Design: Public Goods

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EC426

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Outline

Design
  Fundamentals
  Result

Public Goods
  Characterisation
  Public Goods: voluntarism

PG Mechanisms
  PG: Restricted problem
  Second-best schemes

Conclusions
An approach to design

- Start from same point as in Lecture 1
- Arrow (1951) insight is fundamental to Public Economics
  - helps understand concepts of social welfare (Lecture 1)
  - but also connects to an “information+incentives” problem
- Why can’t the government just do what it likes?
  - maybe exogenous constraints
  - more basic: the problem of “manipulability”
- Begin by making this idea more rigorous
  - connect back to the SWF approach
- Then formalise this in a typical PE application
  - game theoretic approach
  - incomplete information
  - hidden characteristics
Social welfare and individual values

- **A social state:** $\theta \in \Theta$
  
- Individual $i$’s evaluation of the state $v_i (\theta), i = 1, ..., n$
  
  - $v_i$: member of some general class $\mathbb{U}$
  - $\mathbb{U}$: evaluation or utility functions $\Theta \rightarrow \mathbb{R}$

- **A profile:** $[v_1, \ldots, v_i, \ldots v_n]$
  
  - ordered list of functions $v_i$
  - set of all profiles: $\mathbb{V}$

- **SWF problem:** find a constitution $\Sigma : \mathbb{V} \rightarrow \mathbb{U}$ satisfying
  
  - Unrestricted domain
  - Pareto unanimity
  - Independence of Irrelevant Alternatives

- **IF $\mathbb{U}, \mathbb{P}, \mathbb{I}$ hold then $\Sigma$ must be dictatorial (Arrow 1951)**
  
  - except where there are fewer than three social states
  - “dictator” $i^*$: if $v_{i^*} (\theta) > v_{i^*} (\theta')$ then society prefers $\theta$ to $\theta'$
Social choice and manipulation

- Use the same framework as previous slide
  - social state: $\theta \in \Theta$
  - individual preferences $v_i(\cdot) \in \mathbb{U}, i = 1, \ldots, n$
  - profile: $[v_1, \ldots, v_i, \ldots v_n] \in \mathbb{V}$

- A social choice function $\Gamma: \mathbb{V} \rightarrow \Theta$
  - compare this with the constitution $\Sigma$
  - same domain, but different kind of “output”

- Does an individual $i$ have power in the SCF?
  - if all tell the truth about preferences: $\theta = \Gamma(v_1, \ldots, v_i, \ldots v_n)$
  - if $i$ misrepresents preferences: $\hat{\theta} = \Gamma(v_1, \ldots, \hat{v}_i, \ldots v_n)$

- This reveals a fundamental problem
  - if $v_i(\hat{\theta}) > v_i(\theta)$ then there is an incentive to misrepresent
  - the social-choice function $\Gamma$ is manipulable
Implementation

- Is the SCF $\Gamma$ consistent with private economic behaviour?
  - yes if the $\theta$ picked out by $\Gamma$ is also the equilibrium of an appropriate economic game

- A *mechanism* is a partially specified game:
  - rules of game are fixed
  - strategy sets are specified
  - preferences not yet specified

- Plug preferences into the mechanism:
  - does the mechanism have an equilibrium?
  - does the equilibrium correspond to the desired $\theta$?
  - if so, $\theta$ is *implementable*

- Wide range of possible and interesting mechanisms
  - Example: the market as a mechanism
  - Implementation problem: find/design an appropriate mechanism
Design result

• Result on the SCF, \( \Gamma \) (Gibbard 1973, Satterthwaite 1975, Ninjbat 2012)

\[
\text{If the set of social states } \Theta \text{ contains at least three elements; and } \Gamma \text{ is defined for all logically possible preference profiles and } \Gamma \text{ is truthfully implementable in dominant strategies, then } \Gamma \text{ must be dictatorial}
\]

