Divide and conquer: Tax evasion as a global game

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Abstract

Benjamini and Maital (1985) were among the first authors to undertake a systemic approach regarding the tax evasion problem. They modelled the presence of compliance (or non-compliance) "epidemics" as a positive externality that affects an individual’s tax evasion decision.

This article imposes more structure by assuming that the externality is generated by the heterogeneity of individuals’ information sets, in particular with respect to the tax agency’s auditing intensity. This transforms the game into a global game, and hence the technique developed by Morris and Shin (2002a) is used to find an equilibrium which, unlike the one in Benjamini and Maital, is an interior solution and, hence, does not exhibit the epidemic effect.

The main result of the paper is that the agency will always prefer a contingent auditing policy over a non-contingent one. That is, a loss-minimizer/revenue-maximizer agency will condition its audit decision not only on the individual declaration but also on the average economy-wide declaration.

Mention as well: 1. Government chooses to transform the game into a coordination game. 2. Comparative statics.

JEL Classification: H26, D82, D84, C71

Keywords: Tax evasion, Coordination/Global game, Expectations, Externalities, Asymmetric information.

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1 Motivation

Tax agencies worldwide use information on taxpayers’ characteristics (gender, occupation, etc) to estimate their potential income. Based on the data collected, taxpayers are grouped into categories such that similar individuals belong to the same category. The US’s Taxpayer Compliance Measurement Program (TCMP) is an example of these information-gathering activities.

Within each class agencies usually follow a cut-off auditing policy: "Do not audit anyone who declares $T$ or more; audit those who declare below $T$ with probability $P$", where $T$ is the cut-off level and $P$ is the probability that eliminates evasion among those with true income below $T$ (Reinganum and Wilde (1985)). As a result, those with true income above $T$ avoid being audited by declaring the cut-off level while those with true income below $T$ declare truthfully. Thus the agency finds itself not only letting evaders get away with their cheating but also wasting resources by auditing compliant people.

The problem gets worse when taxpayers in a category are subject to common shocks. When the shock is negative, more people than expected by the agency will have incomes below the cut-off level and will hence declare low income, resulting in more resources wasted. When the shock is positive, more people than expected by the agency will have incomes above the cut-off level but each one of them will declare only $T$, thus increasing the number of evaders that go undetected. Thus a category can be systematically mis-targetted by the tax authority: under-audited in good years, over-audited in bad ones.

Can the government do better than following the cut-off rule in this more uncertain scenario? This paper proves that yes, the government can be strictly better-off if it follows an auditing policy contingent on the class-wide average income declaration.

The average declaration introduces a dimension of interaction among the taxpayers usually absent in the literature. Coupled with the fact that a category is composed of fairly homogeneous taxpayers, it improves the agency’s ability to estimate the probability of a given taxpayer being an evader. In particular, it is reasonable to believe that those individuals who declare relatively low are more likely to be evaders than those who declare relatively high, the reason being the above mentioned homogeneity among individuals belonging to the same category: since they share many characteristics, it is only natural to expect they will have fairly similar incomes as well and, consequently, to expect more or less similar income declarations. Any declaration significantly below the class-wide average can then quite confidently be labelled "suspect of potential
evasion". This type of reasoning is presumed (Alm and McKee (2004)) to be behind the method used by the IRS's "Discriminant Index Function" (DIF) to determine which taxpayers are likely to be evaders.

Due to the negative relationship between the "deviation from average declaration" and the "probability of a taxpayer being an evader", the government would be well advised to follow a policy where the auditing intensity among those who declare low increases with the average declaration in the class. This policy introduces an externality among taxpayers and thus transforms the nature of the tax compliance game: each taxpayer has to interact strategically not only with the government -as in most models in the literature-, but also with the other taxpayers in her category. Specifically, when a category experiences a good shock, its taxpayers are forced to participate in a coordination game: if the average declaration is high (meaning that most people comply), the probability of detection is high too and each individual has incentives to comply; if the opposite is true and the average declaration is low (so most people evade), the probability of detection is low as well and each individual has incentives to join the evading majority.

At first sight it might seem counterintuitive to implement a policy where the auditing intensity increases with the average declaration. It may be thought a high auditing intensity when the average declaration is high is nothing but a waste of resources since most people would have declared truthfully. Analogously, a low auditing intensity when the average declaration is low gives rise to the possibility of letting evaders to get away undetected. Certainly one could be inclined to believe that this policy only harms the agency's goal of discouraging evasion. However, the apparent paradox is swiftly eliminated when noted that the auditing intensity only applies to those who declare low (those who declare high are never audited). Indeed, the greater is the average declaration, the greater is the agency's (posterior) belief that the class faces a good shock and, consequently, the greater is its belief that someone declaring low income is an evader. The reasonableness of the optimal policy is thus demonstrated.

This study proves that a policy contingent on the class-wide average declaration is strictly superior to any non-contingent one, including the cut-off policy. Like the latter, the contingent policy is non-increasing in true income. Unlike it, the optimal policy makes the probability of detection among those who declare low an increasing function of the class-wide average declaration. The optimality of the contingent rule, it is important to highlight, is not a consequence of new elements added to the model, but simply an application of the statistical maxim (and the origin of the rational expectations paradigm) that demands an efficient use of all the available information. The end result is a tax authority that takes full advantage of the extra (strategic) uncertainty it generates by following such a rule. Moreover, the policy manipulates the degree of strategic uncertainty by imposing very demanding conditions for coordination or heavy penalties for coordination failures depending on the government's type.

Precisely the government's type (which is its private information and is related to its attitude towards auditing) is the source of the "fundamental" uncertainty which -together with the strategic one generated by the contingent rule-
makes the tax evasion game optimally suited to be modelled as a global game. Thus framed, the (global) tax evasion game reflects more accurately the interaction among its actors than competing models and capture stylized facts usually missing in others, most notably the unique and (usually) interior equilibrium where some people comply while others evade and a tax agency’s comprehensive and efficient use of every piece of available information.

The paper is organized as follows. In section 2 the relevant literature is reviewed. In section 3 the model is introduced and its actors characterized. Section 4 portrays the solution to the taxpayer’s and government’s problems and introduces a government’s objective function based on the accuracy of its targeting policy. Also in section 4 it is proven that the contingent rule is superior to non-contingent ones and comparative statics are shown. Results are discussed in section 5 and conclusions are presented in section ??.

2 Literature review

The paper explores a realm of the tax evasion universe usually neglected by the literature, namely the realm of models which take into account the interaction between taxpayers (See Cowell (1990) and Andreoni et al. (1998) for surveys on the tax compliance literature started by Allingham and Sandmo (1972)).

Those studies that did consider them generally resorted to psychological or social effects embedded in the taxpayer’s utility functions, a school started by Benjamini and Maital (1985). In their seminal article they model an economy where taxpayers face psychological costs for "being different" and hence have incentives to conform to whichever social fashion was in place at the moment of deciding their declarations. This positive externality duly leads to a situation where multiple equilibria are possible: one of them being interior but unstable, the other two being stable corner solutions (one where every taxpayer evades, the other where every one complies). Thus any shock -however small- that perturbs the interior equilibrium unleashes an "epidemic" that will continue until the economy comes to rest at one of the stable equilibria¹. In this paper a similar result (the strategic complementarity between taxpayers’ declarations) is the consequence of the rational behaviour of a cunning tax agency interested in discriminating between compliant and non-compliant taxpayers as explained in section 1. This way unnecessary tinkering of the utility function -frequently and justly accused of accomodating every empirical finding by appropriately adjusted unobservable parameters or variables, is avoided.

¹Gordon (1989) finds stable interior equilibria after some ad hoc manipulation of the expectation formation technology. Myles and Naylor (1996) and Fortin et al. (2004), rely on social norms with utility being an increasing function of conformity. Frey (1992) claims intrinsic (psychological) and extrinsic (enforcement) effects are substitutes and thus more enforcement can "crowd out" intrinsic motivation to comply with taxes without significantly affecting total revenue collection.
The presence of multiple equilibria begs the obvious question about equilibrium selection. Several methods were put forward during the years to cope with this problem (see, for example, Harsanyi and Selten (1988)) but no definitive agreement was reached so far. In the present paper the "global games" approach (Carlsson and van Damme (1993)) is used because it fittingly captures the essential features of the tax compliance game, namely the presence of both fundamental and strategic uncertainty. The fundamental uncertainty stems from the fact that taxpayers do not know the government's type, only its probability distribution. The strategic one arises because taxpayers do not know the actions chosen by the other taxpayers. The technique ensures a unique equilibrium by introducing some heterogeneity among the taxpayers via the presence of private signals about the government's type. Intuitively, the presence of different signals leads to different taxpayers to make different decisions because their expectations over the type of the government will be different as well. Unlike the case without heterogeneity, each taxpayer has now a unique optimal strategy rather than several ones (one for each possible equilibrium), and the equilibrium resulting is (usually) interior and unique.

The government's choice of following a policy rule contingent on the average declaration fundamentally changes the nature of the game, transforming it into a "global" one. This is a contribution to the existing literature on tax evasion—which generally uses the standard "taxpayer v. tax agency" approach—because the rule introduces strategic uncertainty, an element usually absent in most evasion models (the presence of fundamental uncertainty has been analyzed by several studies—see for example Scotchmer and Slemrod (1989), Stella (1991) and Cronshaw and Alm (1995), among others—which usually conclude that more uncertainty is usually good for the government).

The two closest references to this article are Basseto and Phelan (2004) and Alm and McKee (2004). The first analyzes the optimal taxation problem under the assumption that there exist the possibility of "tax riots" (coordination on the "bad equilibrium" from the government's perspective). The focus of the present paper is, however, on the optimal auditing strategy of the government, and so the similarities with Basseto and Phelan (2004) stop here. The second article analyzes in an experiment a coordination game where the government follows an auditing policy contingent on the average declaration in the economy, but this policy is an arbitrary, ad hoc one instead of derived from the rational behaviour of a payoff-maximizing tax agency as is the case here. Further, the authors do not model the government's type, and so their model is just a special case of the one presented in this paper.

In summary, the article fills a gap in the tax evasion literature: one associated with models where the interaction among taxpayers is taken into account without resorting to psychological or social effects. It proves that a rational tax agency interested in distinguishing compliant from noncompliant taxpayers will maximize its payoff-function by implementing a policy contingent on the class-wide average declaration such that high-income taxpayers face a coordination problem with strategic complementarities. This novel way of modelling the tax evasion game is not only appropriate because of its realistic capturing of the
essential features of the game, but also very advantageous as it eliminates the multiple equilibria problem. Finally, the paper introduces an alternative objective function for the tax agency, the "expected loss function", which reflects the quality of its targeting system.

