Wealth Distribution and Taxation

HMRC-HMT Economics of Taxation
http://darp.lse.ac.uk/HMRC-HMT

Frank Cowell, 7 December 2015
Overview...

Why wealth taxation?
Types of tax
Wealth taxation and its alternatives

• Annual wealth tax:
  • mostly on an overall measure of net worth
  • some specific wealth taxes (property taxes)

• Inheritance / estate tax:
  • taxes on transfer of wealth at death
  • inheritance tax: on the beneficiaries of the estate
  • estate tax: on personal representatives of the deceased

• Transfer tax
  • taxes transfer of wealth not necessarily at death

• On other side of balance sheet?
  • “asset-based egalitarianism”
  • start-of-life grants
  • state pension provision
Wealth and W transfer taxes

Wealth transfer taxes

All property taxes
Why wealth taxation?

- **Revenue raising** is unlikely to be major role
  - revenue raised less than 1% of receipts?
  - see OECD Revenue Statistics
- **Efficiency** case for or against wealth taxation is unclear
  - (Cremer and Pestieau 2003)
- **Equity** case for wealth taxation is more promising
  - direct impact of wealth taxation on redistribution must be small
  - in long run taxes may influence savings and bequest behaviour
  - these influence wealth accumulation and inequality
Overview...

Definitions, composition and inequality
Wealth concepts: UK example

- British Household Panel Survey
  - fairly comprehensive
  - suffers from standard participation / attrition problems
- Wealth and assets survey
  - uses survey and administrative data – comprehensive
  - newly emerged, so limited time-series analysis
- HMRC Identified personal wealth
  - emerges directly from the estate multiplier method
  - it is clearly biased (missing wealth, missing persons)
  - differs from balance-sheet concept of wealth
- HMRC Series C: marketable wealth only
  - valuation issues addressed
  - excluded population corrected
- HMRC Series D: includes a valuation of pension rights
- HMRC Series E: includes a valuation of state pension rights
Wealth concepts and inequality

- Identified Wealth Gini=0.58
- Series C Gini=0.67
- Series D Gini=0.59
- Series E Gini=0.49
- pretax income Gini=0.25

Proportion of wealth vs. Proportion of population

7 Dec 2015

Frank Cowell: Economics of Taxation 9.3
# Wealth Distribution: Series C, D, E

<table>
<thead>
<tr>
<th>Year</th>
<th>Top 1%</th>
<th>Top 10%</th>
<th>Top 50%</th>
<th>Gini</th>
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<td>0.19</td>
<td>0.50</td>
<td>0.91</td>
<td>0.65</td>
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<td>1985</td>
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<td>0.65</td>
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<td>1990</td>
<td>0.18</td>
<td>0.47</td>
<td>0.93</td>
<td>0.64</td>
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<tr>
<td>1994</td>
<td>0.19</td>
<td>0.52</td>
<td>0.93</td>
<td>0.67</td>
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</table>

*Marketable wealth (Series C)*

<table>
<thead>
<tr>
<th>Year</th>
<th>Top 1%</th>
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<th>Top 50%</th>
<th>Gini</th>
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<td>1980</td>
<td>0.15</td>
<td>0.43</td>
<td>0.87</td>
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<td>1985</td>
<td>0.14</td>
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<td>1990</td>
<td>0.14</td>
<td>0.41</td>
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<tr>
<td>1994</td>
<td>0.14</td>
<td>0.43</td>
<td>0.89</td>
<td>0.59</td>
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*Marketable wealth and occupational pension rights (Series D)*

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<th>Gini</th>
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<tr>
<td>1980</td>
<td>0.11</td>
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<td>0.79</td>
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<td>1985</td>
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<td>0.36</td>
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<tr>
<td>1990</td>
<td>0.11</td>
<td>0.35</td>
<td>0.83</td>
<td>0.49</td>
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<tr>
<td>1994</td>
<td>0.11</td>
<td>0.36</td>
<td>0.83</td>
<td>0.49</td>
</tr>
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</table>
Wealth and assets survey: 2010/12

