Optimal Policy in a Dynamic Search Model with Risk Aversion

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Abstract

Different policy instruments affecting the labour market do interact among each other. Hence, we propose a joint derivation of the optimal level of unemployment benefits, layoff taxes, hiring subsidies and payroll taxes. This is achieved within a dynamic search model of the labour market which allows for workers’ risk aversion. In a first-best allocation of resources, perfect insurance should be provided against the unemployment risk, layoff taxes are necessary to induce employers to internalise the cost of dismissing an employee but should not be too high in order to allow a desirable reallocation of workers from low to high productivity jobs, hiring subsidies are needed to partially offset the impact of layoff taxes on the rate of job creation and payroll taxes should be approximately equal to zero. We derive an optimal rate of unemployment and show that it is lower when the unemployment risk is partly non-insurable. Also, the optimal level of layoff taxes and hiring subsidies is independent of the amount of public good expenditures, even when payroll taxes are distortionary. Finally, optimal policies are computed numerically when workers have some bargaining power.

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1 Introduction

Dynamic search models of the labour market, in the tradition of Mortensen Pissarides (1994), are now widely used by economists. However, policy analyses relying on such frameworks have essentially been positive rather than normative. Indeed, these models typically assume risk neutral workers, which implies that normative work can hardly go beyond output maximisation. However, the issue of insurance against the unemployment risk is at the heart of the debate on labour market policies. We therefore propose, in this paper, to characterise the optimal labour market institutions in a dynamic search model with risk-averse workers.

There already exists an extensive literature on labour market institutions. However, the different policy parameters are typically considered in isolation, whereas, in fact, they do interact among each other and should therefore be analysed jointly. Thus, a dynamic search model with risk averse workers provides a rather natural framework for a joint derivation of the optimal level of unemployment insurance, layoff taxes, hiring subsidies and payroll taxes. Our analysis being properly microfounded in terms of the individual risk-averse preferences of workers, we are able to derive an optimal rate of unemployment, i.e. a rate of unemployment that is optimal from a welfare perspective.

Solving for the policy that implements a first-best allocation of resources yields closed form solutions which have an intuitive interpretation. Abstracting from moral-hazard concerns, we find that perfect insurance should optimally be provided against the unemployment risk. This could be implemented, if firms unilaterally set wages, by providing generous unemployment insurance. Layoff taxes are necessary to induce firms to internalise the cost of dismissing a worker, which consists of unemployment benefits and of forgone payroll taxes. However, the dismissal cost should not be too high, otherwise it would prevent a desirable reallocation of workers from low to high productivity jobs. By implementing the optimal rate of job destruction, layoff taxes also affect job creation. Hence, hiring subsidies are needed to restore a desirable rate of job creation. Finally, and perhaps surprisingly, payroll taxes should optimally be small and, without time discounting, equal to zero.

We then consider a number of deviations from this first-best benchmark. We show that public goods expenditures for the government have no effect on the optimal level of layoff taxes and hiring subsidies, even when payroll taxes are distortionary. We then turn to a case where full insurance cannot be provided with unemployment benefits and show that an alternative is to reduce
the rate of unemployment, which nevertheless comes at the cost of a lower average productivity. This is achieved by using layoff taxes and hiring subsidies such as to lower job destruction while leaving job creation unaffected. Finally, we consider a number of cases where workers have some bargaining power in the wage formation process, which prevents the provision of full insurance. As the problem is not analytically solvable for realistic forms of wage bargaining, we report instead the optimal policy parameters for a realistic calibration of the model.

This work is closely related to two key papers which consider the optimal setting of different policy instruments jointly, rather than in isolation. Mortensen Pissarides (2003), or Pissarides (2000, chapter 9), analyses policies in a dynamic search model with risk-neutral workers; while Blanchard Tirole (2008) proposes a joint derivation of unemployment insurance and employment protection in a static model with risk-averse workers.

In Mortensen Pissarides (2003), since there is no motive for insurance, the best that the government can do is to maximise output net of recruitment costs. A sufficient condition for this is that the Hosios (1990) condition holds, i.e. that the bargaining power of workers be equal to the elasticity of the matching function. If it does hold, then choosing no-policy is optimal; while, if it does not hold, policy parameters should only be used to implement the rate of job creation and job destruction that would prevail under the Hosios condition. However, in this context, the number of policy parameters is larger than that of optimality conditions. Thus, Mortensen and Pissarides (2003) do not fully characterise an optimal policy, but just derive some restrictions that policy instruments should satisfy. The optimal policy which we fully characterise in this paper under risk aversion is, unsurprisingly, consistent with those restrictions.

Blanchard and Tirole (2008) propose a joint derivation of optimal unemployment insurance and employment protection, i.e. layoff taxes, in a static model with risk-averse workers. Abstracting from moral-hazard concerns, workers should be provided with perfect insurance against the unemployment risk. In their benchmark, which is the static counterpart to the first-best policy derived in this paper, they show that unemployment benefits should be entirely financed by layoff taxes, rather than payroll taxes, in order to induce firms to internalise the cost of unemployment. However, the static nature of their model seems to be a serious limitation.

\footnote{This policy was originally proposed by Feldstein (1976) under the name "experience rating". Other contributions on the topic include Topel Welch (1980), Cahuc Malherbet (2004) and Mongrain Roberts (2005).}
They only consider the job destruction side of the economy, therefore abstracting from potential adverse effects of layoff taxes on job creation. In fact, the work of Mortensen and Pissarides (2003) suggests that if a firing tax is being implemented, then a hiring subsidy is also needed to restore an efficient rate of job creation. Also, and more fundamentally, a static approach entails an entirely negative view of unemployment, whereas in a dynamic setting an unemployed worker is a useful input in the matching process. Matching functions typically exhibits strategic complementarities between the number of unemployed and that of vacancies. Thus, without a good balance between the two, the labour market does not function very well, which prevents a desirable reallocation of workers from low to high productivity jobs. In fact, to maximise output in an economy without governmental intervention, the Hosios condition actually maximises the rate of job destruction.

The literature on dynamic search models of the labour market only comprises few papers allowing for risk-averse workers. Notable exceptions include Cahuc Lehmann (2000), Fredriksson Holmlund (2001) and Lehmann van der Linden (2007) which focus on the moral hazard problem of unemployment insurance and on the associated optimal trade-off between insurance and incentives. As we abstract from moral hazard concerns, these contributions could be seen as complementary to what is being done in this paper.

Acemoglu and Shimer (1999, 2000) made another key contribution which considers risk-averse workers in the context of directed search. They assume that the formation of high quality matches is a time consuming process. However, with insufficient unemployment insurance, risk-averse workers prefer to shorten their unemployment spell, even if this comes at the cost of a lower wage rate. Under directed search, firms respond by offering lower productivity jobs which can be filled quicker. Thus, they conclude that unemployment insurance could generate productivity gains.

Although, in our paper, match quality is unrelated to the length of unemployment, there are some similarities between our findings and that of Acemoglu and Shimer (1999, 2000). If full insurance against unemployment cannot be provided, it is optimal to decrease the productivity threshold below which matches are destroyed, in order to decrease the unemployment risk. But this comes at the cost of a lower average productivity. More fundamentally, our work suggests that, by allowing the government to use several policy instruments, rather than just unemployment benefits, its is possible to get rid of the conflict between productivity and risk sharing. Indeed, in our first-best benchmark, the level of output is maximised and perfect insurance is provided.
Alavarez and Veracierto (2000) use a calibrated search model with risk-averse workers to investigate the effects of different labour market policies. However, their approach is entirely positive and does not attempt to characterise optimal policies.

Finally, the work of Coles and Masters (2006) is closely related to ours. They show that there is some complementarity between the provision of unemployment insurance and that of job creation subsidies. The idea is that, by boosting the job creation rate, subsidies exert a downward pressure on unemployment and, hence, on the cost of providing unemployment insurance. However, their model does not have an endogenous job destruction margin and, therefore, cannot be used to determine the optimal level of employment protection.

This paper begins by a brief reminder of the key features of the Mortensen-Pissarides (1994) framework, on which our subsequent work relies. The following section is dedicated to the derivation of the first-best policy, which then serves as a benchmark. The fourth section investigates how public good expenditures affect optimal labour market policies. We then investigate how uninsurable unemployment risks should be dealt with. Finally, we turn to the possibility that workers have some bargaining power. This paper ends with a conclusion.

2 Search Model

Before solving for optimal policies, we need to describe the main characteristics of the dynamic search model that is considered throughout this paper. The structure of the economy corresponds to the standard Mortensen-Pissarides (1994) framework. Production requires that vacant jobs and unemployed workers get matched, which occur at rate \( m = m(u, v) \), where \( u \) stands for the number of unemployed and \( v \) for that of vacancies. For simplicity, each firm can employ, at most, one worker and the mass of workers is normalized to one, so that \( u \) also stands for the rate of unemployment. The matching function \( m \) is increasing in both arguments, exhibits decreasing marginal product to each input and satisfies constant returns to scale. It follows from this last assumption that the key parameter of interest, which summarizes labour market conditions, is market tightness defined as the ratio of vacancies to unemployment, \( \theta = v/u \).