3 Model

3.1 Taxpayers

The focus of this study is on the mechanics of the tax compliance game within a class of taxpayers (also referred to as "agents" or "individuals"), so the population is assumed to be partitioned into several categories or classes, each one of them composed of a large number of taxpayers (normalized to 1 in each class).\(^2\)

All taxpayers within a class are assumed to have the same income. This assumption, though somewhat radical, captures the fact that categories are composed of fairly homogeneous individuals.

Income \(y\) is assumed discrete and restricted to only two values, namely 0 and 1 (after normalization), such that \(y = 0\) with probability \(\gamma \in (0, 1)\). This reflects the existence of common shocks that affect every taxpayer in the class in the same way: with probability \(\gamma\) the class faces a "bad year", with probability \(1 - \gamma\) it faces a good one.\(^3\)

An agent’s realized income is her private information but the income distribution is common knowledge, so each taxpayer knows the income of every other taxpayer. The class’s realized income is, however, not known by the government.

Agents are assumed risk neutral and, consequently, utility maximization becomes for them disposable-income maximization\(^4\). This implies the model is more appropriate for modelling tax evasion by firms than by individuals, though even in the latter case one would expect the qualitative results to hold when the degree of risk aversion is low.

\(^2\)The partition of the population into categories is not modelled into this paper. However, two important issues related to this topic are worth mentioning. First, only variables observable by the government can be used to define classes, and so neither income nor declared income are valid options. Second, some dynamic flavour can be added to the model by using past values of relevant variables as criteria to define categories. See Macho-Stadler and Perez-Castrillo (2002) and Scotchmer (1987), where the partition process is investigated.

\(^3\)In a richer model, taxpayer \(i\)'s income can be decomposed into a common (class-wide) component, \(y \in \{0, 1\}\), and an idiosyncratic one, \(v_i \sim N(0, \sigma^2)\), with \(\sigma^2\) positive and small. Formally,

\[
y_i = y + v_i
\]

As long as the variance of the idiosyncratic shock is small, the qualitative results of the paper remain valid.

\(^4\)Risk aversion complicates the analysis without adding further insight to it. Risk-averse individuals are expected to have more incentives to comply than risk-neutral ones so, if anything, the presence of risk-aversion will only reinforce the results of the article.
The tax system is a standard one, based on self-reports: First, taxpayers submit tax returns stating their incomes and (voluntarily) pay taxes according to their declarations. Then, the government audits some taxpayers and collects fines from everyone found to have evaded taxes.

Hence a taxpayer who is not discovered gets utility

\[ U^n = y_i - td_i \]  

where \( t \in (0, 1) \) is the tax rate and \( d_i \in \{0, 1\} \) is the taxpayer’s declared income \( (d_i \in \{0, 1\} \) because \( y \in \{0, 1\}, \) and so any declaration other than 0 or 1 is clearly untruthful.\(^5\)

On the other hand, a taxpayer who is caught evading taxes gets utility

\[ U^c = y_i - td_i - f_i \]  

where \( f_i \) is the fine an evader has to pay if caught. This fine is defined (following Yitzhaki (1974)) as

\[ f_i := \begin{cases} 
(1 + \varsigma) t (y_i - d_i) & \text{if Agent } i \text{ evades and is caught} \\
0 & \text{otherwise}
\end{cases} \]  

where \( \varsigma \in (0, 1) \) is the surcharge rate to be paid on top of the tax rate on every dollar of evaded tax\(^6\). Thus a high-income taxpayer who declares low gets utility \( U = 1 - (1 + \varsigma) t \) if caught and \( U = 1 \) if not. The respective values for a low-income taxpayer are \( U = 0 \) and \( U = -t.\)\(^7\)

Taxpayers are audited by the government with probability \( p \in [0, 1] \) and it is assumed that an audit always discover the true income of the taxpayer audited.

Hence, a taxpayer’s expected utility is given by

\[ EU = pU^c + (1 - p) U^n \]  

Taxpayers are assumed to know every parameter of the problem (i.e., \( t \) and \( \varsigma \)) except for \( p \), and so they have to estimate the latter as best as they can. An

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\(^5\)The tax system can be easily transformed into a progressive one by using the following change of variables

\[ y := Y - B \]  

where \( Y \in \{B, B + 1\} \) is a taxpayer’s gross income, \( B \) is the exemption level and \( y \) is taxable income. The simplest progressive tax system is

\[ T = \begin{cases} 
 t (Y - B) & \text{if } Y \geq B \\
0 & \text{if } Y < B
\end{cases} \]  

where \( D \in \{B, B + 1\} \) is the taxpayer’s declaration. Thus, if it happens to be a bad year then everyone in the class is exempt, but if it turns out to be a good year then everyone is liable to pay taxes.

\(^6\)The IRS applies rates between 20% (misconduct) and 75% (fraud) (Andreoni, Erard, and Feinstein (1998)), so \( \varsigma \in (0, 1) \) covers the relevant range. It is assumed that \( (1 + \varsigma) t < 1 \), such that the fine if caught evading does not exhaust a high-income person’s income.

\(^7\)The fact that disposable income is negative when a low-income taxpayer declares high is not a problem if the normalization \( y := Y - B, B > t \) is applied (see footnote 5).
agent’s best estimate is given by her conditional expectation over \( p \), given the information available to her at the time of the decision:

\[
E_i(p) := E(p \mid I_i)
\]  

(8)

where \( I_i \) is the taxpayer’s information set.

Hence, taxpayer \( i \) will maximize expected utility of the form\(^8\)

\[
E_i(EU) = E_i(p) U_c + [1 - E_i(p)] U_a
\]

(9)

### 3.2 Government

The objective of the government (also referred to as the "tax agency" or "tax authority") is to minimize the number of errors made when auditing.

An error occurs when either the government audits an "unprofitable" tax return ("error due to zeal", \( Z \)) or when it does not audit a "profitable" one ("error due to negligence", \( N \)). A tax return is profitable if the gain from auditing it (the extra revenue collected from fines) is greater than the cost of the audit. The opposite is true for unprofitable tax returns. Defining \( a \) as the profitability of a agent \( i \)'s tax return, with \( a \) taking the value 1 if the return is profitable and 0 if it is not, then \( a \) is defined as

\[
a_i := \begin{cases} 
1 & \text{if } f_i > c \\
0 & \text{if } f_i < c
\end{cases}
\]

(10)

where \( f_i \) is the fine agent \( i \) has to pay if audited (see equation 6) and \( c \) is the cost of an audit\(^9\). Hence, \( a_i = 0 \) if the shock faced by the class is a negative one and \( a_i = 1 \) if it is a positive one.

\(^8\)The presence of a public good \( G \) does not change the analysis because the amount provided is a function of the total revenue collected by the government and each taxpayer is too small to affect it. This means that an agent cannot influence the amount of public good she receives and so, when deciding how much income to report, only takes into account how her declaration affects her disposable income. That is, the only reason for compliance is the possibility of a fine ("stick"); the "carrot" (the public good) is just not effective: \( G \) is just a positive externality, an unexpected windfall for the taxpayer. Hence, her decisions about how much to contribute continue to be based purely on financial considerations and the probability of being fined. See Cowell and Gordon (1988) for a formal derivation of this result and section A.1 in the appendix for an extension of the model including public goods.

\(^9\)The assumptions of discrete income-levels, risk-neutral taxpayers and 100%-effective audits are all very common in the literature. On discrete income-levels see for example Reinganum and Wilde (1985) and Cronshaw and Alm (1995). On risk neutrality and 100%-effective audits see Graetz et al. (1986). For a study where audits do not always discover true income, see Macho-Stadler and Pérez-Castrillo (1997).

\(^{10}\)It is assumed that

\[
0 < (1 + \zeta) t - c < t
\]

(11)

These inequalities ensure that the net revenue from catching an evader (the expression in the middle) is greater than if she had not been caught (expression on the left) but smaller than if she had declared truthfully (expression on the right). That is, conditional on facing a high-income taxpayer, the agency finds auditing a profitable activity (first inequality), but not as profitable as to be preferred over compliance (second inequality).
An error due to negligence is then defined as

\[ N_i := \begin{cases} 1 & \text{if } a_i = 1 \text{ and agent } i \text{ is not audited} \\ 0 & \text{otherwise} \end{cases} \quad (12) \]

Analogously, an error due to zeal is defined as

\[ Z_i := \begin{cases} 1 & \text{if } a_i = 0 \text{ and agent } i \text{ is audited} \\ 0 & \text{otherwise} \end{cases} \quad (13) \]

These errors clearly have a negative effect on the government’s utility: negligence errors mean that revenue should have been collected but was not, zeal errors that resources were wasted on audits that will not yield any revenue\(^\text{11}\).

It is straightforward to make a connection between these two error concepts and the statistical "Type I" and "Type II" ones. When a tax agency receives a low declaration, four situations can arise depending on the taxpayer’s true income and the government’s auditing decision (see table 14):

- the taxpayer truly has low income and is audited (NE cell): Zeal error
- the taxpayer truly has low income and is not audited (NW cell): No error
- the taxpayer has high income (evades) and is audited (SE cell): No error
- the taxpayer has high income (evades) and is not audited (SW cell): Negligence error

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<table>
<thead>
<tr>
<th>Government’s action</th>
<th>Don’t audit</th>
<th>Audit</th>
</tr>
</thead>
<tbody>
<tr>
<td>True income</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>No error</td>
<td>Zeal</td>
</tr>
<tr>
<td>High</td>
<td>Negligence</td>
<td>No error</td>
</tr>
</tbody>
</table>
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(14)

Thus, if one assumes that the null hypothesis is "true income is high", then Type I errors ("reject when should have accepted") corresponds to Negligence (do not audit a profitable tax return). That is, if the null hypothesis says true income is high, a Type I error requires the government to act as if the taxpayer had a low income. If the government believes the taxpayer’s income is low it will not audit her, the end result being a negligence error. Along similar lines the link between Type II and Zeal errors can be established (and of course, the pairings are exchanged if the null hypothesis is "true income is low").