- Total Physical Wealth (0.45)
- Total Financial Wealth (0.84)
- Private pension wealth (0.73)
- Total Property Wealth (0.64)
- Total wealth including private pension (0.61)
Financial wealth: WAS 2006/8 to 2010/12

- Financial Wealth 2010-12 (0.84)
- Financial Wealth 2006-08 (0.82)
Property wealth: WAS 2006/8 to 2010/12

- Total Property Wealth 2010-12 (0.64)
- Total Property Wealth 2006-08 (0.63)
Pension wealth: WAS 2006/8 to 2010/12

- Pension Wealth 2010-12 (0.73)
- Pension Wealth 2006-08 (0.77)
### BHPS: Wealth inequality 1995-2005

<table>
<thead>
<tr>
<th>Year</th>
<th>Percentiles</th>
<th></th>
<th>Mean</th>
<th>Gini coefficient (%)</th>
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<td>50</td>
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<tr>
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<td>Mean</td>
<td>Gini coefficient (%)</td>
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<td>1995</td>
<td>Housing wealth</td>
<td>0</td>
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<td>Financial wealth</td>
<td>-1.9</td>
<td>3</td>
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<td>-0.1</td>
<td>37</td>
<td>190</td>
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<td>2000</td>
<td>Housing wealth</td>
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<td></td>
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<td>51</td>
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<td>Housing wealth</td>
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<td>102</td>
<td>306</td>
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<td>69</td>
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<tr>
<td></td>
<td>Net worth</td>
<td>0</td>
<td>113</td>
<td>385</td>
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- Source: Bastagli and Hills (2013)

- Source: Bastagli and Hills (2013)
### BHPS: the effect of house prices

<table>
<thead>
<tr>
<th></th>
<th>Percentiles</th>
<th>Mean</th>
<th>Gini coefficient (%)</th>
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<td></td>
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<td>90</td>
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<tr>
<td><strong>1995</strong></td>
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<tr>
<td>Housing wealth</td>
<td>0</td>
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<tr>
<td>Financial wealth</td>
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<td>77</td>
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<tr>
<td>Net worth</td>
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<td>217</td>
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<td><strong>2005 (actual house prices)</strong></td>
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<tr>
<td>Housing wealth</td>
<td>0</td>
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<td>350</td>
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<tr>
<td>Financial wealth</td>
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<td>80</td>
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<tr>
<td>Net worth</td>
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<td>146</td>
<td>427</td>
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<td><strong>2005 (adjusted house prices)</strong></td>
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<tr>
<td>Housing wealth</td>
<td>0</td>
<td>48</td>
<td>144</td>
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<tr>
<td>Net worth</td>
<td>−0.6</td>
<td>61</td>
<td>223</td>
</tr>
</tbody>
</table>

- Source: Bastagli and Hills (2013)
### Household portfolio composition – LWS

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<tr>
<td><strong>Non-financial assets</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Principal residence</td>
<td>64</td>
<td>64</td>
<td>64</td>
<td>68</td>
<td>61</td>
<td>74</td>
<td>52</td>
<td>45</td>
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<tr>
<td>Real estate</td>
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<td>20</td>
<td>23</td>
<td>17</td>
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<td>9</td>
<td>14</td>
<td>17</td>
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<tr>
<td><strong>Financial assets</strong></td>
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<td></td>
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<td></td>
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<td>Deposit accounts</td>
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<td>11</td>
<td>9</td>
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<td>10</td>
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<td>1</td>
<td>6</td>
<td>n.a.</td>
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<td>Mutual funds</td>
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<td>9</td>
<td>n.a.</td>
<td>n.a.</td>
<td>9</td>
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<tr>
<td><strong>Total assets</strong></td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
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<td><strong>Total debt</strong></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td>Home secured</td>
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<td>15</td>
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<td>n.a.</td>
<td>18</td>
<td>n.a.</td>
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<tr>
<td><strong>Total net worth</strong></td>
<td>74</td>
<td>84</td>
<td>82</td>
<td>96</td>
<td>65</td>
<td>79</td>
<td>78</td>
<td>79</td>
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</table>