The rate at which vacant firms meet unemployed workers is given by:

\[
\frac{m(u, v)}{v} = m\left(\frac{u}{\theta}, 1\right) = m\left(\frac{1}{\theta}, 1\right) = q(\theta),
\]  

(1)
where $q$ clearly is a decreasing function of $\theta$. Similarly the rate at which unemployed find jobs is:

$$\frac{m(u,v)}{u} = m(1, \theta) = \theta q(\theta). \quad (2)$$

We will subsequently refer to the elasticity of the matching function defined by:

$$\eta(\theta) = -\frac{\theta}{q(\theta)} \frac{dq(\theta)}{d\theta}. \quad (3)$$

The other main feature of the Mortensen-Pissarides model is that the productivity of a match is subject to idiosyncratic shocks. Production starts at maximal productivity, normalized to 1. The idea is that recruiting firms are prosperous and initially provide their employees with the best technology available. At Poisson rate $\lambda$, the match is hit and a new productivity $x \in [\psi, 1]$ is randomly drawn from c.d.f. $G(x)$. The match dissolves if the new productivity is below a threshold $R$. More details will be given as the optimal policy is being derived.

3 First-Best Policy

The optimal policy is derived in two steps. First, we find the optimal allocation. Then, we see how it could be implemented in a decentralized economy with search frictions.

3.1 Optimal Allocation

The optimal allocation maximises a utilitarian social welfare function subject to a resource constraint and to the search frictions that characterise the labour market. It is therefore the solution to the following problem:

$$\max_{\theta,R,b,w} \int_0^\infty e^{-\rho t} \left[ (1 - u)v(w) + uv(z + b) \right] dt$$

subject to

$$\dot{u} = \lambda G(R)(1 - u) - \theta q(\theta) u \quad (5a)$$

$$\dot{y} = \theta q(\theta) u + \lambda (1 - u) \int_0^1 s dG(s) - \lambda y \quad (5b)$$

$$(1 - u)w + ub = y - c\theta u \quad (5c)$$

where $\rho$ stands for the planner’s (or workers’) discount rate, $w$ for the net wage that an employee receives, $z$ for the value of leisure, $b$ for unemployment benefits, $y$ for aggregate output of the economy and $c$ for the flow cost of posting.
a vacancy. The instantaneous utility function of risk-averse workers is denoted by \( v(\cdot) \), which is increasing and concave.

The planner’s objective is to maximize intertemporal social welfare, which, following a utilitarian criteria, is composed, at each instant, of the instantaneous utility of \( u \) unemployed and \( 1-u \) employed workers\(^3\). The first constraint depicts the dynamics of unemployment, driven by the difference between job destruction and job creation. A match dissolves when it is hit by an idiosyncratic shock that generates a new productivity below the threshold \( R \), which occurs at rate \( \lambda G(R) \). This rate of job destruction applies to the mass \( 1-u \) of existing matches. Job creation is simply equal to the rate at which unemployed workers find jobs, \( \theta q(\theta) \), multiplied by the mass \( u \) of job seekers. It should be emphasized that this first constraint captures the fact that even the social planner is subject to matching frictions. The second constraint gives the dynamics of aggregate output, \( y \). At each instant, \( \theta q(\theta)u \) new matches are formed and each of these has a productivity of 1. The \( 1-u \) existing jobs are hit at rate \( \lambda \) by idiosyncratic shocks which destroy their current productivity and replaces it, in case of survival, by a randomly drawn number greater or equal to the threshold \( R \). Finally, maximisation of social welfare is subject to a resource constraint. The expenses, composed of the wages paid to the employed and the benefits paid to the unemployed, cannot exceed total output net of the resources allocated to recruitment, which amount to a flow cost \( c \) paid for each of the \( \theta u \) vacancies. The planner’s control variables are market tightness \( \theta \), threshold productivity \( R \), net wage \( w \) and unemployment benefits \( b \). The state variables are unemployment \( u \) and aggregate output \( y \).

The planner’s problem is straightforward to solve using standard optimal control techniques. The first characteristic of the optimal allocation is perfect insurance for workers:

\[
  w = z + b, \tag{6}
\]

which follows directly from risk aversion, i.e. from the concavity of \( v(\cdot) \). This could be combined with the resource constraint, \( (5c) \), to give the optimal value

\(^2\)In the previous section \( v \) denoted the number of vacancies. However, this variable will not appear in the rest of the text as we focus instead on \( \theta \) and \( u \). Hence, where needed, \( v \) is just replaced by \( \theta u \).

\(^3\)An alternative would be to maximise the weighted average between the expected utility of an employed and of an unemployed worker. Such objective function would be more appropriate for political economy work focusing on the conflict between insiders and outsiders. However, without time discounting, this is identical to the planner’s objective retained in this paper.
of \( w \) and \( b \):

\[
\begin{align*}
    w &= y - c\theta u + zu, \quad (7) \\
    b &= y - c\theta u - z(1 - u). \quad (8)
\end{align*}
\]

Note that perfect insurance necessitates a replacement ratio smaller than one provided that the value of leisure, \( z \), is strictly positive. The optimal value of \( \theta \) and \( R \) is determined by:

\[
\left[1 - \eta(\theta)\right] \frac{1 - R}{\rho + \lambda} = \frac{c}{q(\theta)}, \quad (9)
\]

\[
R = z + \frac{\eta(\theta)}{1 - \eta(\theta)}c\theta - \frac{\lambda}{\rho + \lambda} \int_{R}^{1} (s - R)dG(s), \quad (10)
\]

where \( \eta(\theta) \) denotes the elasticity of the matching function, cf. equation (3). These two optimality conditions are exactly identical to the one derived in Pissarides (2000, chapter 8) for net\(^4\) output maximisation. This is not surprising as, here, perfect insurance does not conflict with output maximisation, i.e. there is not trade-off between the two. The first equation, (9), corresponds to optimal job creation. The cost of job creation to a firm consists of the flow cost of having vacancy, \( c \), multiplied by the expected time that has to be spent before a worker could be found, \( 1/q(\theta) \). The value of a newly created match is equal to \((1 - R)/(\rho + \lambda)\). However, optimally, firms should only get a fraction \( 1 - \eta(\theta) \) of this value, otherwise there is too much job creation and excessive resources are allocated to recruitment. Equation (10) gives optimal job destruction. In the static context of Blanchard Tirole (2008), the optimal threshold is just equal to the value of leisure, i.e. \( R = z \). Making the model dynamic yields two extra terms. First, when a low productivity job is destroyed, the corresponding worker returns to unemployment with the hope of finding a new job with productivity 1. To make this more explicit, the corresponding term of equation (10) could be rewritten, using (9), as:

\[
\frac{\eta(\theta)}{1 - \eta(\theta)}c\theta = \theta q(\theta) \eta(\theta) \left[ \frac{1 - R}{\rho + \lambda} \right] = \theta q(\theta) \left[ \frac{1 - R}{\rho + \lambda} - \frac{c}{q(\theta)} \right]. \quad (11)
\]

This says that, once a job is destroyed, an unemployed worker gets matched at rate \( \theta q(\theta) \) which generates a social value of \( (1 - R)/(\rho + \lambda) \) from which

\(^4\) Under risk neutrality, the optimal policy is to maximise net output given by \( y - c\theta u + uz \).
recruitment costs \( c/q(\theta) \) have to be subtracted. In other words, the threshold \( R \) has to be sufficiently high to induce an efficient reallocation of workers from low to high productivity jobs. The second term that is added in a dynamic context to the expression for the optimal threshold \( R \) corresponds to the option value of a match. Even if current productivity is very low, keeping the match alive preserves the option of being hit by an idiosyncratic shock that restores a profitable productivity. The option value decreases the optimal threshold \( R \).

### 3.2 Implementation

As usual in optimal policy problems, the difficulty is not to find the optimal allocation, but to find a way to implement it in a decentralized economy. We distinguish four stages of interest.

- **Stage 1**: The government chooses the level of unemployment benefits \( b \), payroll taxes \( \tau \), layoff taxes \( F \), and hiring subsidies \( H \).
- **Stage 2**: Entrepreneurs decide whether or not to create a firm with a vacant position.
- **Stage 3**: Once a match is created, the employer and employee agree on a wage rate.
- **Stage 4**: Firms choose a threshold productivity \( R \) below which a match hit by an idiosyncratic shock dissolves.

We now proceed by backward induction and start by determining the threshold \( R \) chosen by risk-neutral employers. The value of a producing firm with productivity \( x \), \( J(x) \), solves the following Bellman equation:

\[
rJ(x) = x - (w + \tau) + \lambda \int_{R}^{1} J(s)dG(s) - \lambda G(R)F - \lambda J(x),
\]

where \( r \) clearly denotes the interest rate, \( w \) the net wage that the worker receives and \( w + \tau \) the gross wage paid by the employer. Note that, in our framework, the planner’s discount rate, \( \rho \), does not have to coincide with the economy’s interest rate. This equation states that, for a firm, the flow return from having a filled job with productivity \( x \) is equal to the instantaneous surplus to which the possibility of a change in productivity should be added. An idiosyncratic shock destroys the value of the firm at the current productivity and replaces it by, either, a corresponding expression if the new productivity is above the
threshold, or, by the cost of layoff\(^5\) if the match is to be destroyed. Note that our assumption of a fixed wage will soon be justified, when we discuss stage 3. It is straightforward to show that \(J(x)\) is increasing in \(x\) and, hence, employers’ chosen threshold is determined by:

\[
J(R) = -F. \tag{13}
\]

This says that, at the threshold, employers are indifferent between closing down and continuing the relationship. Simple algebra\(^6\) on (12) and (13) gives the expression for the value of \(R\) chosen by firms:

\[
R = w + \tau - rF - \frac{\lambda}{r+\lambda} \int_R^1 (s - R) dG(s). \tag{14}
\]

The threshold productivity is smaller than the cost of labour because of the firing tax and of the option value of continuing the match. Note that, for this to be possible, firms must be able to borrow and lend from perfect financial markets, an assumption that is maintained throughout this paper. Equation (14) is our first implementability constraint.