The agency’s loss function is then defined by

\[ L := (1 - \lambda) Z + \lambda N \quad (15) \]

where \( N \) and \( Z \) are the number of errors due to negligence and zeal respectively, and \( \lambda \) is the weight assigned by the government to negligence: the greater is \( \lambda \), the more concerned the government is about letting evaders get away with

\(^{11}\)Similar concepts are used by Andreoni (1991) in order to determine the optimal fine.
their evasion. \( \lambda \) is the agency’s type and taxpayers do not know its realized value, only its probability distribution (in other words, \( \lambda \) is the agency’s private information).

Defining \( \kappa \) as the proportion of high-income people who evade, the government’s objective function - the "Expected Loss (EL) function" - is defined as

\[
EL = (1 - \lambda) \gamma p + \lambda (1 - \gamma) \kappa (1 - p)
\]

The first term represents the loss due to zeal among low-income taxpayers \((\gamma p\) is the probability of auditing a low-income person and \(1 - \lambda\) is the government’s associated disutility). Analogously, the second term represents the loss due to negligence among high-income individuals.

This is the first time this function is considered as a tax agency’s objective function, and as such represents a contribution to the existing literature. It focuses on the agency’s targeting proficiency, allowing for asymmetric weighting of negligence and zeal errors and, thus, for a taxonomy of agencies according to the relative importance attached to one error vis-à-vis the other.

Graphically, the EL function looks like a saddle. The contour map is shown in figure 1 (the arrows indicate the direction in which EL increases).

Figure 1: Expected loss function: contour curves

The minimum \((EL = 0)\) is achieved when \((p, \kappa) = (0, 0)\). Indeed, this point corresponds to the case where nobody evades and nobody is audited: full compliance and zero cost. The maximum can be at one of two points depending on the value of \(\mu := \frac{\gamma(1-\lambda)}{(1-\gamma)\lambda}\). If it is low (below 1), it is at \((p, \kappa) = (0, 1)\), where negligence reaches its maximum \((EL = \gamma (1 - \lambda))\): every high-income person evades and nobody is audited. If \(\mu\) is high (above 1), it is at \((p, \kappa) = (1, \kappa)\), \(\kappa \in [0, 1]\), where zeal is maximal \((EL = \gamma (1 - \lambda))\): every high-income person evades and everybody is audited. Note in particular that \(\mu\) is low when the government assigns a high weight to errors due to negligence \((\lambda\) close to 1), so it is not surprising to find that the maximum \(EL\) in that case corresponds to
the maximum negligence. Along the same lines, when $\mu$ is high, then $\lambda$ is low and maximum $EL$ and maximum zeal coincide$^{12}$.

The policy variable used by the tax agency to minimize its expected loss is the auditing intensity $p$. It is assumed to be composed of two elements: one publicly known, the other based on the agency’s type and thus not known by the taxpayers. This realistically describes taxpayers’ uncertainty about the true probability of detection they face. Formally

$$p = T + f(\lambda)$$

where $T$ is the publicly known component and $f(\lambda)$ the uncertain one. It is important to note that $f(\lambda)$ is an increasing function of $\lambda$. Indeed, a government very concerned with negligence errors (high $\lambda$) will be willing to audit more people than another one very concerned with zeal errors (low $\lambda$) ceteris paribus, and so a larger $\lambda$ leads to a larger $p$. Defining

$$\theta := f(\lambda)$$

equation 17 simplifies to

$$p = T + \theta$$

where $\theta$ is assumed to be uniformly distributed on the real line.

$\theta$ can therefore be considered as a measure of the agency’s auditing "toughness": an agency with a larger $\theta$ is "tougher" than another with a lower one, meaning that the former will audit more people than the latter. Note however that equation 19 allows other interpretations beyond the one related to the government’s concern with negligence errors $\lambda$. Indeed, $\theta$ can reflect other types of governmental asymmetric information, and so can be taken to mean, among other possibilities, the government’s efficiency, its level of corruption or the effort it exerts in auditing taxpayers$^{13}$.

$^{12}$Clearly, alternative objective functions can be used. Though several measures of social welfare (including equity as well as efficiency considerations) are possible candidates, the strongest contender is the government’s expected revenue net of enforcement costs (see, for example, Scotchmer (1987) and Reinganum and Wilde (1986), among many others). The results, however, are not significantly different if the latter function is used (see section ?? in the appendix).

Note, however, that the loss function does consider elements related to expected revenue ($N$ counts the cases where a profitable audit is not undertaken, $Z$ the cases where an unprofitable audit is undertaken).

Also, it is worth mentioning, the loss function can be assumed to be the objective function of a net-revenue- (or even profit-) maximizing tax agency after taking into account the incentive scheme provided by the central government, as in Melumad and Mookherjee (1989).

$^{13}$The latter is a particularly interesting interpretation. According to it, $p$ could be considered to be the "effective" probability of detection and $T$ the "nominal" one, with the difference between the two being the effort $\theta$ exerted by the government. Thus, if the agency exerts "below par" effort (i.e., if $\theta < 0$), the effective probability is lower than the nominal one (the opposite is true if $\theta > 0$). Of course, in this case $\theta$ becomes another policy variable from the point of view of the government: the source of the informational asymmetry is due to hidden action instead of hidden information. The hidden action possibility, however, is not considered in this paper and the analysis will proceed under the assumption of hidden information. For a study (unrelated to tax compliance) where the asymmetry is due to moral hazard see Edmond (2003).
As argued in section 1, the government will be interested in using the extra information embodied in the average declaration when choosing its optimal policy. Moreover, the greater is the class’ average declaration \( \bar{d} \), the more low-declaring agents it will audit: a greater \( \bar{d} \) means that a good shock is more likely to have taken place, and so the agency will be well advised to audit more intensively among those who declare low income as they are more likely to be evading taxes.

For simplicity, it is assumed the policy rule followed by the agency is a step-function of the following form:

\[
p = \begin{cases} 
  p_1 & \text{if } \bar{d} \geq \hat{d} \\
  p_2 & \text{if } \bar{d} < \hat{d}
\end{cases}
\]  

(20)

where \( \hat{d} \in [0, 1] \) is a publicly known constant and \( p_1 \leq p_2 \), such that the positive relationship between average declaration and auditing intensity holds.

It is worth noticing that the expected declaration \( \bar{d} \) is negatively related to the proportion of evaders \( \kappa \). In effect, the larger the fraction of taxpayers who evade, the lower the average declaration. For the present analysis, and to make the link with the global games literature a more transparent one, the variable of interest is going to be \( \kappa \) rather than \( \bar{d} \), and so the policy rule develops into the following expression

\[
p = \begin{cases} 
  p_1 & \text{if } \kappa \geq \hat{\kappa} \\
  p_2 & \text{if } \kappa < \hat{\kappa}
\end{cases}
\]  

(21)

where \( \hat{\kappa} := 1 - \frac{\bar{d}}{1 - \gamma} \) is the new threshold, now in terms of \( \kappa \).

![Figure 2: Policy rule](image)

From equation 19, \( p_1 \) and \( p_2 \) can be re-written as

\[
p_1 = T_1 + \theta \\
p_2 = T_2 + \theta
\]  

(22)

(23)
and hence the policy rule becomes

\[ p = \begin{cases} 
0 & \text{if } T_1 + \theta < 0 \\
T_1 + \theta & \text{if } 0 \leq T_1 + \theta \leq 1 \text{ and } \kappa \geq \hat{\kappa} \\
T_2 + \theta & \text{if } 0 \leq T_2 + \theta \leq 1 \text{ and } \kappa < \hat{\kappa} \\
1 & \text{if } 1 < T_2 + \theta 
\end{cases} \quad (24) \]

The government’s original policy variable \( p \) splinters now into three different ones, namely \( T_1, T_2 \) and \( \hat{\kappa} \), which are related to the two probabilities of detection and the critical level of evasion that triggers the switch from one probability to the other.

Note that the expression in equation 24 does not remove the cut-off rule from the government’s set of available policies. The cut-off rule corresponds to the case where the policy does not depend on the proportion of evaders \( \kappa \) (or, equivalently, the average declaration \( \bar{d} \)). If it is chosen, nobody who declares high income is audited, while those who declare low are audited with probability \( p \), independently of the value of \( \kappa \). Formally this occurs when either \( T_1 = T_2 \) (such that \( p_1 = p_2 \)) or \( \hat{\kappa} = 0 \) (such that only \( p_2 \) matters). Any other combination of the policy variables leads to a contingent rule where the auditing intensity changes with the proportion of evaders among high-income people (i.e., \( p_1 \neq p_2 \) and \( \hat{\kappa} > 0 \)).

It is important to draw attention to the fact that the policy does not require the government’s acquisition of more information beyond the one in the tax returns, only using all the information the provide: the decision about whether to audit or not a given taxpayer should be based not only on what the taxpayer declared, but on the entire pool of tax returns received by the agency. In a two-income economy like the one presented here, all the extra information available is condensed into just one "sufficient statistics"-the average declaration \( \bar{d} \) or the proportion of evaders \( \kappa \)- which can hence be used to parameterize the whole distribution of declarations. Ultimately, taking into consideration the average declaration is nothing but an application of the statistical (and rational expectations) maxim: "use all the information available".

This policy requires, implicitly, the agency’s ability to commit to it. This follows a long tradition in the literature (from Allingham and Sandmo (1972) to Reinganum and Wilde (1985) and many others).

The need for commitment arises from the time-inconsistency problem faced by the government: its incentives change over time, making the agency unwilling to follow the original plan it itself devised. In the tax compliance game, the agency faces different incentives before and after tax returns are submitted: ex-ante, its concern with compliance would lead it to announce a tough policy intended to deter evasion; ex-post, its concern with efficiency will lead it to audit only if the benefit of doing it outweighs the respective costs. This schizophrenic behaviour on the agency’s part is, however, expected by rational taxpayers who, consequently, do not believe the agency’s announcement and engage in evasion. As Melumad and Mookherjee (1989) put it:

"(...) in any policy achieving the full commitment welfare level, no
... fines are collected in equilibrium (...). Consequently, audits do not generate any revenue (...). Their only benefit is that they deter misreporting. But if taxpayers have already reported their income, audits would represent \textit{ex post} deadweight losses. The government would thus not audit any taxpayer at all, and anticipating this, taxpayers would misreport." (italics as in the original)


From among the gamut of solutions put forward to cope with this problem, this paper employs the one suggested by Kydland and Prescott (1977), namely, following a rule.