LWS: Wealth inequality in four countries

<table>
<thead>
<tr>
<th></th>
<th>Gini</th>
<th>Share Top 10%</th>
<th>Share Top 5%</th>
<th>Share Top 1%</th>
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<tbody>
<tr>
<td>UK</td>
<td>0.665</td>
<td>0.456</td>
<td>0.301</td>
<td>0.101</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.893</td>
<td>0.582</td>
<td>0.406</td>
<td>0.175</td>
</tr>
<tr>
<td>Canada</td>
<td>0.747</td>
<td>0.532</td>
<td>0.374</td>
<td>0.151</td>
</tr>
<tr>
<td>US</td>
<td>0.836</td>
<td>0.705</td>
<td>0.575</td>
<td>0.329</td>
</tr>
</tbody>
</table>

Source: Cowell (2013)
LWS: Net worth

[Graph showing net worth distributions for Canada 1999 NW1, US SCF 2000 NW1, UK 2000 NW1, and Sweden 2002 NW1]
## LWS: breakdown by wealth group

<table>
<thead>
<tr>
<th></th>
<th>Gini overall</th>
<th>Share rich</th>
<th>Gini rich</th>
<th>Gini non-rich</th>
<th>between</th>
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<tbody>
<tr>
<td><strong>Top 10%</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>UK</td>
<td>0.665</td>
<td>0.456</td>
<td>0.240</td>
<td>0.608</td>
<td>0.356</td>
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<tr>
<td>Sweden</td>
<td>0.893</td>
<td>0.582</td>
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<td>Canada</td>
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<td>0.532</td>
<td>0.314</td>
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<tr>
<td>US</td>
<td>0.836</td>
<td>0.705</td>
<td>0.525</td>
<td>0.730</td>
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<tr>
<td><strong>Top 5%</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>UK</td>
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<td>Canada</td>
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<td>0.286</td>
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<td>US</td>
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<td>0.492</td>
<td>0.735</td>
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<td><strong>Top 1%</strong></td>
<td></td>
<td></td>
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<tr>
<td>UK</td>
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<td>0.148</td>
<td>0.644</td>
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<tr>
<td>Sweden</td>
<td>0.175</td>
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<td>0.327</td>
<td>0.891</td>
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<td>Canada</td>
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<td>0.246</td>
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<td>0.329</td>
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<td>0.392</td>
<td>0.776</td>
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LWS: Net worth (top 10%)
LWS: breakdown by asset type

<table>
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<tr>
<th>Asset Type</th>
<th>Share of...</th>
<th>Gini Coefficient for...</th>
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<tr>
<td></td>
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<td>Principal Residence</td>
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<td>UK</td>
<td>0.315</td>
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<td>Sweden</td>
<td>0.374</td>
<td>0.234</td>
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<tr>
<td>Canada</td>
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<td>0.181</td>
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<td>0.523</td>
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<td>Sweden</td>
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<td>0.382</td>
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<td>Canada</td>
<td>0.675</td>
<td>0.537</td>
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<tr>
<td>US</td>
<td>0.801</td>
<td>0.683</td>
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</table>
LWS: Total Assets

[Graph showing distribution of total assets across different countries, with lines representing Canada 1999, US SCF 2000, UK 2000, and Sweden 2002 total assets.]
LWS: Total *Financial* Assets

![Graph showing distribution of total financial assets](image)

- **Canada 1999 TFA**
- **US 2000 TFA**
- **UK 2000 TFA**
- **Sweden 2002 TFA**

7 Dec 2015
Functional form for wealth distribution

• Distinctive shape of empirical wealth distribution
• Upper tail appears to conform to Pareto model
• Pareto distribution
  • \( F(x) = 1 - \left[ \frac{x}{\bar{x}} \right]^\alpha \)
  • \( f(x) = \alpha \bar{x}^\alpha x^{-\alpha-1} \)
• Simple interpretation
  • \( \alpha \) captures “weight” of tail
  • \( \bar{x} \) “locates” the distribution
• Inequality
  \[
  \frac{\text{average}}{\text{base}} = \frac{\alpha}{\alpha - 1}
  \]
  \[
  \text{Gini} = \frac{1}{2\alpha - 1}
  \]
Figure 9. UK Net Worth: Pareto diagram
## Pareto estimates