Let us now turn to determination of the wage rate that occurs at stage 3. The creation of a match generates a surplus that needs to be shared between the two parties. The size of this surplus is affected by current productivity. But, from equation (6), optimality requires that the net wage paid to a worker, \(w\), is equal to the wage equivalent of being unemployed, \(z + b\). This leads to following proposition:

**Lemma 1** A necessary condition to implement the first-best allocation is that the bargaining power of workers is nil and that all the surplus from matches is captured by firms. This ensures that, as desired:

\[
w = z + b. \tag{15}
\]

The intuition for this result is straightforward. If workers have some bargaining power, they will obtain a mark-up over and above their outside option which is the income they get while unemployed. But this prevents the provision of

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\(^5\) Throughout this paper, we maintain the assumption that firms are able to pay the layoff tax. Blanchard and Tirole (2008) investigate the consequences of having employers constrained by shallow pockets. See also Tirole (2008) for a deeper analysis on the topic.

\(^6\) An analytic expression for the function \(J(.)\) could be obtained by taking the difference between equation (12) evaluated at \(x\) and the same equation evaluated at \(R\). This expression for \(J(.)\) could then be substituted into (12) evaluated at \(R\). Finally, (14) is obtained by plugging (13) in.
full insurance which is a characteristic of a first-best allocation. It should be noted that, in their benchmark case, Blanchard and Tirole (2008) also assume that workers are wage takers. Thus, our first-best benchmark, derived in this section, is a dynamic counterpart to theirs. Clearly, with a binding resource constraint, (5c), and perfect insurance, the optimal value of \( w \) and \( b \) is still given by (7) and (8), respectively.

In the context of this paper, the requirement that workers have no bargaining power could also be seen as part of the optimal policy to be implemented\(^7\). For example, the labour market could be organized in such a way that firms and workers first meet without exchanging any information on the wage rate. Then, firms make a take-it-or-leave-it offer to workers. This also suggests that, here, a minimum wage would be detrimental to insurance. Excessive monopsony power of firms should rather be dealt with traditional policy instruments such as taxes and unemployment insurance. Finally, note that the above proposition entails the following corollary:

**Corollary 1** *The first-best allocation cannot be implemented when the Hosios condition holds, i.e. when the bargaining power of workers is equal the elasticity of the matching function \( \eta(\theta) \).*

If the model does not allow for risk aversion, as in Mortensen Pissarides (2003), choosing not to intervene is optimal whenever the Hosios condition holds. By contrast, when it does not hold, policy instruments are only used to offset the distortions generated by the gap between the bargaining power of workers and the elasticity of the matching function. As our optimal allocation requires net output maximisation and zero bargaining power for workers, the optimal policy will involve such corrections for the failure of the Hosios condition to hold.

Stage 2 is solved by assuming free-entry. Vacancies keep being created by entrepreneurs until the return from doing so reduces to zero. More formally, the value of a vacant position, \( V \), solves:

\[
rv = -c + q(\theta) [J(1) + H - V].
\]

This states that the return from a vacancy consists of the flow cost of recruitment, \( c \), and of the possibility of filling the position at rate \( q(\theta) \) which yields the value of an active firm with productivity 1. The employer also qualifies for

\(^7\) In the context of Nash bargaining, one solution proposed by Lehmann and van der Linden (2007) consists in setting a marginal rate of income taxation equal to 100%.
a hiring subsidy, $H$, when he hires a worker. Free entry implies that:

$$V = 0.$$  \hspace{1cm} (17)

The amount of job creation could thus be determined by plugging (17) into (16) and by using the value of $J(1)$ which could be deduced from (12) and (13). This gives:

$$\frac{1 - R}{r + \lambda} - F = \frac{c}{q(\theta)} - H.$$  \hspace{1cm} (18)

The right hand side corresponds to the expected cost of recruiting a worker; while the left hand side is just the value of a new match to a firm, $J(1)$. Equation (18) is our second implementability condition.

At stage 1, the government needs to choose the optimal policy. The corresponding implementability condition simply consists of the usual government budget constraint:

$$(1 - u)\tau + (1 - u)\lambda G(R)F = ub + u\theta q(\theta)H.$$  \hspace{1cm} (19)

Revenues consist of payroll taxes paid by employed workers and of firing taxes applied to the flow of layoffs; while the expenses are the payment of benefits to the unemployed and of hiring subsidies to the flow of newly created jobs.

It is now straightforward to find the optimal policy by matching the implementability conditions to the equations that characterize the first-best allocation. More specifically, (18) should be combined with (9) and (14) with (10). This gives:

$$F - H = \eta(\theta)\frac{1 - R}{\rho + \lambda} + \frac{\rho - r}{r + \lambda} \frac{1 - R}{\rho + \lambda},$$

$$rF = b + \tau - \frac{\eta(\theta)}{1 - \eta(\theta)}c\theta + \frac{r - \rho}{r + \lambda} \frac{\lambda}{\rho + \lambda} \int_{1}^{R} (s - R)dG(s),$$  \hspace{1cm} (20)

where $\theta$ and $R$ are determined by (9) and (10). These are key equations characterising the optimal policy in our benchmark model. They ensure that job creation and job destruction, respectively, are set at their optimal level.

These conditions have a potentially insightful interpretation. Let us start with the implementation optimal job creation, (20). As we have already discussed, firms should only capture a fraction $1 - \eta(\theta)$ of the surplus from a match; otherwise, entry is too important and too many resources are allocated to recruitment. However, employers have all the bargaining power and this must be offset by setting a firing tax that exceeds the hiring subsidy in order to reduce
job creation to an efficient level. The second term is just a correction in case the planner’s discount rate \( \rho \) differs from the market interest rate \( r \). If the planner is more patient than market participants, \( \rho < r \), then the social value of a new match exceeds the private value as perceived by entrepreneurs. This problem is addressed by raising the hiring subsidy for a given firing tax. Condition, (20), could also be seen as a correction for the failure of the Hosios condition to hold. If it did hold, then, under risk neutrality, efficient job creation would only require that \( F = H \). It should be noted that, in the static framework of Blanchard Tirole (2008), there is no job creation margin and no analogue to equation (20); thus, hiring subsidies are not part of the optimal policy.

Let us now turn to the interpretation of the equation implementing optimal job destruction, (21). As can be seen from (14), a layoff tax only affects the threshold \( R \) if firms discount the future, \( r > 0 \). Indeed, the match will eventually be destroyed and, hence, by not laying off its worker now, a firm knows that it is only postponing the payment of the tax. Thus the relevant cost imposed by the firing tax is \( rF \), rather than just \( F \). A firm that dismisses its worker imposes a double externality on the financing of unemployment insurance. First, the worker will qualify for benefits and, second, he will no longer contribute to its funding by paying payroll taxes. The layoff tax should therefore ensure that employers internalise these external effects. This is the basic insight of Blanchard Tirole (2008)\(^8\). The additional insight that is obtained by extending the analysis to a dynamic context is that layoff taxes should not be too high, otherwise they prevent a desirable reallocation of workers from low to high productivity jobs. This is indeed captured by the third term of equation (21) to which we have already given an intuitive interpretation when the optimal allocation was derived, cf. equation (11). Again, from an output maximisation perspective, the condition for optimal job destruction implicitly corrects for the failure of the Hosios condition to hold. If it did hold, then, with risk-neutral workers, wages would be sufficiently high for this third term to drop out of the equation. Finally, if \( \rho = r \), then the option value of continuing the match is properly taken into account by firms and therefore does not affect the optimal layoff tax. However, a correction is needed if the planner’s discount factor differs from the interest rate. For example, if the planner is more patient than entrepreneurs, \( \rho < r \), then the option value is larger for the social planner than for firms and, hence, the layoff tax needs to be increased.

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\(^8\)In fact, in Blanchard Tirole (2008) payroll taxes do not appear as they should optimally be set equal to zero. However, Cahuc and Zylberberg (2007), who proposed a generalisation to the case where the government needs to raise taxes on income in order to redistribute wealth across heterogenous individuals, did explicitly have them affecting the level of layoff taxes.
It should be noted that, for \( \rho = r \), equations similar to (20) and (21) where derived in Pissarides (2000, chapter 9). However, under risk neutrality these were just restrictions that had to be satisfied by policy parameters in order to compensate for the failure of the Hosios condition to hold. In this framework, risk aversion implies that the value of \( b \), and hence of \( \tau \), are uniquely determined which implies that there exists a unique optimal value of \( F \) and \( H \). As a result, equations (20) and (21) have a more precise interpretation in terms of externalities along the line of Blanchard Tirole (2008).

