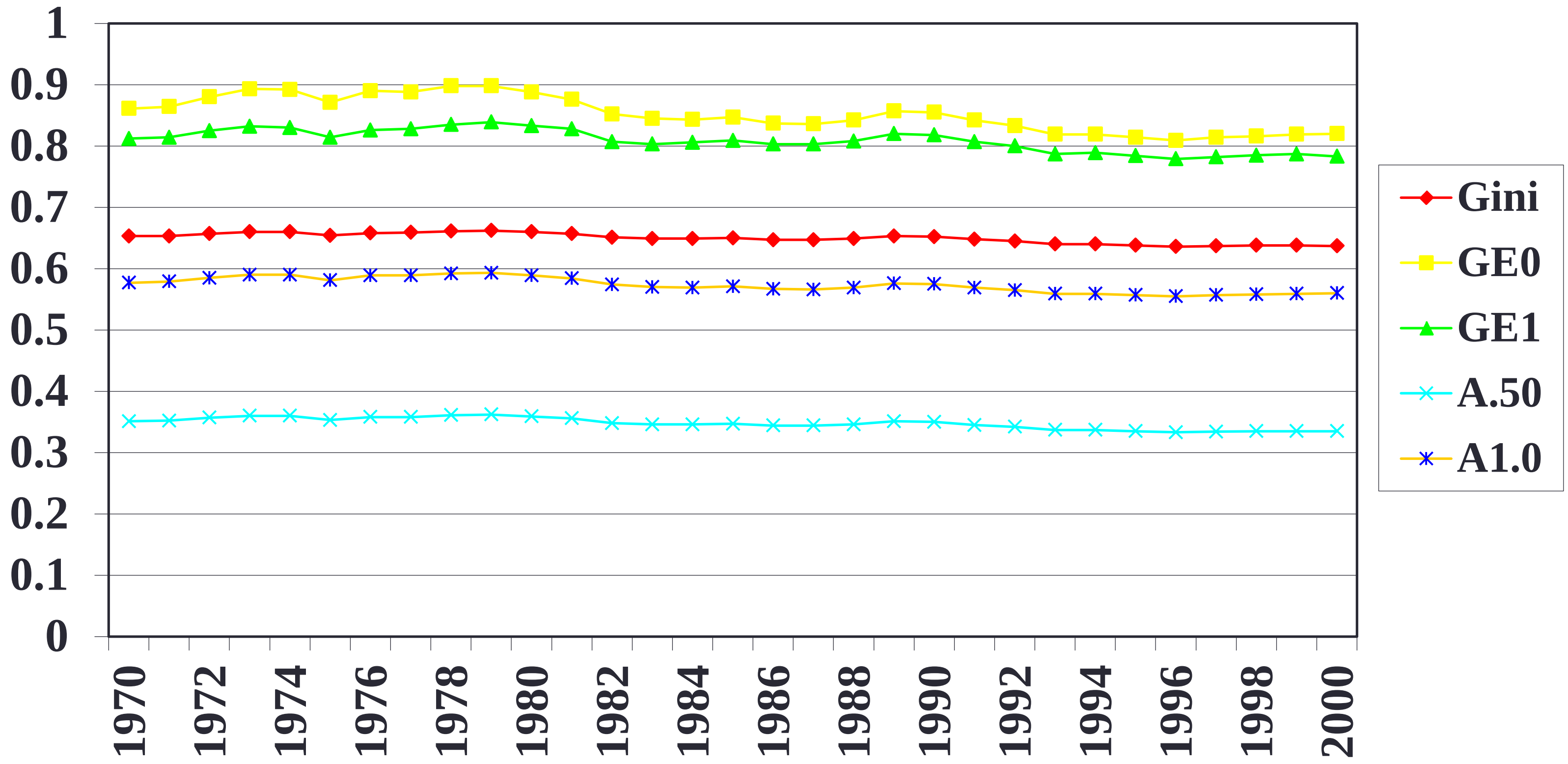
$$I_{\mathrm{GE}}^{\alpha}(\mathbf{x}) = \frac{1}{\alpha^2 - \alpha}\left[\frac{1}{n}\sum_{i=1}^{n}\left[\frac{x_i}{\mu(\mathbf{x})}\right]^{\alpha} - 1\right]$$

- GE class is rich. Some important special cases
  - for α < 1 it is ordinally equivalent to Atkinson (α = 1 – ε)
  - α = 0: $I_{\mathrm{GE}}^{0}(\mathbf{x}) := -\frac{1}{n}\sum_{i=1}^{n}\log(x_i/\mu(\mathbf{x}))$ (mean logarithmic deviation)
  - α = 1: $I_{\mathrm{GE}}^{1}(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n}[x_i/\mu(\mathbf{x})]\log(x_i/\mu(\mathbf{x}))$ (the Theil index)
  - or α = 2 it is ordinally equivalent to (normalised) variance.
- Parameter α can be assigned any positive or negative value
  - indicates sensitivity of each member of the class
  - α large and positive gives a "top-sensitive" measure
  - α negative gives a "bottom-sensitive" measure
  - each α gives a specific distance concept

Example 2: International trends (2)