• Closely related to the Arrow theorem

• Has profound implications for public economics
  • misinformation may be endemic to the design problem
  • may only get truth-telling mechanisms in special cases

• Interested in two types of solution:
  1. “Full information” (“first best”) solutions
     • needs an information-revealing mechanism
  2. Second-best solutions
     • built-in constraints to prevent misrepresentation
### Typology of goods

<table>
<thead>
<tr>
<th>Excludable</th>
<th>Rival</th>
<th>Non-rival</th>
</tr>
</thead>
<tbody>
<tr>
<td>pure private</td>
<td></td>
<td>pure public</td>
</tr>
<tr>
<td>Non-excludable</td>
<td>?</td>
<td></td>
</tr>
</tbody>
</table>

- **Excludable**: is there a way of making people pay for the good?

- **Rival / nonrival**: need extra resources to supply an extra person?
  - For a private good, aggregate consumption is found by summing the consumption of \( n \) individuals
  - For a public good, aggregate consumption equals the consumption of each of the \( n \) individuals
Efficiency: model

- Let $x_i$ be vector of goods consumed by person $i$, $x$ is aggregate vector of goods
- Consumer $i$ has utility function $u_i$
- To find an efficient allocation:
  - max utility of any one person $i$
  - keeping the $n - 1$ others on a fixed utility level $u_\ell (x_\ell) = v_\ell$
  - satisfying production constraint $\Phi(x) = 0$
- Lagrangean is $u_i (x_i) + \sum_{\ell \neq i} \lambda_\ell [u_\ell (x_\ell) - v_\ell] - \mu \Phi(x)$
- For a private good $j$ consumed by person $i$ we have the FOC
  $$\lambda_i \frac{\partial u_i (x_i)}{\partial x_{ij}} = \mu \frac{\partial \Phi(x)}{\partial x_j}$$
- Public good $j = 1$ consumed by everyone equally. The FOC:
  $$\sum_{\ell=1}^{n} \lambda_\ell \frac{\partial u_\ell (x_\ell)}{\partial x_{\ell 1}} = \mu \frac{\partial \Phi(x)}{\partial x_1}$$
Efficiency: result

- Use FOC for a max to characterise efficiency conditions:

- If goods $j$ and $k$ are both private

\[
\frac{\partial u_i(x_i)}{\partial x_{ik}} \div \frac{\partial u_i(x_i)}{\partial x_{ij}} = \frac{\partial \Phi(x)}{\partial x_{k}} \div \frac{\partial \Phi(x)}{\partial x_{j}}
\]

for every agent $i$: $\text{MRS}_i = \text{MRT}$

- If good $j$ is private and good 1 is public

\[
\sum_{i=1}^{n} \frac{\partial u_i(x_i)}{\partial x_{i1}} \div \frac{\partial u_i(x_i)}{\partial x_{ij}} = \frac{\partial \Phi(x)}{\partial x_{1}} \div \frac{\partial \Phi(x)}{\partial x_{j}}
\]

\[
\sum_{i=1}^{n} \text{MRS}_i = \text{MRT}
\]
Efficiency conditions

- Derived from the FOCs
- If “wrong condition” applied to PGs – get under-provision
Strategic view

- Consider two types of Public-good game. In each case:
  - players (Greek, Roman)
  - actions ([+], [−]): (contribute, not-contribute) to public good
  - payoffs denoted by letters for each player

<table>
<thead>
<tr>
<th>Game 1</th>
<th>Game 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roman</td>
<td>Roman</td>
</tr>
<tr>
<td>[+]</td>
<td>[+]</td>
</tr>
<tr>
<td>(\beta, B)</td>
<td>(\beta, B)</td>
</tr>
<tr>
<td>(\delta, A)</td>
<td>(\gamma, A)</td>
</tr>
<tr>
<td>[−]</td>
<td>[−]</td>
</tr>
<tr>
<td>(\alpha, D)</td>
<td>(\alpha, C)</td>
</tr>
<tr>
<td>(\gamma, C)</td>
<td>(\delta, D)</td>
</tr>
</tbody>
</table>