The level of payroll taxes is simply determined by the remaining implementability constraint, i.e. by the government budget constraint, (19). Using the fact that, in steady state, the job creation flow is equal to the job destruction flow, \( (1-u)\lambda G(R) = u\theta q(\theta) \), we obtain:

\[
\tau = \frac{u}{1-u} \left[ b - \theta q(\theta)(F - H) \right].
\]

The difference between \( F \) and \( H \) is given by (20). Hence, and perhaps surprisingly, the level of payroll taxes is mainly determined by the job creation side of the economy. The actual value of \( F \) required for optimal job destruction, (21), has not impact on payroll taxes as \( H \) optimally adjusts one-for-one to leave \( F - H \) unchanged. This suggests that the static Blanchard Tirole (2008) model, which does not have a job creation margin, is not appropriate to determine the share of unemployment insurance expenses that should be financed with payroll taxes.

Further insights could be gained by replacing \( F - H \) by its value from (20), which, after some straightforward rearrangement using (9), yields:

\[
\tau = \frac{u}{1-u} \left( b - \theta q(\theta) \left[ \frac{1-R}{\rho + \lambda} - \frac{c}{q(\theta)} \right] + \theta q(\theta) \frac{r - \rho}{r + \lambda} \frac{1-R}{\rho + \lambda} \right).
\]

The flow of unemployment benefits, \( b \), constitutes the social cost of having an unemployed worker. The second term represents the corresponding social benefit. Indeed, at rate \( \theta q(\theta) \), an unemployed finds a job which generates a social value equal to the expected profits from production net of the recruitment costs. If \( r > \rho \), the value of a match to an entrepreneur is smaller than its social value. This should be offset by having sufficiently large hiring subsidies. But this is costly to the government and, hence, payroll taxes need to be raised accordingly.

Since the optimal rate of unemployment should ensure that the social benefits from joblessness is not too distant from its social cost, we expect the first
two terms in (23) to be close to each other. In fact, with time discounting, we expect the first term to be slightly larger than the second one since the benefit will only be realized in the future. This intuition could be formally confirmed by solving for the optimal value of $b$ from equation (8), using the steady state level of unemployment and output as well as the optimal value of $\theta$ and $R$ from (9) and (10). Substituting the resulting expression for $b$ in (23) gives:

$$\tau = \frac{\rho}{\rho + \lambda} u \left[ \frac{y}{1 - u} - R \right] + \frac{r - \rho}{r + \lambda} \lambda G(R) \frac{1 - R}{\rho + \lambda}. \quad (24)$$

Hence, without time discounting, i.e. $\rho = r = 0$, payroll taxes are not part of the first-best policy. In this case, both unemployment insurance and hiring subsidies should be financed, exclusively, from layoff taxes. This is in accordance with one of the main finding from Blanchard and Tirole (2008), who obtain, in their benchmark specification, that payroll taxes should optimally be set equal to zero. Their intuition is that, when laying off a worker, firms should fully internalize the cost of providing unemployment benefits. When applied to our dynamic framework, this intuition is actually misleading. It might suggest that payroll taxes should be positive since layoff taxes should not be too high in order to allow a desirable reallocation of workers from low to high productivity jobs and, furthermore, part of the revenue from layoff taxes is absorbed by hiring subsidies. The correct intuition is that the optimal rate of unemployment is such that the social cost is equal to the social benefit of having an unemployed worker. The key element is that, with free entry and zero bargaining power to workers, the social benefit is entirely captured by the government as fiscal revenue. Similarly, the social cost, i.e. the unemployment benefits, is a government expense. Hence, the two cancel out of the budget constraint and payroll taxes could be set equal to zero.

The optimal policy could now be fully characterised.

**Proposition 1** When workers have zero bargaining power, the first-best allocation could be implemented by choosing the policy instruments $b$, $\tau$, $H$ and $F$ that satisfy equations (8), (19)$^9$, (20) and (21), respectively.

Knowing that the first-best allocation is implementable, we could derive the equilibrium rate of unemployment by setting $\dot{u} = 0$ in the corresponding dynamic equation, (5a). This yields the well known expression:

$$u = \frac{\lambda G(R)}{\lambda G(R) + \theta q(\theta)}. \quad (25)$$

---

$^9$In steady-state, $\tau$ could be determined from (24) instead of (19).
This equation nevertheless has an interesting new interpretation in our framework. Whereas, for optimal values of $\theta$ and $R$, this is the efficient\textsuperscript{10} rate of unemployment with risk-neutral workers; here, given the microfoundations laid in terms of risk-averse workers, this is the optimal rate of unemployment. Not only could unemployment be too low from an output maximisation perspective, it could also be too low from a welfare point of view, which is conceptually very different.

4 Public Good Financing with Distortionary Taxes

A characteristic of employment protection in our model is that it generates some revenue to the government. In an environment in which all taxes are distortionary, a natural question to ask is whether layoff taxes should be higher when governmental expenses are higher. To answer this question we add a participation margin to the previous model. People who choose to remain out of the labour force enjoy a dollar value of leisure equal to $l$. The distribution of $l$ across agents in the economy is given by the c.d.f $H(l)$. Thus, there exists a threshold $\bar{l}$ such that people choose to work if and only if their value of leisure $l$ is smaller or equal to $\bar{l}$. In a decentralized economy, the value of the threshold $\bar{l}$ is determined endogenously by workers. Hence, this extensive margin makes payroll taxes a distortionary way of raising revenue.

4.1 Optimal Allocation

As in the previous section, we first solve for the optimal allocation and then see how it could be implemented. The population is normalized to 1. We denote by $I$ the number of people out of the labour force, by $N$ the number of employed workers and by $U$ that of unemployed. We clearly have $1 = I + N + U$ and $I = 1 - H(\bar{l})$. Thus, $N + U = H(\bar{l})$. The optimal allocation is the solution to:

\[
\max_{\{\theta, R, b, w, l\}} \int_0^\infty e^{-\rho t} \left[ N v(w) + [H(\bar{l}) - N] v(z + b) + \int_\bar{l}^\infty v(l) dH(l) \right] dt \quad (26)\]

\textsuperscript{10}Following the literature on the topic, here, by "efficient" we mean "output maximising". It should nevertheless be noted that, in the context of this paper, a truly efficient allocation achieves a desirable amount of risk sharing. Hence, the only efficient allocation is the one that maximises the planner’s problem.
subject to 

\[
\dot{N} = \theta q(\theta) \left[ H(\bar{l}) - N \right] - \lambda G(R) N \tag{27a}
\]

\[
\dot{Y} = \theta q(\theta) \left[ H(\bar{l}) - N \right] + \lambda N \int_{\bar{l}}^{l} sdG(s) - \lambda Y \tag{27b}
\]

\[
Nw + \left[ H(\bar{l}) - N \right] Y = Y - c\theta \left[ H(\bar{l}) - N \right] - E \tag{27c}
\]

where \( E \) stands for the resources allocated to public good provision. Note that the dynamic evolution of employment \( N \) is used as a constraint, (27a), instead of that of unemployment \( U \). In fact, here, the number of unemployed, \( U = H(\bar{l}) - N \), is not a state variable as non-working agents who decide to enter the labour force have to transit through unemployment. Conversely, with less than full insurance, marginal workers who decide to leave the labour force must be unemployed. Our formulation implicitly assumes that this is still the case with perfect insurance. In other words, \( U \) is not a state variable as it jumps when the control variable \( \bar{l} \) jumps.

The optimality conditions are identical to what we previously had. Perfect insurance is still desirable, which combined with the resource constraint, (27c), gives:

\[
H(\bar{l})w = Y - c\theta \left[ H(\bar{l}) - N \right] - Nz - E, \tag{28}
\]

\[
H(\bar{l})b = Y - c\theta \left[ H(\bar{l}) - N \right] + \left[ H(\bar{l}) - N \right] z - E. \tag{29}
\]

The optimal values of \( \theta \) and \( R \) are still determined by equations (9) and (10). The only novelty is the condition for the optimal value of the participation threshold \( \bar{l} \), which in steady state is:

\[
\frac{v(w) - v(\bar{l})}{v'(w)} = \frac{\rho}{\rho + \lambda} \left[ (1 - R)G(R) + \int_{\bar{l}}^{l} \left( s - R \right) dG(s) \right] \frac{N}{H(\bar{l})} - \frac{E}{H(\bar{l})}. \tag{30}
\]

Without public good expenditures, \( E = 0 \), and with perfect insurance, we would expect to obtain \( \bar{l} = w = z + b \). But, as can be seen from the first term on the right hand side of (30), such is not the case when the planner discounts the future, i.e. when \( \rho > 0 \). The intuition for \( w = z + b > \bar{l} \) is that, initially, when a person enters the labour force, he becomes unemployed and qualifies for unemployment benefits which is costly for the government. Conversely, if we had assumed that the transition was directly from outside the labour force to employment, without intervening unemployment, we would have obtained that \( w = z + b < \bar{l} \) since, in this case, the marginal worker is producing and therefore relaxes the resource constraint, (27c). Anyway, the first term of (30) is not very interesting for our purpose and would vanish if we either assume \( \rho = 0 \) or that
workers who enter the labour force have a probability \( u \) of becoming unemployed and \( 1 - u \) of becoming employed, with \( u = (H(\bar{I}) - N)/H(\bar{I}) \) denoting the rate of unemployment. The interesting term in (30) is the second one when \( E > 0 \). When public expenditures need to be funded, it is desirable to have more people working, \( \bar{I} > w = z + b \) in order to spread its costs among a larger population. In other words, the social value of someone participating, \( \bar{I} \), is larger than the private value that this person derives, \( w = z + b \). As will become clear, this is why, with a participation margin, it is not possible to achieve a first-best allocation in a decentralized economy.