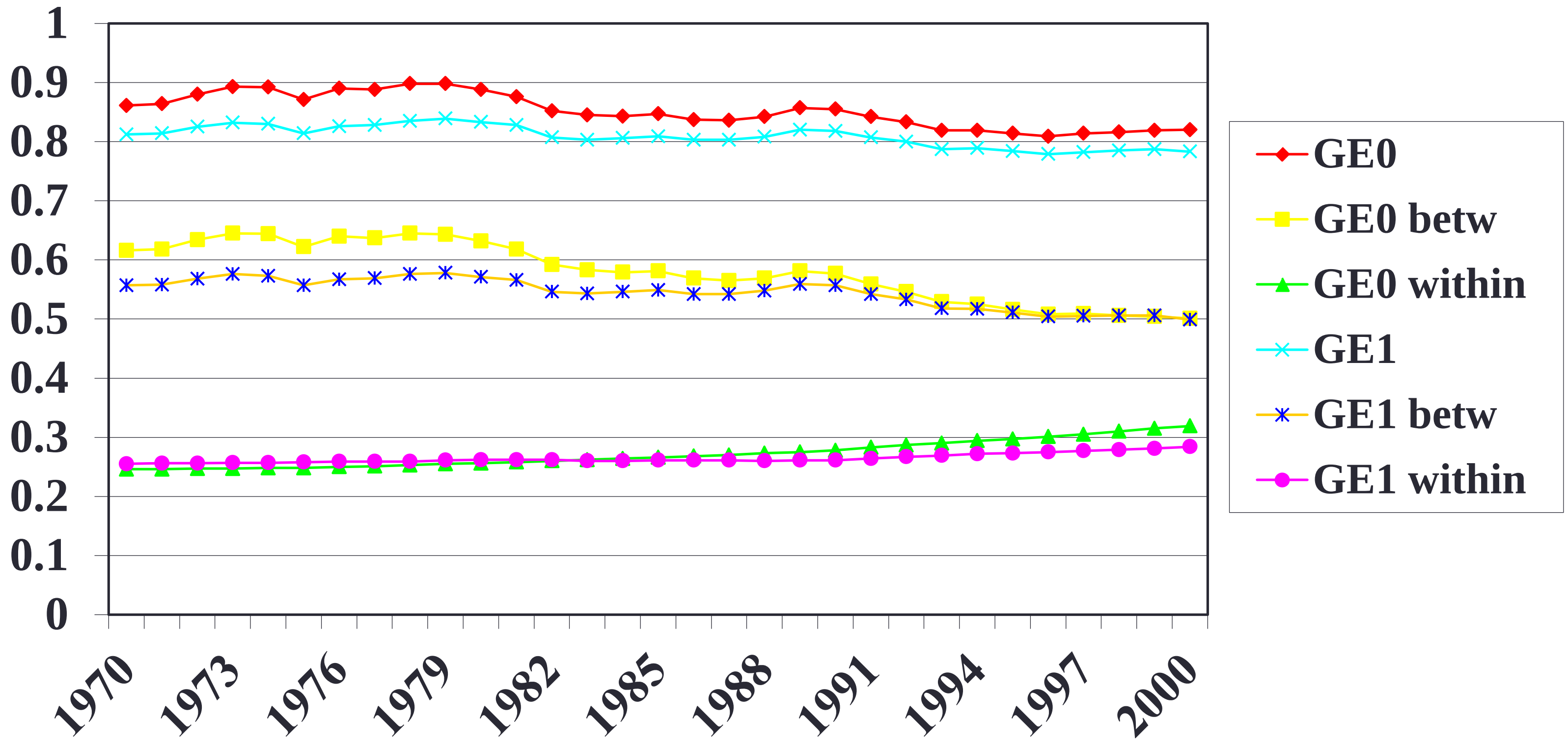
- Break down overall inequality to analyse trends:
  - $I = \Sigma_j w_j I_j + I_{\text{between}}$
  - do this with *any* inequality measure *I*?
  - what weights should we use?
- Traditional approach takes each country as separate unit
  - shows divergence – increase in inequality
  - but, in effect, weights countries equally
  - debatable that China gets the same weight as very small countries
- New conventional view (Sala-i-Martin 2006)
  - within-country disparities have increased
  - not enough to offset reduction in cross-country disparities
- Components of change in distribution are important
  - "correctly" compute world income distribution
  - decomposition within/between countries is then crucial
  - what drives cross-country reductions in inequality?
  - large growth rate of the incomes of the Chinese

Inequality: World experience



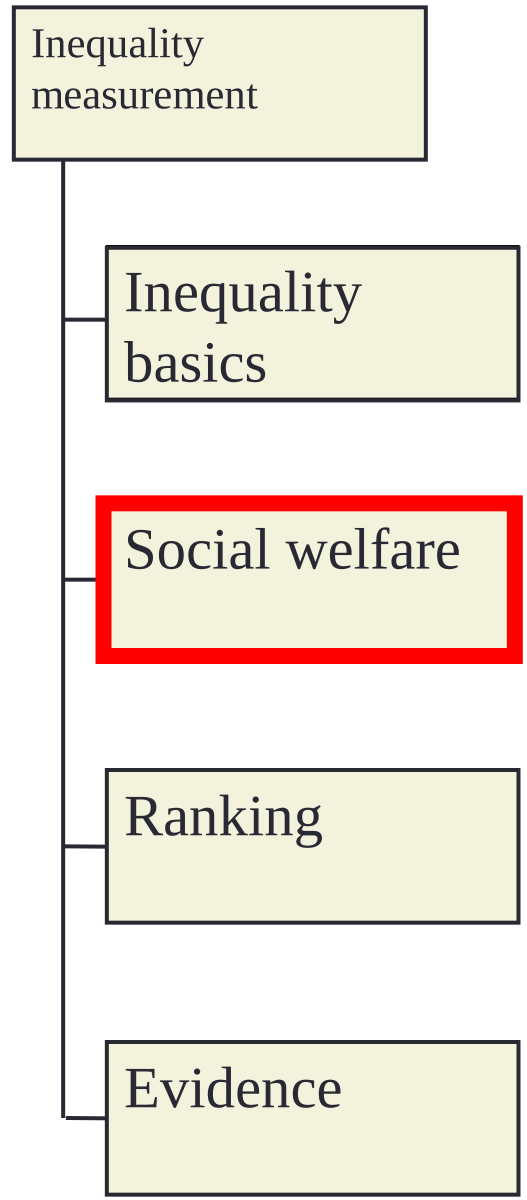
Source: [Sala-i-Martin (2006)]

Inequality: World experience: (2)



Source: Sala-i-Martin (2006)

Overview...