Though the rule is, by definition, just another announcement of the government, it is easy to monitor by the general public and hence can force an agency concerned about its reputation to follow it. A necessary condition for this method to succeed is that the policy must be contingent on aggregate variables publicly observable, a condition which is indeed satisfied in the present model, since the policy is contingent on the average declaration \( \bar{d} \), which is deterministically related to the usually publicly known tax revenue collected by the government \( t_{1d} \).

Empirically, following a rule seems to be a standard practice among tax agencies worldwide. It is hence not surprising to find that agencies apply different formulae in order to determine the returns to be audited. Without a doubt, the most famous one is the "Discriminant Index Function" (DIF) used by the IRS in the US. Interestingly, it is presumed the relative position of a declaration with respect to the average is an important ingredient of the DIF’s algorithm:

"(...) a taxpayer’s probability of audit is based not only upon his or her reporting choices, but also upon these choices relative to other taxpayers in the cohort. In short, there is a taxpayer-taxpayer game that determines each individual’s chances of audit selection."


Thus, the assumption of a government or tax agency willing to implement—or at least considering the possibility of implementing—an auditing policy contingent on economy-wide variables seems to be empirically justified\(^\text{15}\).

\(^{14}\)Alternatively, following Melumad and Mookherjee (1989), the government can solve the problem by delegating the task of collecting taxes to an agency and offering it an appropriately designed incentive scheme. This method also requires public observability of aggregate variables.

\(^{15}\)Similar arguments are put forward in Young et al. (2001): "The computer program flags any tax return that reports income or takes deductions outside the normal ranges established by the DIF". Young et al. (2001), p. 209.
4 Solving the model

As argued in section 3.2, by following a policy rule the government is assumed able to commit to it, and hence the timing of the game is as follows:

- Players’ private information is realized: the government’s type \( \theta \) and taxpayers’ incomes \( y \) and signals \( s_i \).
- Government chooses its policy variables \((T_1, T_2, \hat{\kappa})\) and announces them.
- Taxpayers choose their income declarations \( d_i \) and voluntarily pay taxes \( t d_i \).
- Audits are undertaken according to the announced policy rule. Fines (if any) are collected by the agency.

Given the timing, the model will be solved by backwards induction, starting with the second-stage game (taxpayer problem) and then moving to the first-stage one (government’s problem).

4.1 Taxpayer problem

Agent \( i \)'s problem consists of choosing how much income to declare in her tax return in order to maximize her expected utility\(^{16}\).

For low-income individuals the analysis is trivial: since there is no reward for over-declaring (see equation 6), they always declare truthfully.

The interesting case is the one where agents have high income. In this scenario, and using equations 9 and 21, each one of them faces a payoff matrix as the one shown in Table 25.

\[
\begin{array}{c|cc}
   & \kappa \geq \hat{\kappa} & \kappa < \hat{\kappa} \\
\hline
Evade & 1 - p_1 (1 + \varsigma) t & 1 - p_2 (1 + \varsigma) t \\
Comply & 1 - t & 1 - t \\
\end{array}
\]

There are three interesting cases to consider:

1. If \( 0 \leq p_1 \leq p_2 \leq \frac{1}{1+\varsigma} \), then Evade is the dominant strategy for every taxpayer. From equations 22 and 23 this condition can be re-written as \( \theta \leq \frac{1}{1+\varsigma} - T_2 \leq \frac{1}{1+\varsigma} - T_1 \). That is, when the government is sufficiently "soft" (so \( \theta \) is sufficiently low) taxpayers always evade.

2. If \( \frac{1}{1+\varsigma} \leq p_1 \leq p_2 \leq 1 \), then Comply is the dominant strategy for every taxpayer. The equivalent condition in terms of \( T \) and \( \varsigma \) is now \( \frac{1}{1+\varsigma} - T_2 \leq \frac{1}{1+\varsigma} - T_1 \leq \theta \). That is, when the government is sufficiently "tough" (so \( \theta \) is sufficiently high) taxpayers always comply.

\(^{16}\)It is assumed, as is standard in the literature, that every taxpayer files a tax return. For an analysis where taxpayers can choose not to file one, see chapter 5 in Cowell (1990).
3. If $0 \leq p_1 \leq \frac{1}{p_1} \leq p_2 \leq 1$, then there is no dominant strategy: both can be optimal, depending on other taxpayers’ declarations. The corresponding condition is $\frac{1}{p_1} - T_2 \leq \theta \leq \frac{1}{p_2} - T_1$. That is, when the government is neither too tough nor too soft (so $\theta$ is neither too high nor too low) taxpayers are uncertain about which action (Evade or Comply) to take: the optimal decision depends on the proportion of individuals who evade.

The latter case is the interesting one and the one that can be analyzed as a global game. The policy rule followed by the government (equation 21) leads to a situation where taxpayers’ actions (declarations) display strategic complementarities, i.e., a taxpayer’s optimal declaration is an increasing function of the average declaration $\bar{d}$ (or, equivalently, a decreasing function of the proportion of evaders $\kappa$). Thus, a cunning tax authority following such a contingent rule transforms the nature of the tax compliance game: it generates a coordination game among high-income taxpayers that is not present in standard tax evasion models.

The rationale for the agency behaving this way is simple. The average declaration is an informative signal about the shock faced by the class: the greater the average declaration, the more likely the class is experiencing a good shock. But a good shock means high incomes for those in the class, and so those who declare very low (compared to the average) are very likely to be evaders. Thus, the government is well advised to audit those who declare low income more frequently the greater is the class’ average declaration $\bar{d}$, because the likelihood of their being evaders increases with $\bar{d}$.

The coordination game generated by the policy rule becomes a global game when the "fundamental" uncertainty about the agency’s type $\theta$ is incorporated to the model. This uncertainty plays a crucial role in eliminating the archetypal multiplicity of equilibria of the coordination game because it permits the emergence of some heterogeneity among otherwise homogeneous taxpayers. The heterogeneity arises from the different information different individuals have about the government’s type and is modelled here through the presence of (noisy) private signals only known by the taxpayer receiving them (one per agent). These signals are assumed unbiased and, consequently, correct on average. Formally, the signal received by agent $i$, $s_i$, is given by

$$s_i = \theta + \varepsilon_i$$

where $\theta$ is the government’s type and $\varepsilon_i$ is a white-noise random variable uniformly distributed over the interval $[-\varepsilon, \varepsilon]$, $\varepsilon$ is a small positive number and $E(\varepsilon_i, \varepsilon_j) = 0 \forall i \neq j$ (the probability distributions of $\theta$ and $\varepsilon_i$ are common knowledge among taxpayers).\(^{17}\)

\(^{17}\)The assumptions about the distributions of $\theta$ and $\varepsilon$ are not restrictive. Though they were chosen for their tractability (indeed, they allow for closed form solutions), Morris and Shin (2002a) show that results using them are a good approximation to the shape of the strategic uncertainty when the variation of $\varepsilon_i$ (the signal’s noise), is small relative to the variation of $\theta$ (the type of the agency), as is the case in this article.
The signals can be interpreted as, for example, the opinions about the government agent \( i \) hears from friends and colleagues (i.e., word of mouth). Alternatively, one can think that the signal is just one and the same for everyone, but every person has different processing abilities, thus leading to different idiosyncratic perceptions for different individuals\(^{18}\).

Thus, though taxpayers do know the values of the government’s policy variables \( T_1, T_2 \) and \( \kappa \) (since they were previously announced by the government -see the timing of the game in section 4), taxpayers’ decisions require estimating the government’s type \( \theta \) and the proportion of evaders \( \kappa \). The first of the two unknowns, \( \theta \), is the source of taxpayers’ *fundamental* uncertainty and reflects their ignorance about the type of government (and hence policies) they face. The second one, \( \kappa \), is the origin of taxpayers’ *strategic* uncertainty and reflects their ignorance about other taxpayers’ actions. Since the presence of these two kinds of uncertainty is the defining characteristic of a global game, analyzing the tax evasion game as one of them is the logical way to proceed.

From equations 8, 22, 23 and 26, agent \( i \)’s expectation over \( p_k, k \in \{1,2\} \) is

\[
E_i(p_k) = T_k + s_i
\]

and so the payoff matrix becomes

<table>
<thead>
<tr>
<th></th>
<th>( \kappa \geq \hat{\kappa} )</th>
<th>( \kappa &lt; \hat{\kappa} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evade</td>
<td>( 1 - E_i(p_1) (1 + \varsigma) t )</td>
<td>( 1 - E_i(p_2) (1 + \varsigma) t )</td>
</tr>
<tr>
<td>Comply</td>
<td>( 1 - t )</td>
<td>( 1 - t )</td>
</tr>
</tbody>
</table>

(28)

Since agents only have two choices (Evade or Comply), their optimal strategy will be of the following type

\[
\text{Optimal strategy } = \begin{cases} 
\text{Evade} & \text{if } s_i < s^* \\
\text{Indifferent} & \text{if } s_i = s^* \\
\text{Comply} & \text{if } s_i > s^* 
\end{cases}
\]

where \( s^* \) is the "switching point", the threshold such that if the signal received by the taxpayer is below (above) it, she will expect the government to be rather soft (tough) and so she will evade (comply).

Following Morris and Shin (2002a), it can be proven that the probability of the proportion of evaders \( \kappa \) being not greater than the threshold \( \hat{\kappa} \) equals \( \hat{\kappa} \) itself. Formally,

\[
\text{prob} (\kappa < \hat{\kappa}) = \hat{\kappa}
\]

(30)

The authors claim this probability can be used as a measure of the degree of strategic uncertainty in the game. Indeed, as \( \hat{\kappa} \) increases, coordinating on the Evade equilibrium becomes more difficult because a larger proportion of evaders is needed for the coordination to succeed.

\[^{18}\text{It is straightforward to extend the model to incorporate public signals as well as private ones. For an analysis with both types of signals, see Morris and Shin (2002b).}\]
From equation 29 it is clear that people who receive a signal exactly equal to the threshold level \(s_i = s^*\) are indifferent between evading and complying. That is

\[ EU_i (Evade) = EU_i (Comply) \]  

(31)

Using the payoffs from table 28, equation 30 and the indifference condition in equation 31, the threshold is found to be

\[ s^* = \frac{1}{1 + \zeta} - [(1 - \hat{\kappa}) T_1 + \hat{\kappa} T_2] \]  

(32)

The switching point, it is straightforward to prove it, is a decreasing function of the surcharge rate \(\zeta\) and the policy variables \(T_1, T_2\) and \(\hat{\kappa}\).