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<thead>
<tr>
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<th>UK</th>
<th>Sweden</th>
<th>Canada</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Top 10%:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>2.55</td>
<td>1.78</td>
<td>1.37</td>
<td>0.48</td>
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<tr>
<td>Robust estimate</td>
<td>1.71</td>
<td>2.10</td>
<td>1.89</td>
<td>1.75</td>
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<td><strong>Top 5%:</strong></td>
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<tr>
<td>OLS</td>
<td>2.90</td>
<td>1.76</td>
<td>1.53</td>
<td>0.52</td>
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<td>Robust estimate</td>
<td>2.08</td>
<td>2.18</td>
<td>2.15</td>
<td>2.06</td>
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<td><strong>Top 1%:</strong></td>
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<tr>
<td>OLS</td>
<td>3.52</td>
<td>1.52</td>
<td>1.94</td>
<td>0.73</td>
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<td>Robust estimate</td>
<td>3.07</td>
<td>1.61</td>
<td>2.58</td>
<td>2.27</td>
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Overview...

Rising inequality or stability?

Wealth Distribution and Taxation

- Wealth taxation
- Wealth distribution
- Wealth trends
- Long-run models
Trends in wealth inequality

- Useful to look at *trends* in distribution
  - what effect of wealth taxation in the past?
  - equalisation?
  - is there a trend toward stability…?
  - ….or divergence?

- For historical and recent wealth trends in US
  - Substantial time coverage:
    - From early 20th century

- For historical wealth trends in UK
  - Atkinson et al. (1989)
  - Similar time coverage…
  - But incomplete series
  - Recent picture from HMRC data

- Recent evidence from Sweden
  - Roine and Waldenström (2009)
Distribution of wealth US 1916-2000

Wealth owned by various groups

percent


- top 2%
- top 1%
- top 0.5%
- top 0.25%
- top 0.1%
- top 0.05%
- top 0.01%
Sweden: top 5 percent
Pareto’s $\alpha$: USA and UK

- Sources: see Cowell (2011) Chapter 4
Wealth trends

- UK Inequality falls in early 20th century
  - roughly from first world war
  - substantive rises in income tax and estate duty
- Reductions in inequality continue through mid-century
- US inequality falls from time of great depression
  - Largely attributable to stock prices
  - Large concentration of corporate stock in wealth of very rich
- But US inequality also carried on falling through to 70s
  - Antitrust legislation?
  - Development of estate tax
  - Changing nature of top groups ([Edlund and Kopczuk 2009](#))
- Sweden
  - From World War I until late 20th century equalisation
  - From around 1980 trend reversed
Overview...

Intergenerational dynamics

Wealth distribution

Wealth taxation

Wealth trends

Long-run models
A way forward

• Wealth taxes may work by influencing long-run distribution
  • direct impact of wealth taxes on redistribution will be small
  • small taxes can have big effect on the equilibrium (Kaplow 2000)

• What kind of model?
  • full GE (DeNardi 2004, Cagetti and DeNardi 2008)
  • Piecemeal focus

• Story of wealth distribution in the long run (Piketty 2000):
  • Specify financial constraints
  • Model preferences / tastes / habits
  • Model exogenous resource flow
  • Specify family formation mechanism

• Preferences: what motivates bequests? (Kopczuk 2010)
  • Altruism
  • Exchange
  • Warm-glow
  • Accident and inertia
Simplified model (1)

- Focus on the role of consumption
  - family features absent
  - interaction between intra-/inter-generational factors
  - (Champernowne-Cowell 1998, Cowell 2012)

- A model of single person-dynasties:
  - person inherits $\theta$ years after attaining adulthood
  - dies $\theta$ years after inheritance
  - leaves all his terminal wealth to one descendant