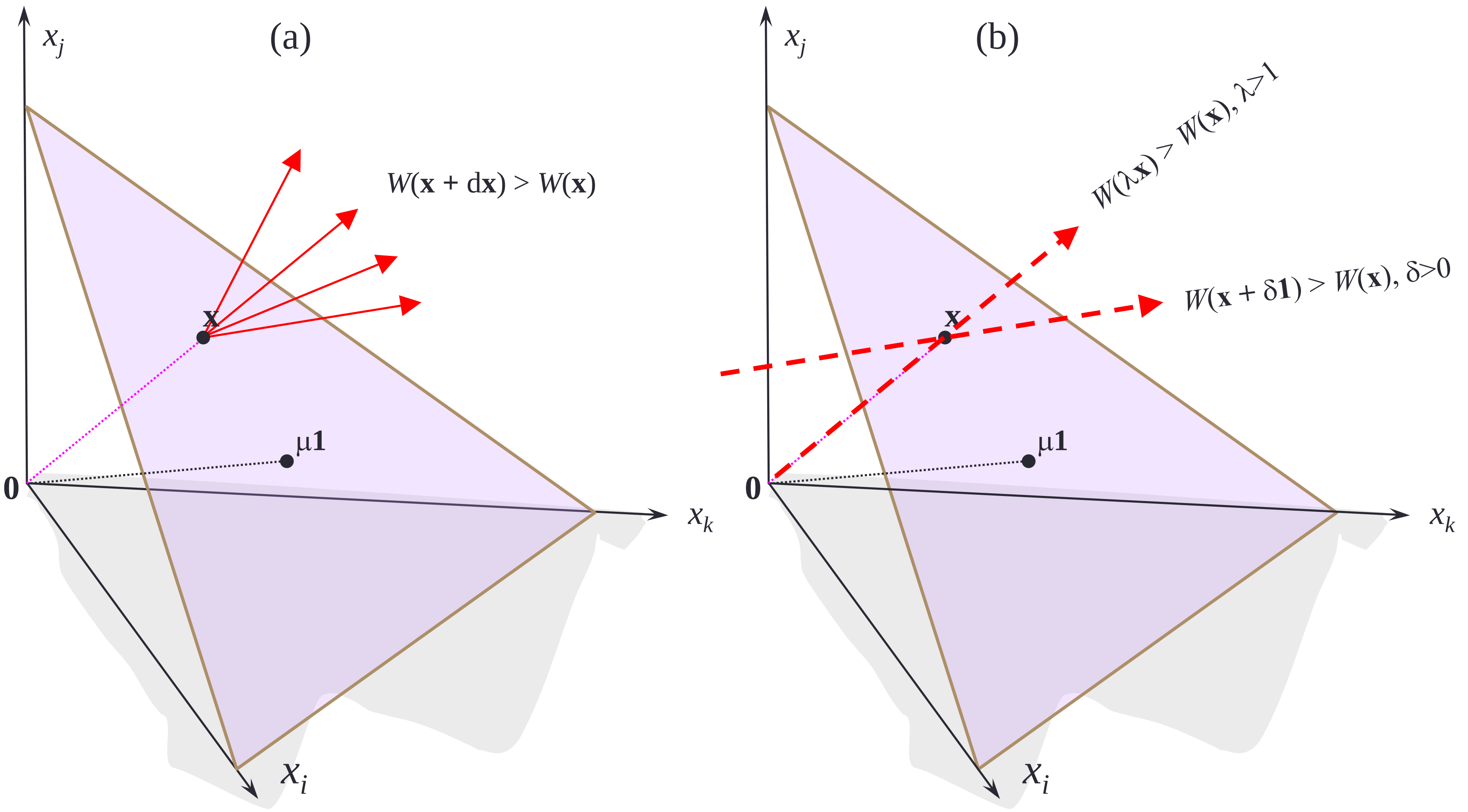


Connecting with Social values

Social-welfare functions

- A standard approach to a method of assessment
- Basic tool is a *social welfare function* (SWF)
  - maps set of distributions into the real line $W = W(\mathbf{x})$
  - i.e. for each distribution we get one specific number
- Properties will depend on economic principles
- Simple example of a SWF: $W = \Sigma_i x_i$
- Principles on which SWF could be based?
  - use counterparts of inequality axioms
  - "reverse them" so welfare increases as inequality decreases
  - we also use…
- **Monotonicity.** $W(x_1,x_2..., x_i+\delta,..., x_n) > W(x_1,x_2,..., x_i,..., x_n)$

