Design: Public Goods

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EC426

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Outline

Design
  Fundamentals
  Result

Public Goods
  Characterisation
  Public Goods: voluntarism

PG Mechanisms
  PG: Restricted problem
  Second-best schemes

Conclusions
An approach to design

- Start from same point as in Lecture 1
- Arrow (1951) insight is fundamental to Public Economics
  - helps understand concepts of social welfare (Lecture 1)
  - but also connects to an “information+incentives” problem

- Why can’t the government just do what it likes?
  - maybe exogenous constraints
  - more basic: the problem of “manipulability”

- Begin by making this idea more rigorous
  - connect back to the SWF approach

- Then formalise this in a typical PE application
  - game theoretic approach
  - incomplete information
  - hidden characteristics
Social welfare and individual values

- **A social state**: $\theta \in \Theta$
- Individual $i$’s evaluation of the state $v_i(\theta), i = 1, ..., n$
  - $v_i$: member of some general class $U$
  - $U$: evaluation or utility functions $\Theta \rightarrow \mathbb{R}$

- **A profile**: $[v_1, \ldots, v_i, \ldots v_n]$
  - ordered list of functions $v_i$
  - set of all profiles: $V$

- **SWF problem**: find a constitution $\Sigma : V \rightarrow U$ satisfying
  - Unrestricted domain
  - Pareto unanimity
  - Independence of Irrelevant Alternatives

- **IF U,P, I hold then \Sigma must be dictatorial** (Arrow 1951)
  - except where there are fewer than three social states
  - “dictator” $i^*$: if $v_{i^*}(\theta) > v_{i^*}(\theta')$ then society prefers $\theta$ to $\theta'$
Social choice and manipulation

- Use the same framework as previous slide
  - *social state*: $\theta \in \Theta$
  - individual preferences $v_i(\cdot) \in U, i = 1, \ldots, n$
  - profile: $[v_1, \ldots, v_i, \ldots v_n] \in V$

- A *social choice function* $\Gamma : V \rightarrow \Theta$
  - compare this with the constitution $\Sigma$
  - same domain, but different kind of “output”

- Does an individual $i$ have power in the SCF?
  - if all tell the truth about preferences: $\theta = \Gamma(v_1, \ldots, v_i, \ldots v_n)$
  - if $i$ misrepresents preferences: $\hat{\theta} = \Gamma(v_1, \ldots, \hat{v}_i, \ldots v_n)$

- This reveals a fundamental problem
  - if $v_i(\hat{\theta}) > v_i(\theta)$ then there is an incentive to misrepresent
  - the social-choice function $\Gamma$ is *manipulable*
Implementation

• Is the SCF $\Gamma$ consistent with private economic behaviour?
  • yes if the $\theta$ picked out by $\Gamma$ is also the equilibrium of an appropriate economic game

• A mechanism is a partially specified game:
  • rules of game are fixed
  • strategy sets are specified
  • preferences not yet specified

• Plug preferences into the mechanism:
  • does the mechanism have an equilibrium?
  • does the equilibrium correspond to the desired $\theta$?
  • if so, $\theta$ is implementable

• Wide range of possible and interesting mechanisms
  • Example: the market as a mechanism
  • Implementation problem: find/design an appropriate mechanism
Design result

- Result on the SCF, $\Gamma$ (Gibbard 1973, Satterthwaite 1975, Ninjbat 2012)

  If the set of social states $\Theta$ contains at least three elements; and $\Gamma$ is defined for all logically possible preference profiles and $\Gamma$ is truthfully implementable in dominant strategies, then $\Gamma$ must be dictatorial

- Closely related to the Arrow theorem

- Has profound implications for public economics
  - misinformation may be endemic to the design problem
  - may only get truth-telling mechanisms in special cases

- Interested in two types of solution:
  1. “Full information” (“first best”) solutions
     - needs an information-revealing mechanism
  2. Second-best solutions
     - built-in constraints to prevent misrepresentation
## Typology of goods