A greater surcharge rate means a more severe penalty per dollar evaded and hence more incentives for taxpayers to comply. This increased desire to comply leads taxpayers to declare high for a larger set of possible signals, which is equivalent to say that the threshold \(s^*\) decreases. Formally, for some people,

\[ s'' < s_i < s^* \]  

(33)

that is, their signal \(s_i\) is smaller than the original threshold \(s^*\) but greater than the new one \(s''\) \((s'' < s^*)\) and so they would have evaded before the increase in the surcharge rate but will comply after it.

It is worth mentioning that, since the declaration space is restricted to \(\{0, 1\}\), the only way to adjust one’s optimal strategy is to change the switching point. Thus, any parametric change that in a continuous-declaration setting would lead to a lower declaration, here will usually lead to a higher switching point (ie, the agent will declare low for a larger range of signals than before).

Along these lines, greater \(T_1\) or \(T_2\) imply greater probabilities of detection \(p_1\) or \(p_2\), which decrease taxpayers’ incentives to evade and so (as stated above) lead to a lower threshold. A greater \(\hat{\kappa}\), similarly, implies that a larger proportion of evaders is needed for the low probability \(p_1\) to be chosen by the government, making coordination on the \(Evade\) equilibrium more difficult to be achieved and thus decreasing the incentives to evade (and hence \(s^*\)).

### 4.2 Government problem

The government’s problem consists of choosing its policy variables \(T_1, T_2\) and \(\hat{\kappa}\) in order to minimize its expected loss. The analysis is however greatly simplified by carrying out a change of variables such that the new policy variables become \(p_1, p_2\) and \(\kappa^{19}\).

\[ ^{19} \text{The definitions of } p_1 \text{ and } p_2 \text{ are immediate from equations 22 and 23.} \]

In order to get \(\kappa\) as a function of \(\hat{\kappa}\), it is important to notice that the proportion of evaders \(\kappa\) is nothing but the proportion of people who got a signal below the threshold \(s^*\) (see equation 29). Noting that the signals are uniformly distributed around \(\theta\) over the interval \([\theta - \varepsilon, \theta + \varepsilon]\)

18
The programme faced by the government can then be written as

$$\min_{\{p_1, p_2, \kappa\}} EL = \gamma p (1 - \lambda) + (1 - \gamma) \kappa (1 - p) \lambda$$

subject to

$$\begin{cases} 0 \leq p_1 \leq \frac{1}{1-\kappa} \\ \frac{1}{1-\kappa} \leq p_2 \leq 1 \\ 0 \leq \kappa \leq 1 \\ 0 \leq \hat{\kappa} \leq 1 \\ \kappa = \frac{\varepsilon + \frac{1}{1-\kappa} - [(1-\hat{\kappa})p_1 + \hat{\kappa}p_2]}{2\varepsilon} \end{cases} \quad (36)$$

where

$$p = \begin{cases} p_1 & \text{if } \kappa \geq \hat{\kappa} \\ p_2 & \text{if } \kappa < \hat{\kappa} \end{cases} \quad (37)$$

Most of the constraints in equation 36 simply reflect the fact that the policy variables are either probabilities ($p_1$ and $p_2$) or proportions ($\kappa$ and $\hat{\kappa}$), and so are defined over the interval $[0, 1]$. The conditions $p_1 \leq \frac{1}{1-\kappa}$ and $\frac{1}{1-\kappa} \leq p_2$ are, on the other hand, the consequences of the coordination game created by the government (see section 4.1). Finally, the last condition is the change of variable from $\hat{\kappa}$ to $\kappa$ mentioned before (see footnote 19).

It is important to notice that another constraint must be taken into account on top of the five ones in equation 36. Indeed, equation 37 (which is simply the policy rule) imposes an extra constraint: either $\kappa \geq \hat{\kappa}$ (and so $p = p_1$ in the expected loss function) or $\kappa < \hat{\kappa}$ (and $p = p_2$).

Using equation 35 the problem can be re-expressed as follows

$$\min_{\{p_1, p_2, \kappa\}} EL = \gamma p (1 - \lambda) + (1 - \gamma) \kappa (1 - p) \lambda$$

subject to

$$\begin{cases} 0 \leq p_1 \leq P \\ P \leq p_2 \leq 1 \\ 0 \leq \kappa \leq 1 \\ L (p_2) \leq \kappa \leq H (p_1) \end{cases} \quad (38)$$

where

$$p = \begin{cases} p_1 & \text{if } \kappa \geq K (p_1, p_2) \\ p_2 & \text{if } \kappa < K (p_1, p_2) \end{cases} \quad (39)$$

(from equation 26), the probability of someone getting a low signal is given by

$$\kappa = \frac{s^* - \theta + \varepsilon}{2\varepsilon} \quad (34)$$

Plugging $s^*$ from equation 32 into equation 34 gives $\kappa$ as a function of $\hat{\kappa}$, namely

$$\kappa = \frac{\varepsilon + \frac{1}{1-\kappa} - [(1-\hat{\kappa})p_1 + \hat{\kappa}p_2]}{2\varepsilon} \quad (35)$$
and

\[
L(p_2) : = \frac{\varepsilon + P - p_2}{2\varepsilon} \quad (40)
\]

\[
K(p_1, p_2) : = \frac{\varepsilon + P - p_1}{2\varepsilon + p_2 - p_1} \quad (41)
\]

\[
H(p_1) : = \frac{\varepsilon + P - p_1}{2\varepsilon} \quad (42)
\]

The first two constraints are changed by the introduction of \( P \), defined as

\[ P := \frac{1}{1+\xi} \quad (43) \]

\( P \) can then be interpreted as the full-deterrence auditing intensity, i.e., the auditing intensity that eliminates evasion.

The third constraint is unaltered while the fourth is the equivalent of \( 0 \leq \hat{\kappa} \leq 1 \) but in terms of \( \kappa \). It is important to notice however that \( \hat{\kappa} = 0 \) when \( \kappa = H(p_1) \) and \( \hat{\kappa} = 1 \) when \( \kappa = L(p_2) \).

The policy rule is also affected, as shown in equation 39.

Collectively, these constraints determine an "attainable set" (also referred to as "policy set") such that every combination of policy variables which belong to such set is implementable, and so can be chosen by the government. Graphically, the policy set can take two forms, depending on whether \( \kappa \geq \hat{\kappa} \) or \( \kappa < \hat{\kappa} \), as shown in figure 3.\(^{20}\)

\[\text{Figure 3: Attainable set}\]

It is important to notice that the \( \kappa = K \) curve depends on both \( p_1 \) and \( p_2 \). This means that the curve will have different positions depending on the values

\[\text{From the formulae of } L, K \text{ and } H \text{ (equations 40, 41 and 42) it can be proven that}\]

\[
\max \{0, L\} \leq K \leq \min \{H, 1\} \quad (44)
\]

\[\text{From the formulae of } L, K \text{ and } H \text{ (equations 40, 41 and 42) it can be proven that}\]

\[
\max \{0, L\} \leq K \leq \min \{H, 1\} \quad (44)
\]

\[\text{From the formulae of } L, K \text{ and } H \text{ (equations 40, 41 and 42) it can be proven that}\]

\[
\max \{0, L\} \leq K \leq \min \{H, 1\} \quad (44)
\]
of these two policy variables. In particular, it is relevant to consider the cases where \( \kappa \geq \hat{\kappa} \) and \( p_2 = P \) (panel (a) of figure 4) and where \( \kappa < \hat{\kappa} \) and \( p_1 = P \) (panel (b) of figure 4).

From the EL contour map (figure 1) and the policy set as defined by the constraints (figure 3), it is clear that no solution can lie within the attainable set, but only on its boundaries. Moreover, except for very special cases (e.g., if \( \lambda = 1 \) or \( \gamma = 0 \) - see panel (a) in section A.3 of the appendix), all possible equilibria will lie at points where the EL contours are downward-sloping. This means that solutions can take place only on either the \( \kappa = K \) curve (when \( \kappa \geq \hat{\kappa} \) and, hence, \( \kappa^* = \hat{\kappa} \)) or the \( \kappa = L \) line (when \( \kappa < \hat{\kappa} \) and, hence, \( \hat{\kappa} = 1 \))\(^{21}\). It also means that the upper bounds of the policy sets can be safely ignored when solving the government problem.

The previous analysis can be formalized by virtue of the following proposition:

**Proposition 1** Except for very particular cases, solutions can only lie on the lower boundaries of the policy set.

**Proof.** From equation 16, the partial derivative of the expected loss with respect to \( \kappa \) is

\[
\frac{\partial EL}{\partial \kappa} = \lambda (1 - \gamma) (1 - p)
\]  
  (45)

which is positive except for very particular cases (namely, \( \lambda = 0 \), \( \gamma = 1 \) and \( p = 1 \)). This means that, for every point \( \pi = (p, \kappa) \) that lies strictly within the policy set, there is another point \( \pi' = (p, \kappa') \) which also belongs to the policy set such that \( EL(\pi) > EL(\pi') \). Thus, the policies chosen by an EL-minimizer

\(^{21}\)Note that the agency will set \( p_2 = 1 \) if \( \kappa \geq \hat{\kappa} \). The largest is \( p_2 \), the lower is the \( \kappa = K \) curve, and so the larger the attainable set (at least in the south-west direction). The government prefers as large an attainable set as possible, as it can reach lower contours. Hence, \( p_2^* = 1 \) is optimal when \( \kappa \geq \hat{\kappa} \).
agency can only lie on either the $\kappa = K$ curve (if $\kappa \geq \hat{\kappa}$) or the $\kappa = L$ line (if $\kappa < \hat{\kappa}$). That is, only the lower boundaries of the policy set matter.

An immediate consequence of proposition 1 is the corollary presented below:

**Corollary 2** If $\kappa \geq \hat{\kappa}$ the optimal value of $p_2$ is $p_2^* = 1$.

**Proof.** The greater is $p_2$, the lower lies the $\kappa = K$ curve, and so the attainable set expands southwards. This expansion allows the government (using proposition 1) to reach lower $EL$ contours, and thus it will set $p_2$ as high as possible, i.e., $p_2^* = 1$.

Having narrowed down the set of possible solutions to those on $\kappa = K$ and $\kappa = L$, the main result of this article is summarized in the following proposition:

**Proposition 3** Regardless of its type, a government will always strictly prefer a contingent rule over a non-contingent (cut-off) one.