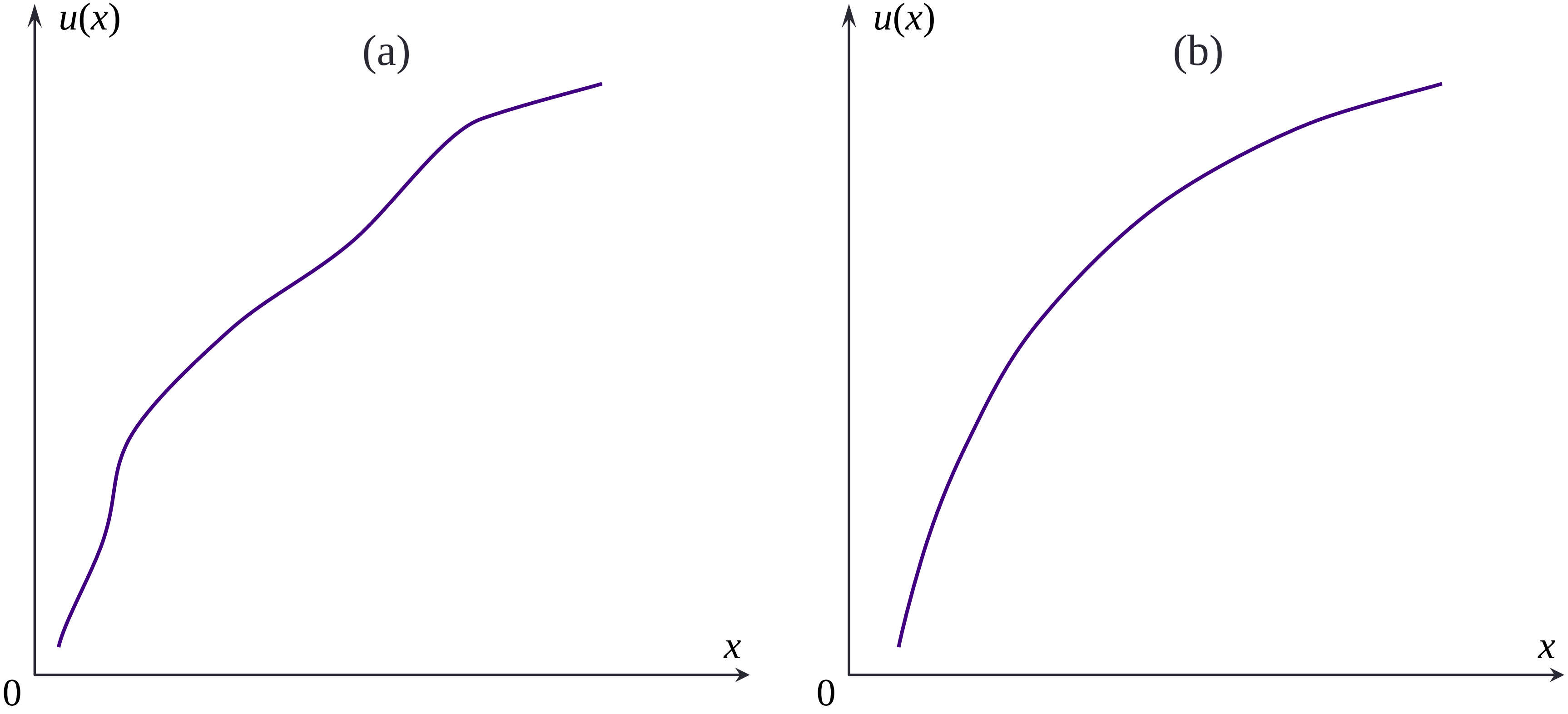
Social welfare and income growth



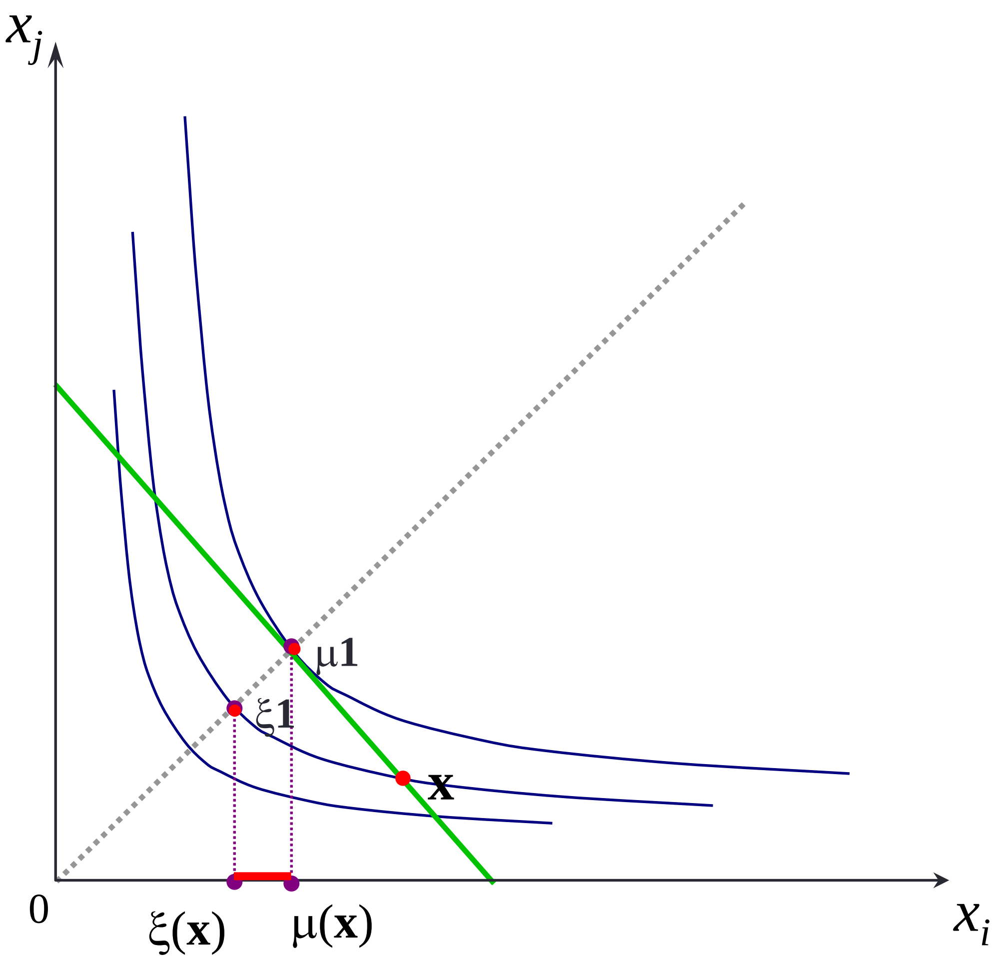
Classes of SWFs

- **Anonymity** and **population** principle:
  - can write SWF in either Irene-Janet form or *F* form
  - may need to standardise for needs etc
- Introduce **decomposability**
  - get class of Additive SWFs $\mathfrak{W}$:
  - $W(\mathbf{x}) = \Sigma_i\, u(x_i)$
  - or equivalently $W(F) = \int u(x)\, \mathrm{d}F(x)$
- If we impose **monotonicity** we get
  - $\mathfrak{W}_1$: $u(\bullet)$ increasing
- If we further impose the **transfer principle** we get
  - $\mathfrak{W}_2$: $u(\bullet)$ increasing and concave

Evaluation functions *u*



SWF and inequality



- *The Irene &Janet diagram*
- *A given distribution*
- *Distributions with same mean*
- *Contours of the SWF*
- *Construct an equal distribution with same social welfare*
- *Equally-Distributed Equivalent income*
- *Social waste from inequality*

- contour: **x** values such that $W(\mathbf{x})$ = const
- Curvature of contour indicates society's willingness to tolerate "efficiency loss" in pursuit of greater equality
- Inequality $1 - \frac{\xi(\mathbf{x})}{\mu(\mathbf{x})}$.

An important family

- Take the $\mathfrak{W}_2$ subclass and impose **scale invariance.**
- Get the family of SWFs where *u* is iso-elastic:

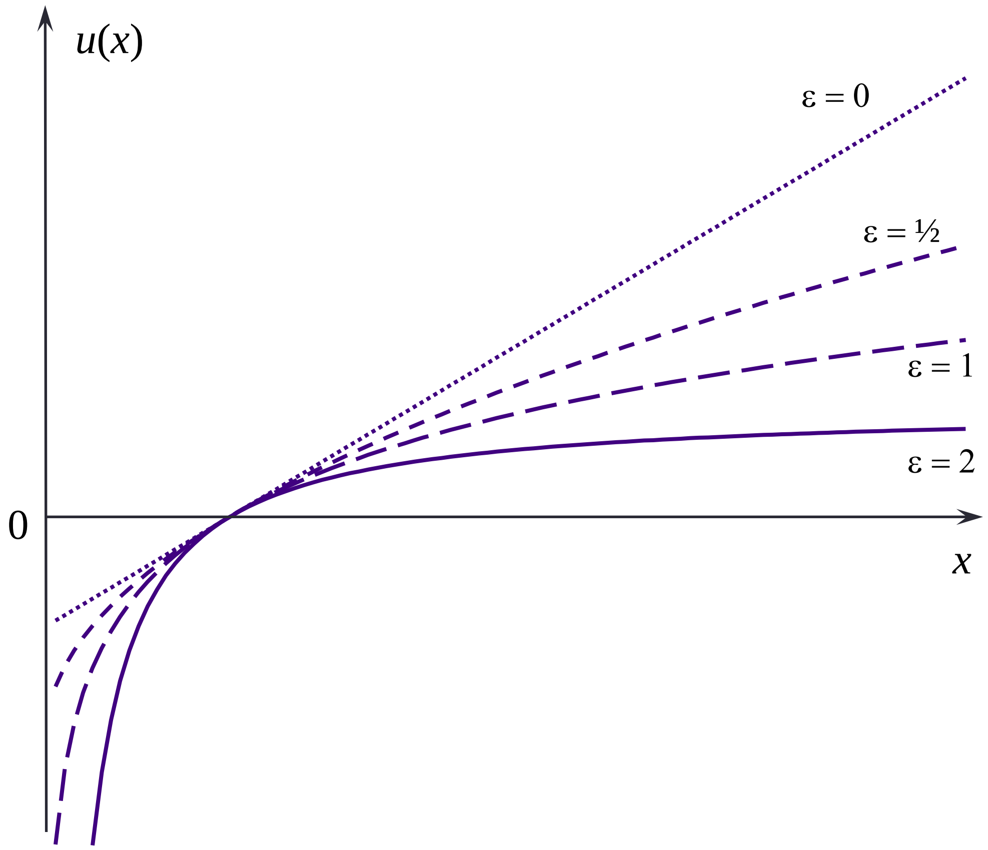
$$u(x) = \frac{x^{1-\varepsilon} - 1}{1-\varepsilon}, \quad \varepsilon \geq 0$$