4.2 Optimal Policy

We now turn to the determination of the optimal policy in an economy where workers have no bargaining power, which ensure perfect insurance \( w = z + b \). The implementability constraints for job destruction and job creation are the same as before, i.e. (14) and (18), respectively. Public good expenditures, \( E \), should be added to the government budget constraint which then becomes:

\[
N\tau + N\lambda G(R)F = \left[H(\bar{I}) - N\right]b + \left[H(\bar{I}) - N\right]\theta q(\theta)H + E. 
\] (31)

The novelty is that workers privately choose whether to participate or not and the government, who needs to raise taxes to finance its expenditures, cannot influence this decision by taxing the leisure of those not participating. This yields a new implementability constraint for \( \bar{I} \), which under perfect insurance is\(^{11}\):

\[
\bar{I} = z + b. \tag{32}
\]

But, this cannot be reconciled with the first-best choice of \( \bar{I} \) given by equation (30). Hence, the first-best allocation is not implementable here. The optimal policy is instead derived by adding the implementability constraints to the planner’s problem. Now, (26) should be maximized under the resource constraints (27a), (27b), (27c), the equilibrium wage when workers have no bargaining power, i.e. \( w = z + b \), and the binding implementability constraint (32). This yields the optimal second-best policy. Strictly speaking, the other implementability constraints, (14), (18) and (19), should also be included. However, they can be safely omitted as they will not be binding since they jointly determine \( \tau \), \( F \) and \( H \) which do not appear elsewhere in the problem.

We have just described how the optimal policy should be derived when

\(^{11}\)We need to assume that the leisure value of unemployment, \( z \), is sufficiently low so that the solution to the problem is well-behaved and non-trivial.
workers have no bargaining power. But note that, in a second-best environment, it is not clear that perfect insurance is still desirable. If the planner was allowed to influence wage bargaining, then the above policy might not be second-best but third-best. To check this, we solve the above problem without imposing any restriction on the net wage rate $w$, which could be treated as a control variable. Note that the implementability constraint for $l$ needs to be changed; (32) should now be replaced by:

$$v(l) = \frac{(\rho + \lambda G(R)) v(z + b) + \theta q(\theta) v(w)}{\rho + \lambda G(R) + \theta q(\theta)},$$

(33)

which says that the utility from not participating has to equal the expected utility from unemployment. It turns out that, with no discounting, $\rho = 0$, perfect insurance is still desirable. With discounting, $\rho > 0$, insurance should not be perfect in order to deter the entry of new workers who would initially all be unemployed, which is costly in terms of resources. This is related to the first term of equation (30), which, as previously argued, is not really interesting. What is important is that, as far as the government expenditures, $E$, are concerned, the impossibility of implementing the first-best level of participation $\tilde{l}$ does not justify any departure from perfect insurance. This is intuitive since the suboptimally low level of participation is due to the existence of a wedge between the social and the private return from work which can only be worsen by underproviding insurance to workers.

Let us now turn to the characteristics of the optimal policy. With perfect insurance, the level of benefits $b$ is given still given by equation (29), which in steady state, $\dot{N} = \dot{Y} = 0$, simplifies to:

$$b = y - c\theta u - (1 - u)z - \frac{E}{H(\tilde{l})},$$

(34)

where $u$ denotes the rate of unemployment and $y$ the level of output per participant, i.e. $Y/H(\tilde{l})$. It turns out that the optimal value of the threshold, $R$, and market tightness, $\theta$, are still determined by the first-best conditions (9) and (10). The implementability constraints for job creation and job destruction being the same as before, the optimal level of hiring subsidies, $H$, and layoff taxes, $F$, are still given by (20) and (21). Finally, the level of payroll taxes is
determined by (31), which, in steady state, could be written as:

\[
\tau = \frac{n}{1 - u} [b - \theta q(\theta) (F - H)] + \frac{E}{N} = \frac{n}{1 - u} [y - c\theta u - (1 - u)z - \theta q(\theta) (F - H)] + \frac{E}{H(l)},
\]

(35)

where the second line was derived by substituting expression (34) for the optimal level of unemployment benefits.

Since \( b + \tau \) is unaffected by the public good expenditures, it is clear from (21) that layoff taxes remain unchanged; furthermore, from (20), hiring subsidies also remain unchanged. This leads to the following proposition:

**Proposition 2** The public good expenditures, \( E \), have no effect on the optimal level of layoff taxes and hiring subsidies.

This result might seem surprising as, in a second-best environment with distortionary taxes, intuition suggests that two small distortions is better than one large one. This should have led us to expect that the public good expenditures should be partly financed from layoff taxes. Such is not the case. In fact, this is a consequence of the ever remarkable Diamond-Mirrlees (1971) production efficiency result according to which optimal taxes never lead to any deviation from production efficiency. In other words, layoff taxes and hiring subsidies should be viewed as Pigouvian instruments used to correct for externalities induced by the decisions of entrepreneurs, not as a general source of revenue for the government\(^{12}\).

5 **Limits to Insurance**

We have so far assumed that workers could be perfectly insured against the risk of becoming unemployed. Following Blanchard Tirole (2008), we now consider the possibility that there is a non-insurable utility cost \( B > 0 \) of being unemployed. This specification is consistent with findings from the happiness literature which has provided some extensive evidence that unemployment has a long-lasting negative effect on life satisfaction; see, for example, Clark Diener

\(^{12}\)The proposition might seem to contradict the findings of Cabuc and Jolivet (2003) who show that public good expenditures increase the optimal size of layoff taxes. However, there model does not allow for government-provided unemployment insurance and the increase in layoff taxes is fully compensated by an increase in hiring subsidies. Hence, the public good expenditures are still financed from taxes on income.
The social planner’s problem now becomes:

\[
\max_{\{\theta, R, b, w\}} \int_0^\infty e^{-\rho t} \left[ (1 - u)v(w) + u[v(z + b) - B] \right] dt
\]

subject to

\[
\dot{u} = \lambda G(R)(1 - u) - \theta q(\theta) u
\]

\[
\dot{y} = \theta q(\theta) u + \lambda(1 - u) \int_R^1 s dG(s) - \lambda y
\]

\[
(1 - u)w + ub = y - \epsilon u
\]

where the constraints remain unchanged. Equations (7), (8) and (9) still characterize the optimal allocation. Note that it is still optimal to have \( w = z + b \) since \( B \) does not affect marginal utilities. The only difference is that the condition for optimal job destruction, (10), is replaced by:

\[
R = z + \frac{\eta(\theta)}{1 - \eta(\theta)} e^\theta - \frac{B}{v'(w)} - \frac{\lambda}{\rho + \lambda} \int_R^1 (s - R) dG(s).
\]

Now that workers cannot be perfectly insured against unemployment, it is desirable to decrease the threshold productivity below which a job is destroyed.

Implementing the optimal wage is not as straightforward as before. Indeed, if workers have zero bargaining power, their wage rate is determined by \( v(w) = v(z + b) - B \), which is clearly not desirable. The optimal policy could nevertheless be implemented when workers have sufficiently low bargaining power by setting a binding minimum wage equal to \( z + b \).

Since the other implementability constraints (14), (18) and (19) are not affected by the utility cost of being unemployed, it is straightforward to derive the optimal policy. Equation (8), (20) and (22) remain unchanged. The only modification is that equation (21) is replaced by:

\[
rF = b + \tau - \frac{\eta(\theta)}{1 - \eta(\theta)} e^\theta + \frac{B}{v'(w)} + \frac{r - \rho}{r + \lambda \rho + \lambda} \int_R^1 (s - R) dG(s).
\]

Layoff taxes need to be raised\(^{13}\) in order to implement the new optimal threshold which is lower than before. Although a similar result has already been derived by Blanchard and Tirole (2008), the interpretation is slightly richer in a dynamic context. The optimal policy implements a lower productivity threshold \( R \) and, hence\(^{14}\), a higher market tightness \( \theta \). This induces a decline in the

\(^{13}\)Strictly speaking, \( F \) is decreasing in \( R \) if and only if \( g(R) [1 - R] < 1 \). For example, this condition is always satisfied for a uniform distribution of idiosyncratic shocks.

