Sacrifice Principle of Taxation

and how it related with ranking of income distribution and benefit principle in taxation
Sacrifice Principle of Taxation

a. What meaning can be given to the concept of equal sacrifice in taxation?

- Equal **absolute** sacrifice principle and equal **proportionate** sacrifice principle

b. What is its relationship to the ranking of distributions in terms of inequality?

c. How does Neill reconcile the sacrifice principle and the benefit principle?
Equal absolute sacrifice principle originally introduced by Mill (1848): everyone should suffer the same **absolute** loss of utility (Young, 1990)

- \( U(x) \): utility given a income level \( x \)
- \( t \): tax
- \( U(x) - U(x - t) = s \); \( s \) is the constant level of sacrifice for all income class.

Calculating \( t \):

\[
t = x - U^{-1}[U(x) - s]
\]
The Concept of Equal Sacrifice in Taxation

- Ideally, everyone has their own utility function
- Impossible in practice
- A more practical method: to consider $U(x)$ as the utility function of a “representative” member of society (Robbins, 1939; Musgrave, 1959)
- Everyone in the society is treated as if they had same utility function
The Concept of Equal Sacrifice in Taxation

- Mill: using Bernoullian utility function

- Sacrificing same amount of utility if the percentage of income sacrificed is same for all.
The Concept of Equal Sacrifice in Taxation

(Equal proportionate sacrifice principle by Cohen Stuart (1889))

Everyone should suffer the same relatively loss in utility.

\[ \frac{U(x-t)}{U(x)} = 1 - r \]; where \( r \) is the rate of loss in utility.
The Concept of Equal Sacrifice in Taxation

Equal absolute sacrifice is the same as Equal proportionate sacrifice in recent theory development.

Ok (1995) demonstrated that, under certain minor restrictions, a progressive tax function equalizes the level of absolute or proportional sacrifice according to a continuous, increasing and concave utility function.
Lorenz-domination in taxation: disposable income under T (tax system) L-dominates that under T* if and only if residual progression of T is higher than that of T* (Jakobsson, 1976)

Residual progression of taxation

\[ \frac{dc}{dy} \frac{y}{c} = \frac{1 - T'(y)}{1 - \frac{T(y)}{y}}; \]

where y is the pre-tax income, c is the post tax income and T(y) is tax under tax system T
According to Young (1987), a continuous, non-decreasing, non-constant utility function $U(x)$ corresponding to the taxation functions under the requirement of equal sacrifice principle (whether absolute or relative) must be scale-invariant.

Scale-invariant utility function means $U(x)$ exhibits constant relative risk aversion.

Constant relative risk aversion in $U(x)$:

$U(x) = \frac{(x^{1-\varepsilon} - 1)}{(1-\varepsilon)}$

where $\varepsilon$ is a constant relative risk aversion.
Residual progression under equal absolute sacrifice approach

- Assume $U(y) - U(y - t) = \text{constant}$
- $U(y) = \frac{y^{1-\varepsilon} - 1}{1-\varepsilon}$

Residual progression of $T$ (differentiate and rearrange)

- $y^{-\varepsilon} - [1 - T'(y)] [y - T(y)]^{-\varepsilon} = 0$
- $RP = \frac{[1 - T'(y)]/[1 - T(y)/y]}{= \frac{[1 - T(y)/y]}{\varepsilon}}$
- $RP > 1$ because $\varepsilon > 1$ (Cowell-Gardiner, 2000)

If there is no tax, $T(x) = 0$, residual progression is 1

Tax system under equal sacrifice principle L-dominates no taxation
Sacrifice Principle and Benefit Principle

- Benefit principle: the taxes which an agent pays should reflect the benefit that he receives from the mix of goods and services supplied by the public (Neil, 2000)
Sacrifice Principle and Benefit Principle

- Combination of benefit and equal sacrifice
  - Consider a set of \( n \) agents with the same indirect, cardinal utility function, \( U(y; x) \)
  - \( y \) is the agent's income and \( x \) is the quantity of a public good that he consumes
  - From equal sacrifice principle
    - \( U(y_i - t_i, x) - U(y_i, x) = U(y_j - t_j, x) - U(y_j, x); \)
    - \( \forall i, j; \)
Sacrifice Principle and Benefit Principle

✦ Combination of benefit and equal sacrifice

✦ If there is no tax, government cannot afford public good

✦ \( U(y_i - t_i, x) - U(y_i, 0) = U(y_j - t_j, x) - U(y_j, 0); \)
✦ \( \forall i, j; \)

✦ To optimized this equation, we take both sacrifice and benefit principle into consideration


Mill, John Stuart, Principles of Political Economy (1848), London: Longmans Green, 1917


Young, H. P. “Progressive taxation and equal sacrifice,” American Economic Review, 80, 253-266, 1990