**Proof.** The existence of - at least - one optimum is immediate from the fact that the policy variables $p_1$, $p_2$ and $\kappa$ are bounded. Hence, finding that no solution is consistent with a non-contingent rule is sufficient to prove the proposition.

A non-contingent rule requires either $p_1 = p_2$ or $\kappa = H$ (or both). The first option is obvious: if the probabilities are the same below and above $\hat{\kappa}$, the rule does not depend on the number of evaders. The second one is equivalent to having $\hat{\kappa} = 0$, and again leads to a non-contingent rule, since the value of $p_2$ is irrelevant (it only applies to cases where $\hat{\kappa} < 0$), and $p = p_1$ for every $\kappa \in [0, 1]$.

The $\kappa = H$ scenario is ruled out by proposition 1. Hence, it is never optimal for the government to choose a policy with $\hat{\kappa} = 0$.

Consider now the alternative, that is, $p_1 = p_2$. Strategic complementarity requires $p_1 \leq P \leq p_2$, so $p_1$ is equal to $p_2$ if and only if both $p_1 = P$ and $p_2 = P$.

The proof will be by *reductio ad absurdum*. Assume $p_1 = P$ and $p_2 = P$.

The attainable sets are shown in figure 4.

The government will choose $p_1 = P$ only if the slope of the $EL$ contour curve at $(p, \kappa) = (P, \frac{1}{2})$ is not smaller than the slope of the $\kappa = K$ line at the same point. Formally,

$$\frac{\partial \kappa}{\partial p_1}\bigg|_{EL=ct.} \geq \frac{\partial \kappa}{\partial p_1}\bigg|_{\kappa=K}$$

(46)

both evaluated at $(p, \kappa) = (P, \frac{1}{2})$ and taking into account that $p_2 = P$. The expression simplifies to:

$$\frac{\partial \kappa}{\partial p_1}\bigg|_{EL=ct.} \geq - \frac{1}{4\varepsilon}$$

(50)

---

22 Note that $\kappa = K$ is a downward sloping curve. Hence, if $\mu$ is sufficiently low such that the contour curve at $(P, \frac{1}{2})$ is upward sloping, the constraint is satisfied. However, if $\mu$ is so high that the contour curve at $(P, \frac{1}{2})$ is downward sloping, the constraint requires the $\kappa = K$ curve to be shallower than the contour curve at $(P, \frac{1}{2})$.

23 From the definition of $K$ in equation 41,

$$\frac{\partial \kappa}{\partial p_1}\bigg|_{\kappa=K} = \frac{P - \varepsilon - p_2}{(2\varepsilon + p_2 - p_1)^2}$$

(47)
Analogously, the government will choose $p_2 = P$ only if the slope of the EL contour curve at $(p, \kappa) = (P, \frac{1}{2})$ is not greater than the slope of the $\kappa = L$ line at the same point. Formally,

$$\frac{\partial \kappa}{\partial p_2} \bigg|_{EL=ct.} \leq \frac{\partial \kappa}{\partial p_2} \bigg|_{\kappa=L}$$

both evaluated at $(p, \kappa) = (P, \frac{1}{2})$ and taking into account that $p_1 = P$. Using equation 40 the expression simplifies to

$$\frac{\partial \kappa}{\partial p_1} \bigg|_{EL=ct.} \leq -\frac{1}{2\varepsilon}$$

(52)

From equation 16,

$$\frac{\partial \kappa}{\partial p_i} \bigg|_{EL=ct.} = -\frac{\partial \kappa}{\partial \kappa} \bigg|_{EL=ct.} = -\frac{\mu - \kappa}{1 - \kappa} \quad \text{for every} \quad i \in \{0, 1\}$$

(53)

Hence, if both $\frac{\partial \kappa}{\partial p_1} \bigg|_{EL=ct.}$ and $\frac{\partial \kappa}{\partial p_2} \bigg|_{EL=ct.}$ are evaluated at $(p, \kappa) = (P, \frac{1}{2})$, then

$$\frac{\partial \kappa}{\partial p_1} \bigg|_{EL=ct.} = \frac{\partial \kappa}{\partial p_2} \bigg|_{EL=ct.} = -\frac{\mu - \frac{1}{2}}{1 - \frac{1}{2}}$$

(54)

So, using these results, we can find that the solution $(p_1, p_2, \kappa) = (P, P, \frac{1}{2})$ requires

$$-\frac{1}{4\varepsilon} \leq \frac{\partial \kappa}{\partial p_1} \bigg|_{EL=ct.} = \frac{\partial \kappa}{\partial p_2} \bigg|_{EL=ct.} \leq -\frac{1}{2\varepsilon}$$

(55)

But

$$-\frac{1}{4\varepsilon} > -\frac{1}{2\varepsilon}$$

(56)

and so equation 55 is contradictory. Consequently, the assumption that $p_1 = P$ and $p_2 = P$ is incorrect. By *reductio ad absurdum*, it must be the case that $p_1 \neq p_2$.

Hence, no policy with $\kappa = 0$ and/or $p_1 = p_2$ is optimal from the perspective of the government. As a consequence, the optimal policy is a *contingent* one,

Since $p_2 = P$ by assumption, it simplifies to

$$\frac{\partial \kappa}{\partial p_1} \bigg|_{\kappa=\kappa} = -\frac{\varepsilon}{(2\varepsilon + P - p_1)^2}$$

(48)

Evaluating it at $(p_1, \kappa) = (P, \frac{1}{2})$, it becomes

$$\frac{\partial \kappa}{\partial p_1} \bigg|_{\kappa=\kappa} = -\frac{1}{4\varepsilon}$$

(49)

Since the contour curve and the $\kappa = L$ are both downward-sloping, the constraint requires the contour curve to be steeper than $\kappa = L$ at $(P, \frac{1}{2})$. If the opposite were true, the government would choose $p_2 > P$ on the $\kappa = L$ line.

Note that only downward-sloping contour curves can lead to an optimum at $(P, \frac{1}{2})$. If they were upward-sloping, the government would prefer $p_2 > P$ (as $\kappa = 0$ and/or $p_2 = 1$).
where the probability of detection depends on the proportion of evaders in the population.

The proposition highlights the importance of taking into account the interaction among taxpayers, a factor usually absent in the tax compliance literature. It proves that a rational agency will take advantage of the signalling power of the average declaration as an indicator of the shock faced by the taxpayers’ class and will use it for its own benefit by devising an auditing policy that creates a coordination game among taxpayers. As a consequence, it is clearly shown that contingent policies are strictly superior to non-contingent ones (like the cut-off rule) and so proposition 3 constitutes a novel contribution to the existing literature on optimal auditing. The gain from discarding the non-contingent rule and adopting the contingent one can be see graphically in figure 5, where points A and B represent the solutions when the non-contingent and contingent rules are used, respectively. The gain is the decrease in the expected loss from $EL_1$ (at point A with a non-contingent rule) to $EL_0$ (at point B with a contingent one).

\[ \begin{align*}
(a): & \quad \mu \text{ low} \\
(b): & \quad \mu \text{ high}
\end{align*} \]

Figure 5: Contingent v. Non-contingent rule

Note, however, that the compliance level with the contingent rule can be greater or smaller than with the non-contingent one, depending on the type of the government: if $\lambda$ is high -and so $\mu$ is low, see panel (a))- the government is very concerned with negligence and so sets a high auditing intensity and taxpayers react by increasing compliance, i.e., $\kappa$ is low. The opposite is true if $\lambda$ is low.

The following proposition further characterizes the equilibrium solution and sheds light on the mechanisms available to the government.

**Proposition 4** The government’s optimal policy makes coordination on the Evade equilibrium more difficult to be achieved by imposing either
1. extremely demanding conditions for coordination, or

2. penalties of maximal severity in the case of coordination failure.

Proof. Part 2 of the proposition follows immediately from footnote 2: when \( \kappa \geq \hat{\kappa} \), the optimal value of \( p_2 \) is \( p_2^* = 1 \), which means that every person declaring low will be audited if taxpayers fail to coordinate on the *Evade* equilibrium. This implies that the (expected) penalty is maximal \( (pf_i = (1 + \varsigma) t) \), and so the claim is proven.

Part 1 can be derived straightforwardly from proposition 1. It is stated there that, when \( \kappa < \hat{\kappa} \), the solution must lie on the \( \kappa = L \) line. Since this condition implies \( \hat{\kappa} = 1 \) (see discussion of equation 40 above), it means that the low probability of detection \( p_1 \) will be applied if and only if every single taxpayer declares zero income \( (p_2 \) being applied otherwise, see equation 37). Since \( \hat{\kappa} = 1 \) is the most demanding condition for coordination the government can impose, the statement is proven.

This proposition draws attention to the new tools available to the government when using a contingent policy that generates a coordination game: the coordination requirement \( \hat{\kappa} \) and the penalty for coordination failure \( p_2 \). The first one is a measure of how difficult coordination on the *Evade* equilibrium is: the greater is \( \hat{\kappa} \), the larger the fraction of evaders needed for the government to implement the low auditing intensity \( p_1 \). As \( \hat{\kappa} \) increases, taxpayers feel less confident about the likelihood of successful coordination on *Evade*, and hence their incentives to comply increase. The second one represents, from the taxpayers’ perspective, a measure of the penalty they face if coordination fails and they are caught evading. It is the "stick" the agency waves to warn taxpayers that if they intend to abuse the system and are caught the punishment will be extremely severe.

This taxonomy of new tools reveals that, in fact, they are substitutes of each other: when the coordination requirement \( \hat{\kappa} \) (the penalty for coordination failure \( p_2 \)) is maximal, the penalty for coordination failure \( p_2 \) (the coordination requirement \( \hat{\kappa} \)) can be relaxed. Intuitively, it means that a government will either impose moderate penalties that will be applied quite often \( (\hat{\kappa} = 1 \) and \( p_2 < 1 \)) or will impose high penalties that will seldom be applied \( (\hat{\kappa} < 1 \) and \( p_2 = 1 \)).

It is clear that a non-contingent rule cannot take advantage of these tools, and so its inferiority with respect to contingent policies is evident.

At this point, and since the optimal solution is now appropriately characterized, the logical next step consists of considering how this optimal solution changes when the parameters of the problem vary. These parameters are:

- \( P \in (0, 1) \), the full-deterrence auditing intensity. It is the auditing intensity that eliminates evasion.

- \( \varepsilon \in (0, \infty) \), the dispersion of taxpayers’ signals around the government’s type. Its inverse, \( \alpha := \frac{1}{\varepsilon} \), measures the precision of the signals.
• $\gamma \in (0, 1)$, the probability of a bad shock. Measures the uncertainty faced by the government regarding the income of individuals in the economy.