\(^{14}\)If the elasticity of the matching function is not constant, a sufficient condition for \( \theta \) to be
rate of job destruction, \( \lambda G(R) \), and a rise in the rate of job creation, \( \theta q(\theta) \), which unambiguously leads to a lower equilibrium rate of unemployment. It is interesting to note that the optimal job creation condition, (9), is only indirectly affected, through \( R \), by the non-insurable cost of being unemployed, \( B \). This suggests that the planner primarily tries to reduce job destruction while leaving job creation unchanged. This is implemented by an increase in layoff taxes together with a corresponding adjustment in hiring subsidies such as to restore the optimal rate of job creation.

The key new feature of the optimal policy is summarized in the following proposition.

**Proposition 3** A higher non-insurable utility cost of being unemployed, \( B \), is associated with a lower optimal rate of unemployment.

When insurance cannot be perfect, reducing the number of jobless is a substitute to the provision of unemployment benefits. This policy nevertheless comes at a cost, since net output is no longer maximized. Hence, purchasing power will now be lower for both the employed and the unemployed. This case highlights clearly the conceptual distinction between the output maximizing efficient rate of unemployment and the welfare maximizing optimal rate of unemployment.

Finally, we can compute the optimal level of payroll taxes by replacing \( b \) and \( F - H \) by their optimal values in the steady state government budget constraint, (22). This yields:

\[
\tau = -u \frac{B}{v'(w)} + \frac{\rho}{\rho + \lambda} u \left[ \frac{y}{1-u} - R \right] + \frac{r - \rho}{r + \lambda} \lambda G(R) \frac{1 - R}{\rho + \lambda}. \tag{40}
\]

With no discounting, i.e. \( \rho = r = 0 \), the payroll tax rate is negative. The intuition is that the social cost of unemployment now falls short of the corresponding budgetary cost to the government as, without perfect insurance, the social cost of having an unemployed worker is larger than the level of benefits for which he qualifies. However, the social planner still equates the social cost to the social benefit of unemployment and, hence, he induces the budgetary benefit, \( \theta q(\theta)(F - H) \), to exceed the budgetary cost, \( b \). This generates a surplus that allows the existence of negative payroll taxes or, equivalently, positive employment subsidies.

\[ \text{increasing in } B \text{ is } \frac{d\eta(\theta)}{d\theta} > -\eta(\theta) [1 - \eta(\theta)] / \theta. \]
6 Workers with Bargaining Power

Under risk aversion, it is desirable to suppress any fluctuations in income between employment and unemployment. Thus, the implementation of first-best allocations requires workers to have zero bargaining power, as stated in Proposition 1. However, it could be objected that workers fundamentally do have some bargaining power and that this cannot be influenced by the planner. Thus, when solving for the optimal policy, the expression for the wage rate resulting from the bargaining process should be added to the implementability constraints.

We begin this section by briefly considering a simplistic, but tractable and hopefully insightful, case where bargaining power allows workers to obtain a mark-up over and above the income that they get while unemployed. We then turn to more microfounded forms of bargaining, where wages are a function of outside opportunities, which typically depends on the other parameters of the model, i.e. on $R$, $\theta$, $\tau$, $F$, $H$ and, of course, $b$. However, in such environments, optimal policy problems are typically not analytically solvable and the first-order conditions are hardly interpretable. Hence, we perform instead a reasonable calibration of the model and report numerical evaluations of the optimal policy parameters for different values of the bargaining power of workers. An obvious drawback from our analysis in this section is that we do not allow for private savings. Indeed, when workers have some bargaining power, their income fluctuates over time which should induce them to save in a risk-free asset in order to smooth the time profile of their consumption.

6.1 A Simple Case

We begin by considering a very simple reduced form of wage bargaining, which has the merit of being analytically tractable. More precisely, we assume that the wage rate of workers is determined as a mark-up over and above the dollar value of being unemployed:

$$ w = k(z + b) \text{ with } k > 1. $$

(41)

This expression for the wage rate could easily be substituted into the planner’s problem, which could then be solved. As in the previous section, equations (7), (8) and (9) still characterize the optimal allocation and the only modification
affects the optimal job destruction condition, (10), which is replaced by:

\[
R = k(z + b) - b + \frac{\eta(\theta)}{1 - \eta(\theta)} e^\theta - \frac{\lambda}{\rho + \lambda} \int_R^1 (s - R)dG(s)
\]

\[
- \frac{(1 - u)k + u}{(1 - u)kv'(k(z + b)) + uv'(z + b)} [v(k(z + b)) - v(z + b)] .
\]

We could distinguish two effects of wage bargaining on the optimal threshold \(R\). The first term on the RHS captures the fact that, when the wage is higher, labour is more costly and it is not worth allocating a lot of resources to people working with a low productivity. Against this effect, the last term of the RHS pushes the threshold \(R\) down in order to decrease job destruction to provide insurance against income fluctuations to risk-averse workers. The net effect of the mark-up \(k > 1\) on \(R\), and hence on unemployment, is ambiguous. By differentiating the terms involving \(k\), it is nevertheless possible to show that, starting from perfect insurance, i.e. from \(k = 1\), an increase in \(k\) induces a fall in \(R\) provided that less than half the population is unemployed. Combining this with the job creation condition, (9), it implies that a small departure from perfect insurance leads to a fall in the rate of unemployment.

The optimal policy is identical to what we previously had, except for the determination of layoff taxes. Hence, equation (8), (20) and (22) remain unchanged and (21) is replaced by:

\[
\tau = u \left[ - \frac{k(b + z) - (b + z)}{(1 - u)kv'(k(z + b)) + uv'(z + b)} [v(k(z + b)) - v(z + b)] 
\right]
\]

\[
+ \frac{\rho}{\rho + \lambda} u \left[ \frac{y}{1 - u} - R \right] + \frac{r - \rho}{r + \lambda} u \left[ 1 - u \right] - \frac{R}{\rho + \lambda} .
\]

Again, it could be shown that, for a small deviation from perfect insurance, the
term in the main bracket is negative. As in the previous section, the intuition is that the social cost of unemployment is smaller than the budgetary cost to the government. However, the first two terms, \( w - (b + z) \), which did not previously appear, suggest that the gap between the social and budgetary cost of unemployment is not as large as before since employed worker now consume too much resources compared to the unemployed.

### 6.2 Surplus Splitting

With bargaining, wages typically depend on worker’s outside opportunities which are affected by a number of endogenous parameters. In order to address this concern, we first propose to implement the optimal policy in a decentralized economy where wages are determined by surplus splitting as in Mortensen-Pissarides (1994). Thus, workers get a proportion \( \beta \) of the dollar amount of the surplus from the match. It could, fairly, be objected that worker’s risk aversion should explicitly be taken into account in the determination of the bargained wage rate. We will turn to this possibility in the next subsection. However, splitting the surplus in fixed proportions does not seem completely implausible and this is indeed the form of wage bargaining that was considered by Blanchard and Tirole (2008) in an extension to their benchmark model.

Wages are bargained over each time a productivity shock occurs. The initial net wage, denoted \( w_0(1) \), is different from others since, in case no agreement is reached, the firm does not receive the hiring subsidy but does not have to pay the firing tax\(^\text{15} \). By contrast, subsequent bargaining is not affected by the subsidy, which is sunk, but does respond to the cost of laying off a worker. The resulting net wage is denoted by \( w(x) \) for a match of productivity \( x \). The corresponding expressions are:

\[
\begin{align*}
    w_0(1) &= \beta \left[ 1 + c \theta - \tau - \lambda F + (r + \lambda)H \right] + (1 - \beta) \left[ z + b \right], \\
    w(x) &= \beta \left[ x + c \theta - \tau + rF \right] + (1 - \beta) \left[ z + b \right],
\end{align*}
\]

where we assume that workers and firm both discount future income at rate \( r \).

\(^{15}\)The layoff tax nevertheless enters the expression for the initial wage rate as it affects the firm’s expected profits from a newly created match.

\(^{16}\)Also, note that similar expressions are carefully derived in Mortensen Pissarides (2003) and in Pissarides (2000, chapter 9).
firing tax decreases it; while, subsequently, the hiring subsidy is sunk and the firing tax put workers in a stronger position. The job destruction condition, determined by \( J(R) = -F \), is now given by:

\[
R = z + b + \frac{\beta}{1 - \beta}e\theta - rF - \frac{\lambda}{r + \lambda} \int_{R}^{1} (s - R) dG(s);
\]  

while the job creation condition, resulting from free entry \( V = 0 \), is:

\[
(1 - \beta) \left[ \frac{1 - R}{r + \lambda} + H - F \right] = \frac{c}{q(\theta)}. \]  

Note that these two expressions generalize the implementability conditions that we previously had. Indeed, for \( \beta = 0 \), (47) and (48) reduce to (14) and (18), respectively.

As it is clearly impossible to implement the first-best allocation in this context of fluctuating wages, the optimal policy should be solved directly under the implementability constraints, (47), (48) and (19). The corresponding optimization problem is displayed in the appendix. As it is not analytically solvable, we now perform a calibration of the model.