- Resources
  - all get the same earnings, $e(\theta)$ at age $\theta$
  - but people may differ in terms of inherited wealth

- Taxation:
  - could introduce income or wealth taxes in model
  - here focus on wealth-transfer taxation
  - simple piecewise linear tax on bequests
  - wealth left in excess of $W^*$ taxed at rate $\tau$
Link between generations

$n - 1$

$B_{n-1} - \text{tax}$

$n$

$n + 1$

$n + 2$
Intra-generational – naïve consumption

- Role of consumption is crucial
- Naïve consumption behaviour:
  - income is given by $y(\theta) = e(\theta) + rW(\theta)$
  - consumption aims at a target level: $c(\theta) = \min \{\bar{c}, y(\theta)\}$
  - earnings are fixed $\bar{c} > \bar{e}$
- Wealth accumulation:
  - given initial wealth $W(0)$: $W(\theta) = \max \{W(0)e^{r\theta} - B^*[e^{r\theta} - 1], 0\}$.
  - rising/falling wealth: $W(0) \geq B^*$
  - $W(0)$ will be determined by intergenerational link
Inter-generational

- Role of taxation is crucial:
  - bequest tax: \( \max \{\tau[B - W^*], 0\} \)
- Bequest determined by intragenerational component
  - terminal wealth: \( B_n = W(\bar{\theta}) \)
  - tax determines starting wealth (inheritance) for next generation
  - from tax function: \( W(0) = \min\{B_n, [1 - \tau]B_n + \tau W^*\} \)
- Get a model of bequest dynamics:
  - connect \( n \) and \( n+1 \): \( B_{n+1} = \max\{\min\{B_n, [1 - \tau]B_n + \tau W^*\} e^{r\bar{\theta}} - B^* [e^{r\bar{\theta}} - 1], 0\} \)
  - use the difference operator \( \Delta B_n := B_{n+1} - B_n \)
  - \( \Delta B_n := \max\{\min\{B_n [e^{r\bar{\theta}} - 1], [[1 - \tau]e^{r\bar{\theta}} - 1] B_n + \tau W^* e^{r\bar{\theta}}\} - B^* [e^{r\bar{\theta}} - 1], -B_n\} \).
Bequest Dynamics

• Overall equation:
  \[
  \Delta B_n := \max \{ \min \{ B_n [e^{r\bar{\theta}} - 1], [1 - \tau]e^{r\bar{\theta}} - 1 \} B_n + \tau W^*e^{r\bar{\theta}} \} - B^* [e^{r\bar{\theta}} - 1], - B_n \}.
  \]

• Break this down by tax regime

• For low bequests (below \( W^* \))
  • dynamics: \( \Delta B_n = [B_n - B^*] [e^{r\bar{\theta}} - 1] \)
  • equilibrium: \( B_n = B^* \)

• For high bequests (above \( W^* \))
  • dynamics: \( \Delta B_n = [1 - [1 - \tau]e^{r\bar{\theta}}] [B^{**} - B_n] \)
  • equilibrium: \( B^{**} := \frac{\tau W^* - B^*[1 - e^{-r\bar{\theta}}]}{\tau - 1 + e^{-r\bar{\theta}}} \)

  \[
  \frac{\partial B^{**}}{\partial \tau} = - \frac{[1 - e^{-r\bar{\theta}}][W^* - B^*]}{[\tau - 1 + e^{-r\bar{\theta}}]^2} < 0
  \]
Bequest Dynamics: naïve consumption

- The phase diagram
- Paths to riches
- Paths to ruin
- Find equilibria ($\Delta B = 0$)
- Now cut tax…
Wealth distribution overall

- Distribution of $W$ amongst wealthy
- Take into account lower equilibrium
Alternative consumption version