<table>
<thead>
<tr>
<th>Excludable</th>
<th>Rival</th>
<th>Non-rival</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure private</td>
<td></td>
<td>Pure public</td>
</tr>
<tr>
<td>Non-excludable</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **Excludable**: is there a way of making people pay for the good?
- **Rival / nonrival**: need extra resources to supply an extra person?
  - For a private good, aggregate consumption is found by summing the consumption of $n$ individuals.
  - For a public good, aggregate consumption equals the consumption of each of the $n$ individuals.
Efficiency: model

- Let \( x_i \) be vector of goods consumed by person \( i \), \( x \) is aggregate vector of goods
- Consumer \( i \) has utility function \( u_i \)
- To find an efficient allocation:
  - max utility of any one person \( i \)
  - keeping the \( n - 1 \) others on a fixed utility level \( u_\ell(x_\ell) = v_\ell \)
  - satisfying production constraint \( \Phi(x) = 0 \)
- Lagrangean is \( u_i(x_i) + \sum_{\ell \neq i} \lambda_\ell [u_\ell(x_\ell) - v_\ell] - \mu \Phi(x) \)
- For a private good \( j \) consumed by person \( i \) we have the FOC
  \[ \lambda_i \frac{\partial u_i(x_i)}{\partial x_{ij}} = \mu \frac{\partial \Phi(x)}{\partial x_j} \]
- Public good \( j = 1 \) consumed by everyone equally. The FOC:
  \[ \sum_{\ell=1}^{n} \lambda_\ell \frac{\partial u_\ell(x_\ell)}{\partial x_{\ell1}} = \mu \frac{\partial \Phi(x)}{\partial x_1} \]
Efficiency: result

- Use FOC for a max to characterise efficiency conditions:
  - If goods $j$ and $k$ are both private
    \[
    \frac{\partial u_i(x_i)}{\partial x_{ik}} \bigg/ \frac{\partial u_i(x_i)}{\partial x_{ij}} = \frac{\partial \Phi(x)}{\partial x_k} \bigg/ \frac{\partial \Phi(x)}{\partial x_j}
    \]
    for every agent $i$: $\text{MRS}_i = \text{MRT}$
  - If good $j$ is private and good 1 is public
    \[
    \sum_{i=1}^{n} \frac{\partial u_i(x_i)}{\partial x_{i1}} \bigg/ \frac{\partial u_i(x_i)}{\partial x_{ij}} = \frac{\partial \Phi(x)}{\partial x_1} \bigg/ \frac{\partial \Phi(x)}{\partial x_j}
    \]
    \[
    \sum_{i=1}^{n} \text{MRS}_i = \text{MRT}
    \]
Efficiency conditions

- Derived from the FOCs
- If “wrong condition” applied to PGs – get under-provision
Strategic view

- Consider two types of Public-good game. In each case:
  - players (Greek, Roman)
  - actions ([+], [−]): (contribute, not-contribute) to public good
  - payoffs denoted by letters for each player

**Game 1**

<table>
<thead>
<tr>
<th></th>
<th>Roman [+]</th>
<th>Roman [−]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greek [+]</td>
<td>β, B</td>
<td>δ, A</td>
</tr>
<tr>
<td>Greek [−]</td>
<td>α, D</td>
<td>γ, C</td>
</tr>
</tbody>
</table>

- Three efficient outcomes
- But none of these is a NE

**Game 2**

<table>
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<th></th>
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<td>α, C</td>
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</tr>
</tbody>
</table>

- Three efficient outcomes
- Maybe implementable?
Voluntary provision

- Two-good model: $i$’s utility is $\psi(g) + x_i$

- $i$ is endowed with income $y_i$, contributes an amount $z_i$
  - so private consumption is $x_i = y_i - z_i$

- $g$ produced from total contributions $g = \phi(z)$, $z := \sum_{\ell=1}^{n} z_\ell$

- Suppose every agent $i$ makes a Cournot assumption:
  - assumes that $\sum_{\ell=1}^{n} z_\ell - z_i$ is a constant, $\bar{z}_i$
  - perceives problem as “choose $z_i$ to max $\psi(\phi(\bar{z}_i + z_i)) + y_i - z_i$”