  - has same form as CRRA utility function
- Parameter ε captures society's inequality aversion.
  - Similar to individual risk aversion (Atkinson 1970)

$$\xi(\mathbf{x}) = \left[\frac{1}{n}\sum_{i=1}^{n} x_i^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \varepsilon > 0$$

$$I_{\mathrm{A}}^{\varepsilon}(\mathbf{x}) := 1 - \left[\frac{1}{n}\sum_{i=1}^{n}\left[\frac{x_i}{\mu(\mathbf{x})}\right]^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}.$$

Isoelastic *u* for different values of ε



Where do values in SWF come from?

- Consensus?
  - the problem of the "Arrow Theorem..."
- Personal concern for distribution
  - people may have two sets of values, private and public
  - may treat distribution as a "public good"
- The PLUM principle
  - "People Like Us Matter"
  - interest groups determine SWF – will they be consistent?
- Based on individual rationality under uncertainty
  - argument by analogy between welfare and risk analysis (Atkinson 1970)
  - social welfare based on individual utility (Harsanyi 1953, 1955)
  - several versions of this argument (Amiel et al 2009)

Harsanyi: Impartial observer

- Consider preferences over set of lotteries
  - think of lotteries concerning life prospects
  - individual i's preferences V_i satisfy EU axioms $i = 1,\ldots,n$
- Observer sympathetic to the interests of each member of society
  - makes value judgments
  - assumes interpersonal comparisons of utility
  - j imagines himself being person i
- To get a representative person, continue the thought experiment
  - j imagines he has an equal chance of being any person in society
  - equal consideration to each person's interests
- Impartial observer j calculates average EU of each lottery:

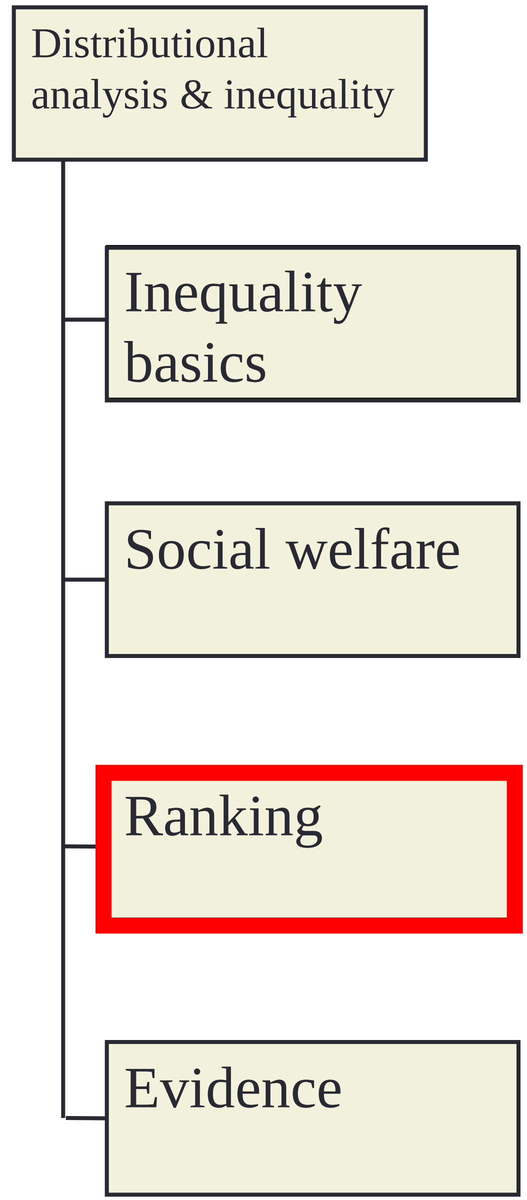
$$V_j(\mathbf{p}) = \frac{1}{n} \sum_{i=1}^{n} V_i(\mathbf{p})$$

  - I.e. person j's expected utility

Implications of Harsanyi

- The “representative person” induces a probabilistic approach
  - *social* welfare is inherited from *individual* expected utility
  - are “expectations” an appropriate basis for social choice?
- Are preferences known behind the “Veil of ignorance”?
  - not some VoI approaches (John Rawls)
  - Harsanyi assumes that representative person knows others’ utilities
- Model assumes equal probability
  - independent of income, wealth, social position etc
  - do people have prior information?
  - subjective probabilities may be inconsistent
- Do people view risk and distributional choices in the same way?
  - Cowell and Schokkaert (2001)
  - Carlsson et al (2005)
  - Kroll and Davidovitz (2003)

Overview...



Alternative approaches within Distributional Analysis

Ranking and dominance

- Introduce two simple concepts
  - first illustrate using the Irene-Janet representation
  - take income vectors **x** and **y** for a given *n*
- First-order dominance:
  - $y_{[1]} > x_{[1]},\ \ y_{[2]} > x_{[2]},\ \ y_{[3]} > x_{[3]}$
  - Each ordered income in **y** larger than that in **x**
- Second-order dominance:
  - $y_{[1]} > x_{[1]},\ y_{[1]}+y_{[2]} > x_{[1]}+x_{[2]},\ \ y_{[1]}+y_{[2]}+\ldots+y_{[n]} > x_{[1]}+x_{[2]}\ldots+x_{[n]}$
  - Each cumulated income sum in **y** larger than that in **x**
- Generalise this a little
  - represent distributions in *F*-form (anonymity, population principle)
  - q: population proportion ($0 \leq q \leq 1$)
  - $F(x)$: proportion of population with incomes $\leq x$
  - $\mu(F)$: mean of distribution F