• $\lambda \in [0, 1]$, the weight assigned to negligence in the loss function. It represents the type of the government and it is its private information.

The parameters influence the problem in different ways: while $\lambda$ and $\gamma$ affect only the government’s objective function, $\varepsilon$ and $P$ affect only the constraints.

A by-product of proposition 1, the analysis can be made easier by using a simplified policy set as the one in figure 6. It must be stressed, however, that the simplified attainable set is a combination of the ones shown in figure 3. That is, for $p < P$, it is identical to the one in panel (a) and so $p = p_1$, but for $p > P$, it corresponds to the one in panel (b) and so $p = p_2$. Taking this caveat into account, figure 6 highlights the inverse relationship between the equilibrium values of the auditing intensity $p^*$ and the proportion of evaders $\kappa^*$: as $p^*$ increases (decreases) along the boundaries of the attainable set, $\kappa^*$ increases (decreases). The interpretation is straightforward: as the auditing intensity increases, the fraction of evaders decreases, exactly the kind of relationship we would expect between these two variables when considering an optimal auditing strategy.

![Figure 6: Simplified attainable set](image)

The effects of the government’s type $\lambda$ and the probability of a good shock $(1 - \gamma)$ are shown in propositions 5 and 6 below.

**Proposition 5** The more concerned with negligence the government is (the greater $\lambda$ is),

1. the greater is the auditing intensity $p$, and
2. the lower is the proportion of evaders $\kappa$.

A graphical illustration is presented in section A.4 of the appendix. It shows how the optimal solution changes as $\lambda$ increases from very low (panel (f)) to very
high (panel (a)). Intuitively, a government mostly interested in avoiding errors due to negligence does not want to let evaders get away with their evasion and is (almost) not concerned at all about wasting resources auditing low-income taxpayers who truthfully declare low income. Both factors work in the same direction: they give the agency incentives to increase the auditing intensity, and hence $p$ is high. The increased auditing intensity reduces taxpayers’ utility associated with evasion, thus increasing their incentives to comply and leading to a low proportion of evaders $\kappa$.

Interestingly, propositions 4 and 5 reveal that the preferred policy of a government depends on its type. Indeed, a government very concerned with negligence (high $\lambda$) will prefer to minimize the probability of coordination on the Evade equilibrium and consequently will choose $\hat{\kappa} = 1$, while setting $p_2$ just high enough to discourage evasion. On the other hand, a government mostly concerned with zeal (low $\lambda$) will prefer to avoid auditing and so will set $p_1$ low and $\hat{\kappa} < 1$, but will also set $p_2 = 1$ so it punishes severely those who evade if coordination fails.

An analogous result applies for $\gamma$:

**Proposition 6** The more likely the probability of a good shock $(1 - \gamma)$ is,

1. the greater is the auditing intensity $p$, and
2. the lower is the proportion of evaders $\kappa$.

From the diagrams in section A.4 of the appendix it can be seen how the optimal solution changes as the probability of a bad shock $(1 - \gamma)$ increases from very low (panel (f)) to very high (panel (a)). The intuition behind this result is straightforward: the greater the probability of a good shock, the more likely that someone who declares low is an evader. The government will then have incentives to audit more intensively among those who declare low (higher $p$), and this will lead to increased compliance (lower $\kappa$).

Moving on to the comparative statics of those parameters that affect the policy set ($P$ and $\varepsilon$), it is important to determine, first of all, how they modify it.

Consider first the full-deterrence auditing intensity $P$. Its effect on the attainable set (shown in figure 7) is such that a lower $P$ (i.e., a higher surcharge rate $\zeta$, since $P := \frac{1}{1 + \zeta}$) expands the policy set. Given this fact, the comparative statics can be summarized in the following proposition:

**Proposition 7** A higher surcharge rate $\zeta$ (hence, a lower full-deterrence auditing intensity $P$) leads to

1. a lower expected loss $EL^*$,
2. a lower auditing intensity $p^*$, and
3. a lower proportion of evaders $\kappa^*$.
Figure 7: Effect of $P$ on the attainable set

The result is well known in the literature: as the penalty rate $\zeta$ increases, the benefits of evasion decrease, thus decreasing the proportion of evaders $\kappa$ and, with it, the expected loss $EL$. Since fines and audits are substitutes of each other, the government will react by decreasing the auditing intensity $p$, since audits are costly activities. That is, the government switches to a policy where the possibility of detection is lower (lower $p$) but the fine if caught evading is greater (greater $\zeta$). This result follows the idea behind proposition 4, namely, that of substitutability between the size and likelihood of punishment.

The second parameter that affects the policy set is $\varepsilon$, the dispersion of signals around the government’s type, which measures the degree of fundamental uncertainty faced by taxpayers. It may also be useful to consider its inverse, $\alpha := \frac{1}{\varepsilon}$, which quantifies the precision or informativeness of signals: a larger $\varepsilon$ is associated with less precise/informative signals.

The effects of $\varepsilon$ on the policy set (shown in figure 8) are more complex than those generated by the surcharge rate $\zeta$. An increase in $\varepsilon$ can have different effects on the attainable set depending on the value of $p^*$: it is enlarged if $p^*$ is low ($p^* < 2P - 1$), but it contracts if $p^*$ is high. Proposition 8 summarizes the results.

**Proposition 8** More noisy signals (ie, a lower precision $\frac{1}{\varepsilon}$) lead to

1. a lower (higher) expected loss $EL^*$ if the government is very concerned with zeal (negligence) errors,
2. a lower (higher) auditing intensity $p^*$ if the government is very concerned with zeal (negligence) errors, and
3. a lower (higher) proportion of evaders $\kappa^*$ if the government is very concerned with zeal (negligence) errors.

Similar results are found in, for example, Mookherjee and Png (1992) and Andreoni (1991) (the latter in a different setting).
Figure 8: Effect of $\varepsilon$ on the attainable set

The intuition underlying this proposition underlines the importance of information in this model.

Consider first a government very concerned with zeal errors (ie, $\lambda \to 0$ and so $EL \to Z \to \gamma p$): it will be interested in auditing very little. Now, if the signals were very precise ($\varepsilon \to 0$), individuals will know the government will audit just a few people, thus increasing their incentives to evade. On the other hand, if the signals convey little information ($\varepsilon \to \infty$), individuals will ignore them and will, by default, believe the government is "average"$^{26}$. This will decrease their incentives to evade (compared to the perfect information case), and so the proportion of evaders $\kappa$ will decrease. Moreover, the increased compliance allows the government decreasing the auditing intensity $p$ even further and this, given that the government is only concerned with zeal errors, implies the expected loss $EL$ decreases.

Analogously, when the government is very concerned with negligence errors (ie, $\lambda \to 1$ and $EL \to N \to (1 - \gamma) \kappa (1 - p)$), it will be very interested in auditing as much as possible. So, if the signals are very precise ($\varepsilon \to 0$), taxpayers will know evasion will be discovered almost certainly, and hence will comply. But if signals are rather uninformative about the agency’s type ($\varepsilon \to \infty$), people will believe the government to be "average" and will hence evade more (compared to the perfect information case). The government will then have further incentives to move the auditing intensity $p$ even closer to 1. These two effects (higher $\kappa$ and lower $p$) work in different directions with respect to the expected loss: a greater $\kappa$ increases $EL$ because the proportion of evaders increases; a higher $p$ decreases it because it increases the proportion of evaders who are caught. The first one, however, dominates the second when the government is very concerned with negligence errors ($\lambda$ is high), and hence the expected loss increases.

Panels (a) and (b) in figure 9 illustrate the proposition’s results (see section

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$^{26}$Average in the sense that the only information about it is the one derived from the prior distribution of types. It is assumed that (as in most interesting cases) $0 = \min (\lambda) < E (\lambda) < \max (\lambda) = 1$. That is, the average government is concerned with both zeal and negligence errors.
.... of the appendix for a derivation of figure 9). When the dispersion of signals increases from $\varepsilon$ to $\varepsilon'$ ($\varepsilon' > \varepsilon$), the relationship between $\kappa^*$ and $p^*$ becomes flatter ($n \rightarrow n'$). The effects on $\kappa^*$ and $p^*$ depend then on whether the initial solution lies to the right or to the left of point $C$ (see panel (a)). A government concerned mostly about zeal errors (low $\lambda$) will usually choose a point like $A$, where the auditing intensity $p$ is low and the proportion of evaders $\kappa$ is high. In this case, more uncertainty leads to less evasion (lower $n$, hence lower $\kappa$) and a lower probability of detection (lower $p$), leading to lower expected loss (panel (b)). A government concerned mostly about negligence errors (high $\lambda$), on the other hand, will generally prefer a policy like $B$, where the auditing intensity $p$ is high and the proportion of evaders $\kappa$ is low. As the precision of the signals decreases, more evasion takes place (higher $n$) and auditing intensity $p$ is increased, but the end result is dominated by the first effect and so the expected loss increases (panel (b)).

![Diagram](image)

Figure 9: Comparative statics: $\varepsilon$

5 Discussion

A question that springs to mind after characterizing the optimal policy and analysing the corresponding comparative statics is: how important is the strategic uncertainty?

The degree of strategic uncertainty (following Morris and Shin (2002a)) is related to the critical threshold $\hat{\kappa}$. Intuitively, the larger is the critical threshold, the more demanding are the coordination requirements and the lower is taxpayers’ confidence about satisfying them. The intuition is consistent with the comparative statics of the switching point $s^*$ in section 4.1: as $\hat{\kappa}$ increases, so does the danger that the attempt at coordination will fail, and so a taxpayer will only declare low if her signal is very low (or, rather, there will be a range...
of signals for which she would have originally chosen to evade but now chooses to comply). Importantly, the strategic uncertainty remains even if the fundamental uncertainty is eliminated, as $\tilde{\alpha}$ is positive even when $\varepsilon \to 0$ (except for very special cases - see panel (a) of figure 8). Thus, the agency would be better off by implementing a contingent rule even if every taxpayer knew its type with 100% precision.

Another topic worth discussing is the heterogeneity within a class of taxpayers. Clearly the assumption made here of perfect income correlation within a class is an extreme one, but it is just intended to capture the fact that usually taxpayers belonging to the same category are homogeneous in most aspects, including income. Even more important, what really matters for the analysis is the fact that incomes within a class are more homogeneous than the signals received by its members, such that the differences among them are mainly due to disparate perceptions of the government’s type. The assumption of perfect uniformity allows observing the effect of the fundamental uncertainty unadulterated by the presence of income heterogeneity, and so the analysis is greatly simplified.