- FOC for perceived problem is $\psi'(g) \phi'(z) - 1 = 0$
- So we get $\psi'(g) = \frac{1}{\phi'(z)}$; in other words $MRS_i = MRT$

- But for efficiency we need $\sum_{i=1}^{n} MRS_i = MRT$
Each party makes Cournot assumption

- Nash equilibrium is at intersection of the reaction functions
- Efficient outcomes given by locus of common tangencies
Willingness To Pay and the Public Good

- $p_i$ reflects person $i$’s Willingness To Pay: $\text{MRS}_i = \text{WTP}_i$
- Sum of WTP equal MC of producing public good $g^*$
Lindahl

- Introduce concept of “tax price” for funding public goods
- $p_i$: “tax price” for $i$ of PG, set by government (Lindahl 1919)
- These “prices” must satisfy $p_i = WTP_i$ and $\sum_{i=1}^{n} p_i = MRT$

- What if $i$ realises that “price” depends on announced WTP?
- WTP announced strategically: $i$ announces $\hat{WTP}_i$ knowing that
  - $p_i = \hat{WTP}_i$
  - amount of public good is $g = \phi (\text{const} + p_i g)$
  - private consumption is $x_i = y_i - p_i g$

- Suppose $i$ announces $\hat{WTP}_i$ to maximise utility $\psi(g) + x_i$
  - then this becomes exactly the problem of voluntary contribution!
Way forward

• Lindahl results in the same suboptimal outcome as voluntarism

• What can be done?
  • public provision through regular taxation
  • change the problem
  • change perception of the problem

• Alternatives to elementary model of individual rationality:
  • truthful revelation as a social norm (Johansen 1977)
  • reciprocity motives in the utility function (Guttman 1987)
  • co-operative outcome in a repeated game (Pecorino 1999)

• More promising: alternative institutional mechanisms
  • Pivot mechanisms in a restricted choice problem
  • “Forced” reciprocity
  • Provision-point mechanisms
  • Lotteries
A binary project

- An all-or-nothing choice (Clarke 1971, Groves and Ledyard 1977)

- $\Theta = \{0, 1\}$. A project in $\Theta$ completely characterised by
  - each person $i$’s endowment $y_i$ of private good
  - payment $z_i$ by $i$ if project goes ahead ($\sum_i z_i = z, \phi(z) = 1$)
  - a system of penalties

- All have zero income effect (ziff) utility: $\tau_i \psi(g) + x_i$
  - where $\psi$ is increasing, concave, $\tau_i$ is a taste parameter
  - $\tau_i$ reflects Willingness To Pay
Preferences for a binary project

\[ v_1(\theta^\circ) > v_1(\theta'); \quad v_2(\theta^\circ) < v_2(\theta') \]
A criterion for the project

- Let $CV_i$ be compensating variation for $i$ if project goes ahead
- If $S := \sum_{\ell=1}^{n} CV_\ell$ then appropriate criterion seems to be $S > 0$
  - gainers could compensate losers
  - but $S$ is unobservable (information on preferences is private)
- So, use instead the announced $CV$, $\hat{CV}_i$, and define
  \[ \hat{S} := \sum_{\ell=1}^{n} \hat{CV}_\ell, \quad \hat{S}_i := \hat{S} - \hat{CV}_i, i = 1, \ldots, n \]
- If $\hat{S}$ and $\hat{S}_i$ have opposite signs then person $i$ is *pivotal*
- Now consider the following criterion
  1. Approve (reject) the project if $\hat{S} \geq 0$ ($\hat{S} < 0$)
  2. If $i$ is pivotal, then impose a penalty of $\hat{S}_i$ on person $i$
- Mechanism guarantees that truth-telling is a dominant strategy
Payoff to $i$ under the mechanism

$$
\begin{array}{cc}
\hat{S} < 0 & \hat{S} \geq 0 \\
(\theta^\circ \text{ chosen}) & (\theta' \text{ chosen}) \\
\hat{S} - i < 0 & v_i(\theta^\circ) \\
\hat{S} - i \geq 0 & v_i(\theta') - \hat{S} - i \\
\end{array}
$$