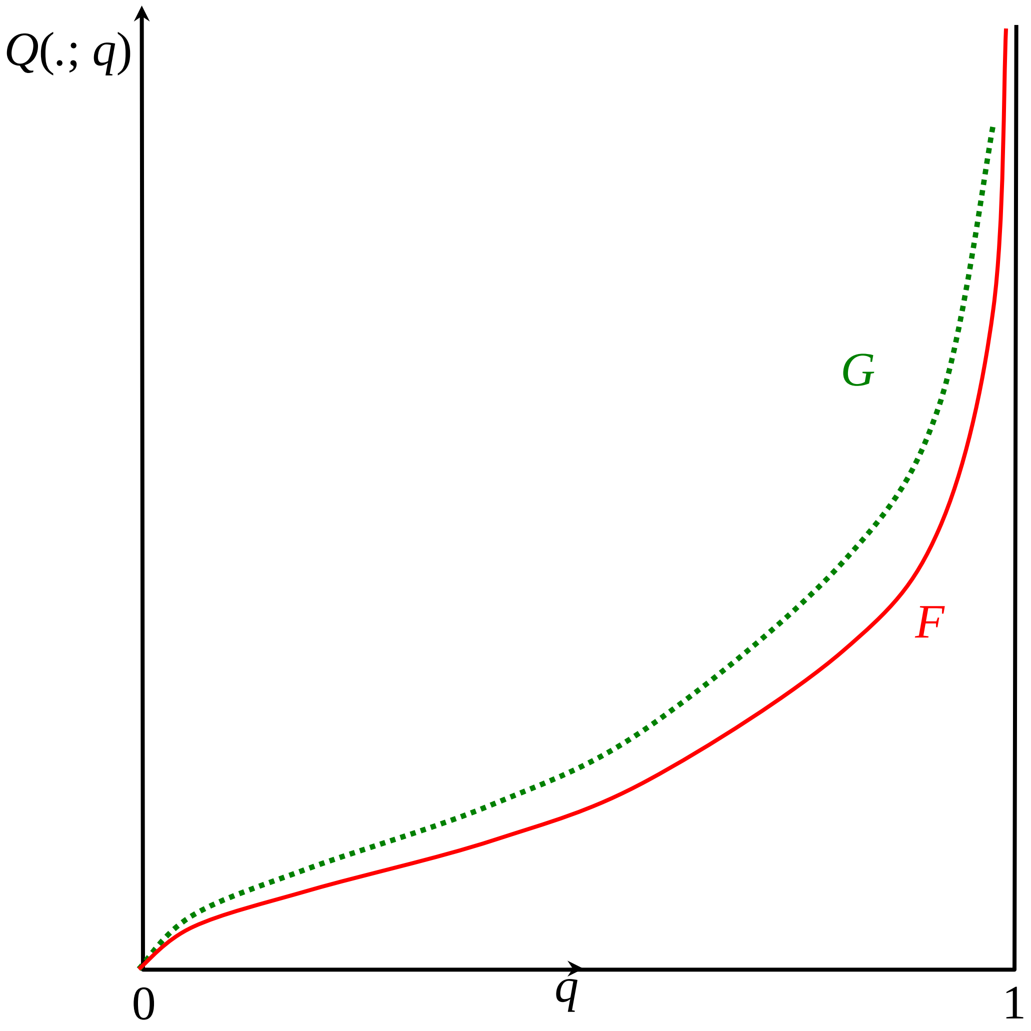
1st-Order approach

- Basic tool is the *quantile*, expressed as

  $$Q(F;\, q) \;:=\; \inf\, \{x \mid F(x) \geq q\} \quad = x_q$$

  - "smallest income such that cumulative frequency is at least as great as q"
- Use this to derive a number of intuitive concepts
- Also to characterise the idea of 1st-order (quantile) dominance:
  - "G quantile-dominates F" means:
    - for every q, $Q(G;q) \geq Q(F;q)$
    - for some q, $Q(G;q) > Q(F;q)$
- A fundamental result:
  - G quantile-dominates F iff $W(G) > W(F)$ for all $W \in \mathfrak{W}_1$

Parade and 1st-Order dominance



- *Plot quantiles against proportion of population*
- *Parade for distribution F again*
- *Parade for distribution G*

- In this case *G* clearly quantile-dominates *F*
- But (as often happens) what if it doesn't?
- Try second-order method