We use the same functional forms and parameter values as used in Mortensen Pissarides (2003), except for risk aversion which does not appear in their model. Thus, we take a Cobb-Douglas matching function, which reduces to:

\[
q(\theta) = \theta^{-\eta}. \]  

It clearly implies a constant elasticity, \( \eta \), of matching. The distribution of idiosyncratic shocks is assumed to be uniform on \([\psi, 1] \); hence its c.d.f. is:

\[
G(x) = \frac{x - \psi}{1 - \psi}. \]  

Finally, we assume a standard constant relative risk aversion (CRRA) instantaneous utility function with coefficient \( \phi \):

\[
v(x) = \frac{x^{1 - \phi}}{1 - \phi}. \]  

The chosen exogenous parameter values are shown in the following table:

<table>
<thead>
<tr>
<th>r</th>
<th>ρ</th>
<th>λ</th>
<th>c</th>
<th>z</th>
<th>η</th>
<th>ψ</th>
<th>φ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0.02</td>
<td>0.1</td>
<td>0.3</td>
<td>0.35</td>
<td>0.5</td>
<td>0.65</td>
<td>3</td>
</tr>
</tbody>
</table>
We now report the results of the calibration for four different values of the bargaining power of workers, $\beta$. The initial case, $\beta = 0$, corresponds to the first-best benchmark. The results are displayed in the following table:

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
</tr>
</thead>
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<td>1.39</td>
<td>0.66</td>
<td>0.24</td>
</tr>
<tr>
<td>$R$</td>
<td>0.901</td>
<td>0.897</td>
<td>0.878</td>
<td>0.833</td>
</tr>
<tr>
<td>$u$ (%)</td>
<td>4.98</td>
<td>5.64</td>
<td>7.42</td>
<td>9.72</td>
</tr>
<tr>
<td>$y$</td>
<td>0.937</td>
<td>0.929</td>
<td>0.906</td>
<td>0.867</td>
</tr>
<tr>
<td>Average Wage</td>
<td>0.926</td>
<td>0.881</td>
<td>0.864</td>
<td>0.829</td>
</tr>
<tr>
<td>$b$</td>
<td>0.576</td>
<td>0.434</td>
<td>0.365</td>
<td>0.323</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.0007</td>
<td>-0.0016</td>
<td>-0.0054</td>
<td>-0.0074</td>
</tr>
<tr>
<td>$F$</td>
<td>0.706</td>
<td>0.620</td>
<td>0.594</td>
<td>0.581</td>
</tr>
<tr>
<td>$H$</td>
<td>0.295</td>
<td>0.229</td>
<td>0.0612</td>
<td>-0.225</td>
</tr>
<tr>
<td>$F - H$</td>
<td>0.411</td>
<td>0.390</td>
<td>0.533</td>
<td>0.806</td>
</tr>
<tr>
<td>Welfare Loss (%)</td>
<td>0</td>
<td>0.36</td>
<td>1.78</td>
<td>4.91</td>
</tr>
<tr>
<td>Gross Job Flow</td>
<td>0.0682</td>
<td>0.0665</td>
<td>0.0602</td>
<td>0.0473</td>
</tr>
<tr>
<td>$(1 - u)\tau/ub$ (%)</td>
<td>2.33</td>
<td>-6.04</td>
<td>-18.55</td>
<td>-21.41</td>
</tr>
</tbody>
</table>

Welfare loss is computed as equivalent to a proportional decline in consumption in the first-best case. For example, when $\beta = 0.5$, welfare is equal to what it would be in the first best case, $\beta = 0$, with consumption decreased by 1.78%. In steady state, the gross job flow is given by $uq(\theta)$ or, equivalently, by $(1 - u)\lambda G(R)$. Finally, the last row reports the share of unemployment insurance expenses financed by payroll taxes.

When workers have more bargaining power, a smaller proportion of the match surplus is captured by firms. Hence, the expected recruitment costs must adjust downward, which is achieved by reduced entry. This explains the negative relationship between $\beta$ and $\theta$. As insurance is not perfect and as the length of unemployment increases due to reduced entry of new firms, the reservation threshold $R$ also declines. The reduction in the rate of job creation being larger than that of job destruction, unemployment increases with $\beta$. Output, which in steady state can be written as $y = (1 - u) \left[ G(R) + \int_{R}^{1} sdG(s) \right]$, declines because a smaller number of people work, unemployment is higher, and the average productivity of workers is also reduced due to a lower reservation threshold. As the bargaining power of workers gets stronger, the payment of unemployment benefits becomes less efficient at providing insurance. Indeed, if the outside option of employees is more attractive, wages must be higher
thereby defeating the purpose of the unemployment insurance provision. This explains the falling level of benefits as $\beta$ increases.

The level of layoff taxes and hiring subsidies declines with $\beta$. The primary reason is that they push wages up which is undesirable for consumption smoothing. Furthermore, by increasing the gap between the two, $F - H$, the planner inflicts a desirable downward pressure on new wages, $w_0(1)$. But this comes at the cost of reducing entry, and increasing unemployment, even further. Lower unemployment benefits and a larger gap between firing taxes and hiring subsidies leads to a decline in payroll taxes, which then become negative. Another way to see this result is that, due to imperfect insurance, the social cost of unemployment is larger than its budgetary cost to the government while the corresponding social and budgetary benefits are almost equal to each other.\footnote{When $r = \rho = 0$, the social benefit from a match is equal to the firm’s surplus from producing net of recruiting costs $J_0(1) - J(R) - \frac{\varphi(R)}{q(R)}$. After some manipulations, it can be checked, using the job creation condition (48), that this is equal to the budgetary benefit from a match $F - H$.}

6.3 Nash Bargaining with Risk Aversion

In the previous subsection, we have assumed that, at each instant, the dollar amount of the surplus from the match is split in fixed proportions between the worker and the firm. However, this leads to substantial wage fluctuations which seem inconsistent with workers’ risk aversion. Since the Mortensen-Pissarides model cannot be solved under risk aversion, we consider a slight modification.

We assume that workers and firms bargain initially over a wage rate that remains fixed throughout the duration of the match. It should be emphasized that such wage bargaining is \textit{ex-ante} optimal given the risk aversion of workers and the risk neutrality of firms. However, it leads to some inefficient separations and might therefore be subject to renegotiation. Indeed, for a fixed wage, when the productivity of the match is equal to $R - \epsilon$ where $J(R) = -F$, for $\epsilon > 0$ sufficiently small, the firm chooses to layoff the worker while the worker is enjoying a strictly positive surplus from the match and would be willing to accept a wage cut to avoid becoming unemployed. It might nevertheless be possible to rationalize the existence of fixed wages in a world where workers cannot observe the productivity of a match or where firms want to build a reputation in the context of repeated interactions.

Wages being constant over time, the value of a match for a firm is still determined by equation (12) and the job destruction condition remains given by (14). Similarly for the value of a vacant position, (16), and for the job
creation condition, (18), which remain unchanged. The Bellman equations corresponding to the expected utility of an unemployed, \( U \), and of an employed worker, \( W \), are:

\[
\begin{align*}
\rho U &= v(z + b) + \theta q(\theta) [W - U], \\
\rho W &= v(w) + \lambda G(R) [U - W],
\end{align*}
\]

where, as before, \( v(.) \) stands for the instantaneous utility of consumption. The constant wage, \( w \), is determined initially by Nash bargaining:

\[
w = \arg \max_{w_i} [W_i - U]^\beta [J_i(1) + H - V]^{1-\beta},
\]

where \( i \) is used to stress the fact that the wage bargained in match \( i \) does not affect the value of outside options, i.e. the values of \( U \) or \( V \). We take into account the fact that, if an agreement is not reached, the employer does not receive the hiring subsidy. Note that, when solving the Nash bargaining problem, (54), it is important to treat the threshold \( R \), determined by (14), as a function of the wage rate. The worker’s net salary is therefore implicitly determined by:

\[
\frac{v(w) - v(z + b)}{v'(w)} \left[ 1 + \frac{\beta}{1 - \beta} \frac{c}{q(\theta)} \frac{r + \lambda}{\rho + \lambda G(R)} \right] = \frac{r + \lambda G(R) + \beta}{\rho + \lambda G(R) 1 - \beta} \frac{c}{q(\theta)}.
\]

The optimal policy could then be derived by adding this equation as a constraint to the original problem. Thus, the planner should maximize (4) with respect to \( \theta, R, b \) and \( w \) subject to (5a), (5b), (5c) and (55). The three remaining implementability constraints, (20), (21) and (22), could be left out since they jointly determine \( H, F \) and \( \tau \) which do not appear elsewhere in the planner’s problem.
Calibrating the model as before, we obtain the following results:

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>1.88</td>
<td>1.62</td>
<td>1.16</td>
<td>0.71</td>
</tr>
<tr>
<td>( R )</td>
<td>0.901</td>
<td>0.898</td>
<td>0.887</td>
<td>0.857</td>
</tr>
<tr>
<td>( u ) (%)</td>
<td>4.98</td>
<td>5.28</td>
<td>5.90</td>
<td>6.56</td>
</tr>
<tr>
<td>( y )</td>
<td>0.937</td>
<td>0.933</td>
<td>0.924</td>
<td>0.907</td>
</tr>
<tr>
<td>( w )</td>
<td>0.926</td>
<td>0.933</td>
<td>0.937</td>
<td>0.936</td>
</tr>
<tr>
<td>( b )</td>
<td>0.576</td>
<td>0.452</td>
<td>0.366</td>
<td>0.291</td>
</tr>
<tr>
<td>( \tau )</td>
<td>0.0007</td>
<td>-0.0080</td>
<td>-0.0191</td>
<td>-0.0350</td>
</tr>
<tr>
<td>( F )</td>
<td>0.706</td>
<td>0.726</td>
<td>0.792</td>
<td>0.947</td>
</tr>
<tr>
<td>( H )</td>
<td>0.295</td>
<td>0.258</td>
<td>0.170</td>
<td>0.011</td>
</tr>
<tr>
<td>( F - H )</td>
<td>0.411</td>
<td>0.468</td>
<td>0.621</td>
<td>0.936</td>
</tr>
<tr>
<td>Welfare Loss (%)</td>
<td>0</td>
<td>0.22</td>
<td>0.94</td>
<td>2.56</td>
</tr>
<tr>
<td>Gross Job Flow</td>
<td>0.0682</td>
<td>0.0671</td>
<td>0.0636</td>
<td>0.0553</td>
</tr>
<tr>
<td>((1-u)\tau/ub) (%)</td>
<td>2.33</td>
<td>-31.72</td>
<td>-83.11</td>
<td>-171.32</td>
</tr>
</tbody>
</table>

Again, the case \( \beta = 0 \) corresponds to the implementation of the first-best policy.