- Person $i$'s true valuations are $v_i(\theta^\circ), v_i(\theta')$
- Person $i$ announces $\hat{v}_i(\theta^\circ), \hat{v}_i(\theta')$
- Limitations:
  - the amounts $\hat{S} - i$ have to be computed for all $n$ persons
  - mechanism applies only to binary projects
A binary choice

- Tax/subsidy to persuade people to reciprocate? (Gradstein 1998)

- All have same ziff utility, endowed with 1 unit of private good
  - $i$’s contribution choice is represented as $z_i \in \{0, 1\}$
  - $i$ has unobservable cost of contribution $c_i$
  - Utility: $\psi(g) + 1 - z_i c_i$

- Public good production: $g = \phi(q)$
  - where $q$ is proportion of contributors

- For an efficient outcome want low-cost agents to contribute
  - for some cut-off value $\bar{c}$,
    
    $$z_i = \begin{cases} 
    1 & \text{if } c_i \leq \bar{c}, \\
    0 & \text{otherwise}
    \end{cases}$$

- If provision is left to private action there will be underprovision
Binary choice: tax/subsidy

- Government knows the distribution $F(\cdot)$ of contribution costs
  - can condition on the threshold value $\bar{c}$

- Design tax/subsidy scheme based on observables:
  - subsidy $s > 0$ if you contribute
  - tax $t > 0$ if you don’t contribute

- Person with critical cost $\bar{c}$ gets utility:
  - $\psi(g) + 1 - \bar{c} + s = \psi(g) + 1 - t$
  - implies $s + t = \bar{c}$

- Those with costs $c_i < \bar{c}$ will contribute; those with $c_i > \bar{c}$ will not
- Breakeven achieved if $sF(\bar{c}) = t[1 - F(\bar{c})]$
  - the necessary tax to achieve this is $t = \bar{c}F(\bar{c})$
Provision-point mechanism

- Voluntary contribution *plus* target value $z^*$ *plus* refund scheme
  - target $z^*$ exceeded: a rebate in proportion to your contribution
  - if $z^*$ is not reached, all contributions refunded

- If total contributions are $\bar{z}$ and agent $i$’s share is $\pi_i$, utility is

$$\psi(\phi(z^*)) + \pi_i [\bar{z} - z^*] + y_i - z_i, \quad \text{if } \bar{z} \geq z^*$$
$$y_i \quad \text{otherwise}$$

- Each agent *appears* to have an incentive to report truthfully

- Issues arising:
  - $z^*$ must be exogenous
  - but how is $z^*$ determined?
  - better than voluntarism in practice? (Rondeau et al. 2005)
Lottery mechanism

- If it is a fair lottery with fixed prize \( P \) then amount of public good is \( g = \phi(\bar{z} - P) \)
  - probability of winning is \( \pi_i = z_i / \bar{z} \)
  - expected utility is \( \psi_i(g) + \pi_i P + y_i - z_i \)

- Again \( i \) makes the Cournot assumption when maximising
  - FOC gives \( \psi'_i(g) = \beta(P) / (z_i - P) \) where \( \beta(P) := 1 - \bar{z}P/z_i^2 < 1 \)
  - A higher \( P \) results in more public good being provided

- Fixed-prize lottery introduces an offsetting externality
  - each time you buy a lottery ticket you affect others’ chances of winning (Morgan 2000, Morgan and Sefton 2000)
Summary

- Design principles associated with social-choice problem
  - Arrow and Gibbard-Satterthwaite theorems connected
  - associated with an imperfect-information problem

- Public goods combine special properties
  - more than one “cause for market failure”
  - easy to solve the characterisation problem
  - implementation problems are much harder

- Mechanism design depends on:
  - the type of public good
  - the economic environment (Morgan 2000, Rondeau et al. 2005)
Coming up...

- Much more on design principles and application
  - apply principles to taxation
    - [lecture 3]
- Introduce “hidden-action” problems
  - moral hazard social insurance
    - [lecture 6]
- More on “hidden-characteristics” problem
  - adverse selection and health insurance
    - [lecture 7]
- Develop analysis of externalities
  - key component of public goods
    - but also wider importance
    - [lecture 9]
Bibliography I


Econometrica 45, 783–809.