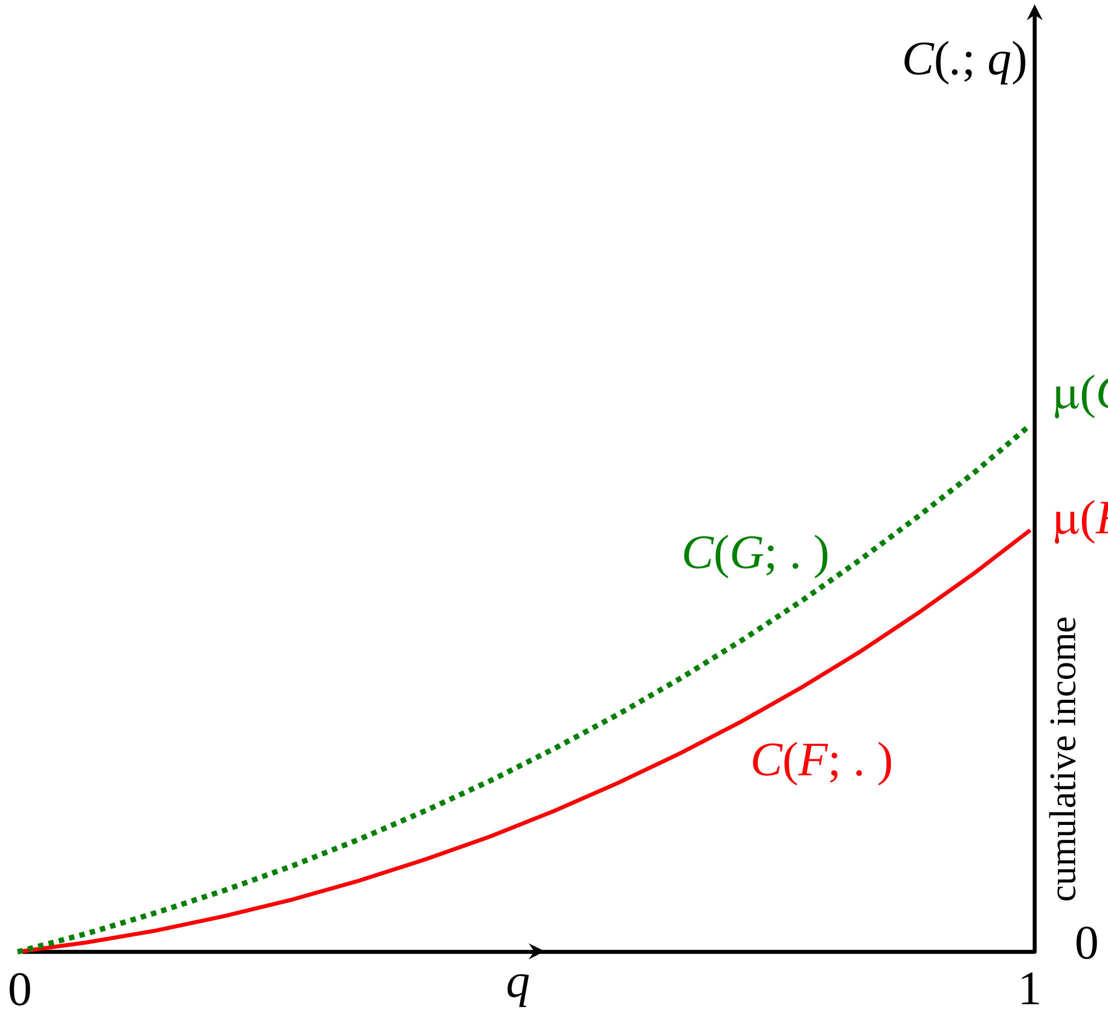
2^{nd}-Order approach

- Basic tool is the *income cumulant*, expressed as

$$C(F;\, q) \;:= \int^{Q(F;\, q)} x \,\mathrm{d}F(x)$$

  - "The sum of incomes in the Parade, up to and including position q"
- Use this to derive a number of intuitive concepts
  - the "shares" ranking, Gini coefficient
  - graph of C the *generalised Lorenz curve*
- Also to characterise the idea of 2^{nd}-order (cumulant) dominance:
  - "G cumulant-dominates F" means:
    - for every q, $C(G;q) \geq C(F;q)$,
    - for some q, $C(G;q) > C(F;q)$
- A fundamental result (Shorrocks 1983):
  - G cumulant-dominates F iff $W(G) > W(F)$ for all $W \in \mathfrak{W}_2$

GLC and 2nd-Order dominance



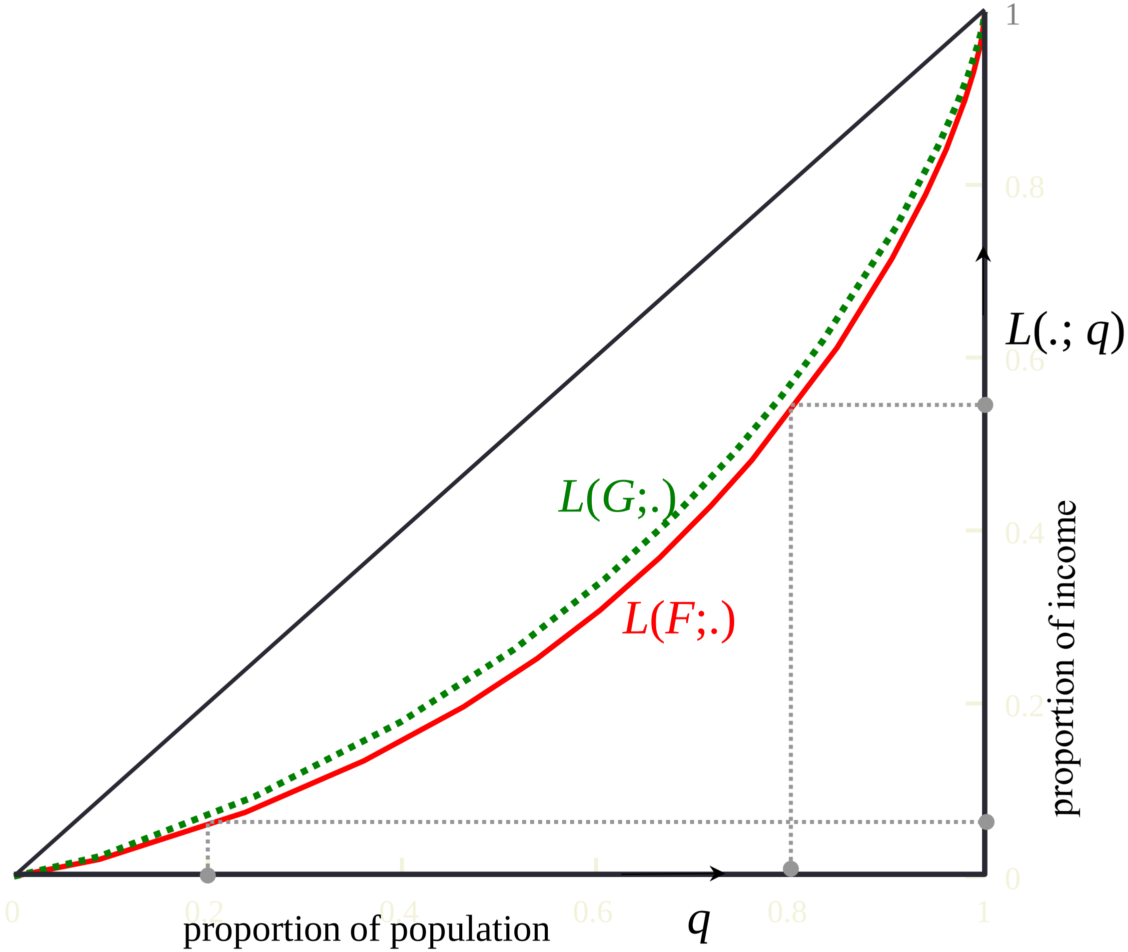
- *Plot cumulations against proportion of population*
- *GLC for distribution F*
- *GLC for distribution G*

- Intercept on vertical axis is at mean income

2^{nd}-Order approach (continued)

- The *share* of the proportion q of distribution F is $L(F;q) := C(F;q) / \mu(F)$
  - “income cumulation at q divided by total income”
- Yields Lorenz dominance, or the “shares” ranking:
  - “G Lorenz-dominates F” means:
    - for every q, $L(G;q) \geq L(F;q)$
    - for some q, $L(G;q) > L(F;q)$
- Another fundamental result (Atkinson 1970):
  - For given μ, G Lorenz-dominates F iff $W(G) > W(F)$ for all $W \in \mathfrak{W}_2$

Lorenz curve and ranking



- Plot shares against proportion of population
- Perfect equality
- Lorenz curve for distribution F
- Lorenz curve for distribution G