As \( \beta \) increases, \( \theta \) and \( R \) both decline in order to partially offset the increase in the gap between \( w \) and \( b + z \). Indeed, a higher market tightness puts workers in a stronger bargaining position which is detrimental to insurance. From the job destruction condition, (14), a decline in wage decreases the reservation threshold \( R \). As with surplus splitting, the decline in the rate of job creation being stronger than that of job destruction, unemployment increases with \( \beta \). Similarly, output falls. Unemployment benefits and recruitment costs fall so much that, despite the resource constraint (5c), wages increase slightly with bargaining power \( \beta \).

The layoff tax increases in order to impose a downward pressure on the threshold \( R \). The gap between the firing tax and hiring subsidy, \( F - H \), increases as well since, as can be seen from the job creation condition (18), the recruitment costs decline, due to falling \( \theta \), while the profits from a new match increase, thanks to a lower \( R \). Another justification for this result is that wages are bargained over before the match is formed. Hence, the layoff tax puts a downward pressure on wages while the hiring subsidy has the opposite effect. This leads the former to be increasing, and the latter decreasing, in \( \beta \). Again, payroll taxes are negative as the social cost of unemployment exceeds its budgetary cost while the social and budgetary benefits are equal to each other.

Wages could be determined by directed search, rather than by Nash bar-
gaining. In such environment, competitive market makers jointly choose the wage rate and the length of queues, equal to \(1/\theta q(\theta)\), such as to maximise the expected utility of an unemployed worker subject to a free entry condition for firms; or more formally:

\[
\max_{\theta,w} \rho U \text{ subject to } V = 0. \tag{56}
\]

This yields exactly the same equation as (55) with \(\beta\) replaced by \(\eta\). Thus, in the table above, directed search corresponds to the case where \(\beta = \eta = 0.5\). As implied by Corollary 1, directed search and the associated Hosios condition fail to ensure optimality in an economy with risk-averse workers as they do not entail adequate risk sharing.

## 7 Conclusion

In this paper, we have investigated optimal policies in a dynamic search model with risk-averse workers. More precisely, we have focused on a joint derivation of the optimal level of unemployment benefits, layoff taxes, hiring subsidies and payroll taxes.

From a policy perspective, our main conclusion is that more than full experience rating is desirable. Indeed, not only should layoff taxes pay for unemployment benefits, they should also cover the cost of providing hiring subsidies\(^{18}\). The intuition is that, by setting the social benefit from unemployment equal to its social cost, the government also sets its budgetary benefit equal to its budgetary cost. The former being the difference between layoff taxes and hiring subsidies and the latter consisting of unemployment benefits, payroll taxes are not needed. However, it should be emphasized that these insights crucially rely on a free entry assumption, which would be interesting to relax. Simulations suggest that this main conclusion is robust to cases where workers have some bargaining power.

Layoff taxes and hiring subsidies should nevertheless only be viewed as Pigouvian instruments used to correct some externalities, not as a general source of revenue for the government. This remains true even if payroll taxes are a distortionary source of fiscal revenue.

More generally, our work has shown that, without governmental intervention, labour markets with search frictions generically implement an inefficient

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\(^{18}\)This could be qualified by noting that hiring subsidies could potentially be negative. Simulations nevertheless suggest that such is not the case for reasonable calibrations of the model.
allocation of resources. With risk-neutral workers, inefficiencies are only due to unbalanced search externalities associated with deviations from the Hosios condition. Here, the inefficiency is much deeper and involves a lack of insurance against the risk of becoming unemployed.

A number of important issues are left for further research. As is often the case in the literature on optimal social insurance, we have abstracted from the possibility that agents sign non-trivial private contracts which could be a substitute to government-provided insurance. Indeed, the work of Pissarides (2004) and Fella (2007) suggests that employers might contractually offer some severance compensation. It would naturally be interesting to derive the optimal policy in a world where such contracts exist. For simplicity, we have also abstracted from the moral hazard effect of unemployment insurance. Although workers’ search intensity could easily be built in the framework of this paper, a satisfactory treatment should allow for private savings. More generally, private savings should be taken into account whenever full insurance cannot be publicly provided, such as when workers have some bargaining power. Throughout this paper, we have only considered time invariant policy instruments. In fact, in a dynamic context, it would be interesting to allow the level of unemployment benefits to be affected by the length of unemployment and that of layoff taxes and hiring subsidies to depend on the age of the match.

References


A Surplus Splitting
A.1 Wage Determination

An entrepreneur expects a stream of income $V$ from a vacancy, while an unemployed expects $U$. The initial value of a match for a firm and for a worker are denoted by $J_0(1)$ and $W_0(1)$, respectively. The corresponding subsequent values, after an idiosyncratic shock has reduced the productivity of the match to $x$, are $J(x)$ and $W(x)$. The Bellman equation for the value of a vacancy is:

$$rV = -c + q(\theta) [J_0(1) + H - V],$$

which is identical to what we previously had, cf. equation (16). The corresponding equation for the value of unemployment is:

$$rU = z + b + \theta q(\theta) [W_0(1) - U].$$
The initial wage being denoted by $w_0(1)$, the initial value of match for a firm and for a worker are given by:

$$rJ_0(1) = 1 - (w_0(1) + \tau) + \lambda \int_R^1 J(s)dG(s) - \lambda G(R)F - \lambda J_0(1),$$
$$rW_0(1) = w_0(1) + \lambda \int_R^1 W(s)dG(s) + \lambda G(R)U - \lambda W_0(1).$$

Again, the first expression is similar to equation (12) which we previously considered. Finally, the corresponding values for a subsequent match of productivity $x$, and with wage $w(x)$, are:

$$rJ(x) = x - (w(x) + \tau) + \lambda \int_R^1 J(s)dG(s) - \lambda G(R)F - \lambda J(x),$$
$$rW(x) = w(x) + \lambda \int_R^1 W(s)dG(s) + \lambda G(R)U - \lambda W(x).$$

The surplus splitting rule from which the initial wage, $w_0(1)$, is derived is:

$$(1 - \beta) [W_0(1) - U] = \beta [J_0(1) + H - V],$$

where we take into account that the firm receives the hiring subsidy in case an agreement is reached. The corresponding rule for an existing match that has just been hit by an idiosyncratic shock resulting in productivity $x$, and from which $w(x)$ is derived, is:

$$(1 - \beta) [W(x) - U] = \beta [J(x) + F - V],$$

where we now take into account the fact that, if the match dissolves, the firm needs to pay the layoff tax.

### A.2 Optimal Policy Problem

The optimal policy problem that must be solved by the planner is:

$$\max_{\theta, R, b, \tau, F, H} \int_0^\infty e^{-\rho t} \left[ nv(w_0(1)) + (1 - u - n) \int_R^1 \frac{v(w(x))}{1 - G(R)}dG(x) + uv(z + b) \right] dt$$
subject to  
\[ \dot{u} = \lambda G(R)(1 - u) - \theta q(\theta)u \]
\[ \dot{n} = \theta q(\theta)u - \lambda n \]
\[ \dot{y} = \theta q(\theta)u + \lambda(1 - u) \int_R^1 s dG(s) - \lambda y \]
\[ nw_0(1) + (1 - u - n) \int_R^1 \frac{w(x)}{1 - G(R)} dG(x) + ub = y - c\theta u \]
\[ R = z + b + \tau + \frac{\beta}{1 - \beta} c\theta - rF - \frac{\lambda}{r + \lambda} \int_R^1 (s - R)dG(s) \]
\[ (1 - \beta) \left[ \frac{1 - R}{r + \lambda} + H - F \right] = \frac{c}{q(\theta)} \]
\[ (1 - u)\tau + (1 - u)\lambda G(R)F = ub + u\theta q(\theta)H \]

where \( n \) denotes the number of matches which have not been hit by an idiosyncratic shock yet and with prevailing wage \( w_0(1) \). Clearly, the expressions for the wage rate, (45) and (46), should be substituted into the maximisation problem where needed.