It is clear, however, that such heterogeneity can and does exist in reality, and it is expected to be greater the greater the heterogeneity among the taxpayers of the class regarding characteristics other than income (age, profession, gender, etc.). This, in turn, is usually a consequence of having to group into one class rather dissimilar individuals in order to have a "significant number of members" as to be able to apply statistical and econometric techniques to the data by them generated. Since this is related to the issue of defining classes - one this paper is not concerned with - the only related matter worth discussing here is the type of classes that favours the present model. And since the latter clearly relies on some degree of uniformity within the class, it is evident that its predictions are more likely to fit the data from classes with a large number of rather homogeneous people (e.g., unskilled manufacture workers or non-executive public servants) than the ones from small and/or heterogeneous classes.

6 Conclusions

Tax agencies across the world categorize the population of taxpayers according to observable characteristics that make easier the detection of "outliers" that could hint the presence of evasion. Under these conditions, the literature recommends audit strategies of the cut-off kind, where declarations above a certain level are not audited while those below it are investigated with a given probability (usually the one that discourages evasion among those who have incomes below the threshold).

This strategy loses its appeal, however, when common shocks can affect everyone in the class, leading to suboptimal results both when the shock is positive (simply more people declare the threshold, increasing evasion) and when it is negative (more compliant people are audited, wasting more resources).
This paper makes a contribution to the literature by introducing the possibility of conditioning the auditing policy on the class' average declaration, and proves such a policy is strictly superior to the standard, cut-off one.

The average declaration is a signal of the shock faced by the class, thus improving the information available to the agency without requiring new information to be gathered but just using efficiently every piece of the one present in the tax returns. Indeed, since taxpayers in a given class are more or less homogeneous, a low declaration (compared to the average) can confidently be labelled as suspicious of potential evasion. Given this, the agency will be well advised to follow an audit policy decreasing in declared income but increasing in the average declaration: the higher the average declaration, the higher the probability that the class is experiencing a good shock, and hence the higher the probability of someone who declares (relatively) low income to be an evader. Hence, the optimal probability evidently requires a higher auditing intensity (among low-income declares) the higher the average declaration is: the probability of them being evaders is high enough as to make the audits very likely to be profitable.

This policy introduces into the model an element never considered by the existing literature so far: a coordination game between the taxpayers. The latter hence faces the additional problem of having to decide on which equilibrium to coordinate -"strategic uncertainty-, which means their decisions become more complex and plays a significant role in determining the superiority of the contingent rule over the con-contingent one.

The apparent problem that the contingent rule can create for the agent -that of multiplicity of equilibria in coordination games- is easily dealt with thanks to the presence of a second element of uncertainty in the model. Indeed, taxpayers usually do not know the "type" of tax authority the face, whether it is tough or soft, or its position between these two extremes. This "fundamental" uncertainty leads to the formation of different beliefs among taxpayers, thus generating sufficient heterogeneity among them as to eliminate all equilibria but one. This unique equilibrium is usually interior and stable, what fits the empirical evidence regarding compliance (some people comply while others evade) without having to resort to unmeasurable concepts as "ethics", "stigma" or "conformity".

The model thus highlights the importance of taking into account the interaction among taxpayers and how a cunning agency can profit from it by giving rise to a coordination game among them. The strategic uncertainty about other taxpayers' actions as well as the fundamental one about the government type make the game particularly suitable to be modelled as a global game, with the added benefit that the end result is a unique, stable and interior equilibrium that is consistent with the real world data.

The paper also considers for the first time a tax agency's objective function that measures its targeting proficiency and allows a taxonomy of agencies based on the relative weights attached to the two types of errors that can be made, namely, those of negligence (not auditing profitable tax returns) and zeal (auditing non-profitable ones). Thus, this function makes both possible and plausible the extrapolation of the model's results to other applications -like targeting
of welfare benefits or regulation- where the allocation of resources is based on reports submitted by the agents and verification is costly.
References


Appendix

A.1 Public goods

The presence of public goods does not affect the results because the amount provided is a function of the revenue raised by the government (and, hence, of the proportion of evaders). This means that individual taxpayers cannot affect the level of public goods available to themselves, and as a consequence will decide their declarations based only on their effect on their private consumption.

Formally, the utility function of taxpayer $i$ will be modified as follows

$$u(c_i, G) = c_i + \phi(G)$$

(57)

where $c_i$ is the agent’s private consumption (equal to her disposable income), $\phi(G)$ is the utility if an amount $G \geq 0$ of the public good is provided, $\phi(0) = 0$ and $\phi'(G) > 0 \forall G \geq 0$ (The assumption of quasilinear utility functions is a standard one in the literature on public goods -see, for example Ledyard and Palfrey (2002) and Morgan (2000)). The amount of public good provided depends on the total revenue collected, which is inversely related to the fraction of evaders. Then,

$$G = g(1 - \kappa)$$

(58)

with $g(0) = 0$, $g(1) > 0$ and $g'(1 - \kappa) > 0 \forall \kappa \in [0, 1]$. This setting is general enough as to allow for a non-linear relationship between revenue and public good provision, as seems to be the case when significant initial investments/fixed costs are needed to start production (eg, building a lighthouse). This feature links the analysis with another application of the global game technique: the "contribution" game (see, for example, Morris and Shin (2002a) and Myatt et al. (2002)).

The new payoff matrix is then

<table>
<thead>
<tr>
<th></th>
<th>$\kappa \geq \hat{\kappa}$</th>
<th>$\kappa &lt; \hat{\kappa}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evade</td>
<td>$1 - p_1 (1 + \kappa) t + \phi(G)$</td>
<td>$1 - p_2 (1 + \kappa) t + \phi(G')$</td>
</tr>
<tr>
<td>Comply</td>
<td>$1 - t + \phi(G)$</td>
<td>$1 - t + \phi(G')$</td>
</tr>
</tbody>
</table>

where $G' > G$ because evasion is lower ($\kappa < \hat{\kappa}$ in the first case, $\kappa \geq \hat{\kappa}$ in the second).

It is straightforward to prove the possible cases are exactly identical to those without public goods shown in section 4.1 and that the switching point (equation 32) is unchanged. Thus, it is clear that public goods do not modify the results in the paper.
A.2 Expected net revenue

Expected net revenue (ENR) is given by the expression

\[ ENR = -\gamma pc + (1 - \gamma) \{(1 - \kappa) t + \kappa p \{1 + \zeta \} t - c\} \tag{60} \]

The first term corresponds to the ENR when there is a bad shock and the second to the ENR when the shock is a good one, both weighed by the probability of occurrence of the corresponding shock.

Like the expected loss function, the expected net revenue looks like a saddle (see figure 10; the arrows indicate the direction in which ENR increases). The saddle point is given by \((p, \kappa) = \left( \frac{\zeta}{1 + \zeta \mu - \varepsilon}, \mu' \right)\), with \(\mu' := \frac{\xi}{1 + \zeta (1 + \zeta) \mu - \varepsilon} \in [0, \infty)\). (Note that \(\frac{\zeta}{1 + \zeta \mu - \varepsilon} > 1\) from the assumption in footnote 10). The maximum is achieved when \(p = 0\) and \(\kappa = 0\), that is, when nobody evades and nobody is audited. The minimum when \(p = 0\) and \(\kappa = 1\), that is, when every high-income individual evades and nobody is audited.

![Figure 10: Expected net revenue contours](image)

Since \(L\), \(K\) and \(H\) do not depend on the objective function, the attainable set is exactly identical to the one presented in the text (figure 6). Thus, as can be seen from both figures 1 and 10, maximizing ENR and minimizing EL demand the same actions from the government, and the results obtained in the text for the EL function will apply almost without any modification to the ENR case.

A.3 EL contour maps. Special cases

Three special cases are worth mentioning:
• if $\gamma = 0$ or $\lambda = 1$ (see figure 11, panel (a)): $\mu = 0$. There are infinite minima along the $p = 1$ and $\kappa = 0$ lines. The maximum is at $(p, \kappa) = (0, 1)$.

• if $\gamma = 1$ or $\lambda = 0$ (see figure 11, panel (b)): $\mu \to \infty$. The minimum is at point $(p, \kappa) = (0, 0)$. There are infinite maxima along the $p = 1$ line.

• if $\gamma = \lambda \in (0, 1)$ (see figure 11, panel (c)): $\mu = 1$. The minimum is at point $(p, \kappa) = (0, 0)$. There are infinite maxima along the lines $p = 1$ and $\kappa = 1$.

Figure 11: EL contours. Special cases

A.4 Comparative statics: $\lambda$ and $\gamma$

A.5 Derivation of figure 9

From the attainable set, $p^*$ and $\kappa^*$ have a negative relationship. This is illustrated in panel (a) in figure 13, where $\kappa$ is drawn as a function of $p$. Also shown in this panel are the curves $\kappa p$ and $\kappa (1 - p)$, whose derivation is straightforward once $p$ and $\kappa$ are drawn.

Define

\begin{align*}
  z & : = p \\
  n & : = \kappa (1 - p) \\
  l & : = n + p
\end{align*}

so that $z$ is the proportion of low income people audited, $n$ the proportion of high income people who evade and are not caught, and $l$ the sum of both. Abusing
Figure 12: Comparative statics: $\mu$

terminology they will be called zeal, negligence and expected loss respectively. Though not weighted by the probability of a bad shock ($\gamma$) or the concern of the government about negligence errors ($\lambda$), they provide a reasonable substitute for the true variables $-N, Z$ and $EL-$ and greatly simplify the analysis. The diagrams are accurate for the case when $\lambda = \gamma = \frac{1}{2}$, but suitable changes of scale can be applied for different weights. Ultimately, the qualitative results will not change, and so for simplicity $\lambda$ and $\gamma$ will be safely ignored here.

Using the definitions above, panel (a) can then be simplified to look as in panel (b).

It is important to notice that the relationship between $\kappa$ and $p$ is negative, as is the one between $n$ and $p$. This allows for the comparative statics analyses to be undertaken using panel (b) only, and drawing conclusions on the effects of different parameters on the proportion of evaders $\kappa$ from their effects on the proportion of "successful" (ie, not caught) evaders $n$. 

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Figure 13: Derivation of figure ??