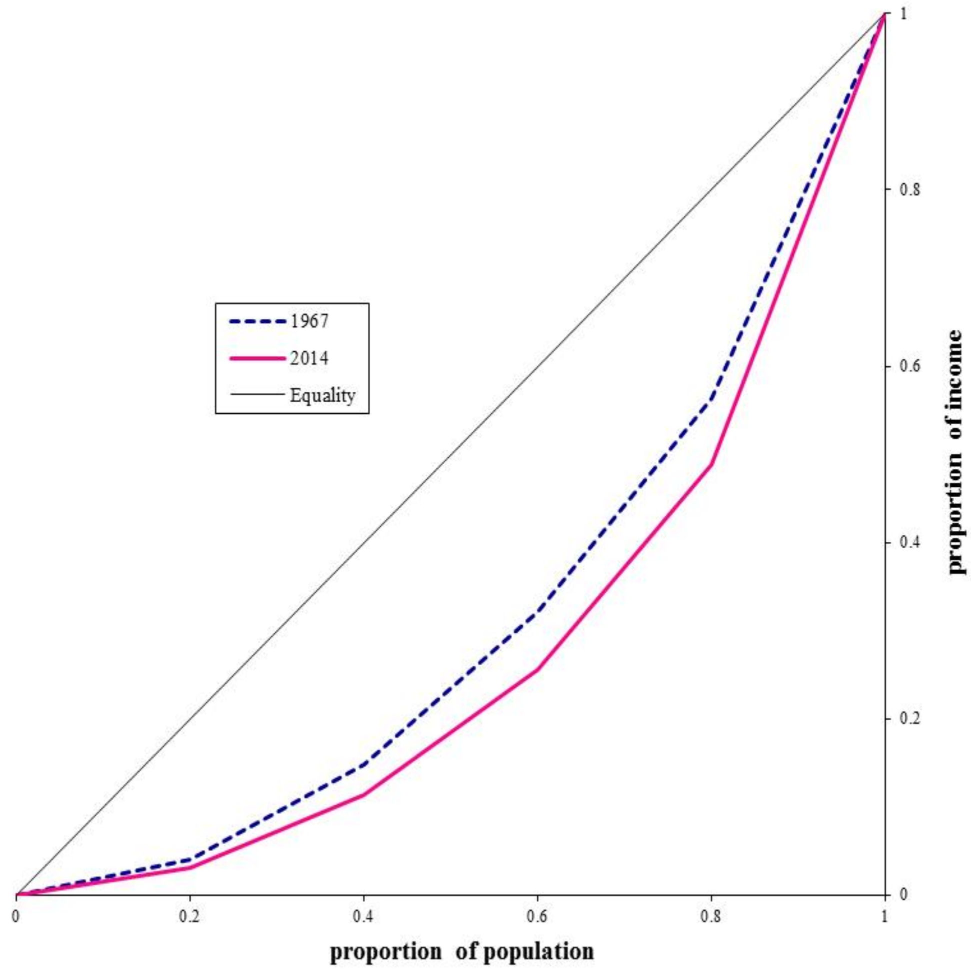
- Here *G* clearly Lorenz-dominates *F*
- *F* displays more inequality than *G*
- But what if L-curves intersect?
- No clear statement about inequality without further information

Mean incomes by groups of households

	1967	2014	Growth
1st quintile	$9,915	$11,676	17.8%
2nd quintile	$27,473	$31,087	13.2%
3rd quintile	$43,865	$54,041	23.3%
4th quintile	$61,372	$87,834	43.1%
5th quintile	$110,447	$194,053	75.7%
Overall	$50,614	*$75,738*	*49.6%*

- DeNavas-Walt et al (2015) Table A-3

Construct the Lorenz curve



- Natural interpretation in terms of shares
- Gives a natural definition of the Gini coefficient

Another application of ranking

- Tax and benefit system maps one distribution into another
  - $c = y - T(y)$
  - y: pre-tax income c: post-tax income
- Use ranking tools to assess the impact of this in welfare terms
- Typically this uses one or other concept of Lorenz dominance
- Linked to *effective tax progression*
  - T is progressive if c Lorenz-dominates y
  - see Jakobsson (1976)
- What Lorenz ranking would we expect to apply to these 5 concepts?

original income
+ cash benefits
gross income
- direct taxes
disposable income
- indirect taxes
post-tax income
+ non-cash benefits
final income

Overview...

- Inequality measurement
  - Inequality basics
  - Social welfare
  - Ranking
  - **Evidence**

Attitudes, values and perceptions

Views on distributions

- Does the theory coincide with people's views of distribution?
- Summarised from Amiel-Cowell (1999)
  - examine proportion of responses in conformity with standard axioms
  - both directly in terms of inequality and in terms of social welfare

	Inequality		*SWF*	
	Num	*Verbal*	*Num*	*Verbal*
Anonymity	83%	72%	66%	54%
Population	58%	66%	66%	53%
Decomposability	57%	40%	58%	37%
Monotonicity	-	-	54%	55%
Transfers	35%	31%	47%	33%
Scale indep.	51%	47%	-	-

Do people care about distribution?

- Large body of experimental evidence:
  - Carlsson et al (2005)
  - Kroll and Davidovitz (2003)
- Subjective well-being (Ebert, U. and Welsch, H. 2009)
  - model well being as a function of personal and environmental data
  - examine which inequality index seems to fit preferences best
- Happiness studies (Alesina et al 2004)
  - Use data on happiness from social survey
  - Construct a model of the determinants of happiness
  - Use this to see if income inequality makes a difference
- Views on distribution also may depend on
  - personal expectations (Ravallion and Lokshin 2000)
  - other issues beyond self interest (Fong 2001, Ohtake and Tomioka 2004)

Inequality aversion and Elasticity of MU

- Direct evidence
  - from happiness studies 1.0 to 1.5 (Layard et al 2008)
  - related to extent of inequality in the country? (Lambert et al 2003)
  - affected by way the question is put? (Pirttilä and Uusitalo 2010)
- Evidence on *risk aversion* as a proxy (Cowell and Gardiner 2000)
  - direct survey evidence: estimated relative risk-aversion 3.8 to 4.3 (Barsky et al 1997)
  - indirect evidence (from estimated life-cycle consumption model) suggests 0.4 to 1.4 (Blundell et al 1994)
  - in each case depends on how well-off people are
- And finally…
  - HMG suggest a value of around 0.7 – 2 (HM Treasury 2011 pp 93-94)

Conclusion

- Axiomatisation accomplished using a few basic principles
  - accord with intuition
  - several survive scrutiny in experiments
- Ranking criteria can provide broad judgments:
  - connections between dominance and welfare judgments
  - capture progressivity of policies toward redistribution
- Inequality measures can incorporate values
  - what shape should they have?
  - how do we specify them empirically?

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