- Consumption proportional to lifetime resources (Becker-Tomes 1979)
- lifetime earnings:
  \[ E = \frac{e^{r\bar{\theta}} - 1}{r} \]
- revised bequest equation:
  \[ B_{n+1} = s \min \{ B_n, [1 - \tau]B_n + \tau W^* \} + E e^{r\bar{\theta}} \]
- Bequest dynamics:
- difference equation:
  \[ \Delta B_n = \min \left\{ B_n \left[ se^{r\bar{\theta}} - 1 \right], [s(1 - \tau)e^{r\bar{\theta}} - 1]B_n + s\tau W^* e^{r\bar{\theta}} \right\} + sE e^{r\bar{\theta}} \]
- equilibrium:
  \[ B^{***} := s \frac{\tau W^* + E}{e^{-r\bar{\theta}} - s[1 - \tau]} \]

\[ \frac{\partial B^{***}}{\partial s} = \frac{\tau W^* + E}{e^{-r\bar{\theta}} - s[1 - \tau]} + \frac{s[1 - \tau][\tau W^* + E]}{[e^{-r\bar{\theta}} - s[1 - \tau]]^2} > 0 \]

\[ \frac{\partial B^{***}}{\partial \tau} = s \frac{e^{-r\bar{\theta}}W^* - s[E + W^*]}{[e^{-r\bar{\theta}} - s[1 - \tau]]^2} \]
Bequest Dynamics: alternative consumption

- The phase diagram
- Paths to equilibrium
- Find equilibria ($\Delta B = 0$)
- Cut tax (high s)
- Cut tax (low s)
Simplified model (2)

- Focus on family formation (Champernowne-Cowell 1998)
  - each generation is a discrete unit
  - pairs always consist of people with equal wealth
  - no-one benefits from more than one bequest
  - bequest is divided equally amongst the $k$ kids ($k$ given)
- Model applies to upper wealth levels – above specified wealth level $W^*$
- For any $W > W^*$, the proportion of testators with $k$ kids is $p_k$:
  - independent of $W$
  - $p_k \geq 0$
  - $\sum_k p_k = 1$
  - $\sum_k kp_k = 2$
  - two examples:
Equilibrium distribution

- Let $F_n, F_{n+1}$ be the wealth distribution in generations $n, n+1$
  - $F_n(W)$ is the proportion of the population in generation $n$ with wealth $\leq W$
  - We have equilibrium if $F_n = F_{n+1} = F$

- Take a person with wealth $W$ in a family where parents had $k$ kids
  - if parental wealth was $W'$ per head bequest must have been $2[1-\tau]W'$
  - so each kid would get $2[1-\tau]W'/k$
  - therefore $W' = kW / 2[1-\tau]$
  - given that there are $p_k$ such families: $F_{n+1}(W) = \sum_k 1/2k p_k F_n(kW / 2[1-\tau])$

- Equilibrium requires
  $$F(W) = \sum_k 1/2k p_k F(kW / 2[1-\tau])$$

- Only functional form that permits a solution for all $W$ is Paretian
  $$F(W) = 1-AW^{-\alpha}$$

- So the equilibrium condition is:
  $$\sum_{k=1}^{K} p_k \left(\frac{k}{2}\right)^{1-\alpha} = [1 - \tau]^{-\alpha}$$
Tax: Equilibrium Wealth Distribution

- Higher tax produces lower long-run inequality
- If tax is too low – no long-run equilibrium
- Quite low tax rates produce values similar to actual economies.

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<th>τ(%)</th>
<th>(i) Narrow α</th>
<th>Gini</th>
<th>(ii) Wide α</th>
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</table>
Equilibrium Distribution $\tau = 10\%$

- $\alpha = 2.113$ (Narrow family spread)
- $\alpha = 1.9002$ (Wide family spread)
Summary

- Dynastic model produces a bifurcation
  - Convergence to equilibrium distribution
  - Inequality within and between groups
  - Source of inequality lies in savings behaviour
- Role of uncertainty captured in savings behaviour
- Family structure affects long-run equilibrium
  - Spread out families reduce effectiveness of taxation
- Tweaking the models would modify this a little
  - Variation in income
  - Out-of-class marriage
  - (Champernowne-Cowell 1998)
- Potentially major role for taxation
References (1)

References (2)