- Three efficient outcomes
- But none of these is a NE
- Three efficient outcomes
- Maybe implementable?
Voluntary provision

- Two-good model: $i$’s utility is $\psi(g) + x_i$

- $i$ is endowed with income $y_i$, contributes an amount $z_i$
  - so private consumption is $x_i = y_i - z_i$

- $g$ produced from total contributions $g = \phi(z)$, $z := \sum_{\ell=1}^{n} z_\ell$

- Suppose every agent $i$ makes a Cournot assumption:
  - assumes that $\sum_{\ell=1}^{n} z_\ell - z_i$ is a constant, $\bar{z} - i$
  - perceives problem as “choose $z_i$ to max $\psi(\phi(\bar{z}_i + z_i)) + y_i - z_i$”

- FOC for perceived problem is $\psi'(g) \phi'(z) - 1 = 0$
- So we get $\psi'(g) = \frac{1}{\phi'(z)}$; in other words $\text{MRS}_i = \text{MRT}$

- But for efficiency we need $\sum_{i=1}^{n} \text{MRS}_i = \text{MRT}$
Outcomes of contribution game

Each party makes Cournot assumption

- Nash equilibrium is at intersection of the reaction functions
- Efficient outcomes given by locus of common tangencies
• $p_i$ reflects person $i$’s Willingness To Pay: $\text{MRS}_i = \text{WTP}_i$
• Sum of WTP equal MC of producing public good $g^*$
Lindahl

- Introduce concept of “tax price” for funding public goods
- \( p_i \): “tax price” for \( i \) of PG, set by government (Lindahl 1919)
- These “prices” must satisfy \( p_i = \text{WTP}_i \) and \( \sum_{i=1}^{n} p_i = \text{MRT} \)

- What if \( i \) realises that “price” depends on announced WTP?
- WTP announced strategically: \( i \) announces \( \hat{\text{WTP}}_i \) knowing that
  - \( p_i = \hat{\text{WTP}}_i \)
  - amount of public good is \( g = \phi (\text{const} + p_ig) \)
  - private consumption is \( x_i = y_i - p_ig \)

- Suppose \( i \) announces \( \hat{\text{WTP}}_i \) to maximise utility \( \psi(g) + x_i \)
  - then this becomes exactly the problem of voluntary contribution!
Way forward

• Lindahl results in the same suboptimal outcome as voluntarism

• What can be done?
  • public provision through regular taxation
  • change the problem
  • change perception of the problem

• Alternatives to elementary model of individual rationality:
  • truthful revelation as a social norm (Johansen 1977)
  • reciprocity motives in the utility function (Guttman 1987)
  • co-operative outcome in a repeated game (Pecorino 1999)

• More promising: alternative institutional mechanisms
  • Pivot mechanisms in a restricted choice problem
  • “Forced” reciprocity
  • Provision-point mechanisms
  • Lotteries
A binary project

- An all-or-nothing choice (Clarke 1971, Groves and Ledyard 1977)

- $\Theta = \{0, 1\}$. A project in $\Theta$ completely characterised by
  - each person $i$’s endowment $y_i$ of private good
  - payment $z_i$ by $i$ if project goes ahead ($\sum_i z_i = z, \phi(z) = 1$)
  - a system of penalties

- All have zero income effect (ziff) utility: $\tau_i \psi(g) + x_i$
  - where $\psi$ is increasing, concave, $\tau_i$ is a taste parameter
  - $\tau_i$ reflects Willingness To Pay
Preferences for a binary project

\[ v_1(\theta^\circ) > v_1(\theta') \; ; \; v_2(\theta^\circ) < v_2(\theta') \]
A criterion for the project

- Let $CV_i$ be compensating variation for $i$ if project goes ahead.
- If $S := \sum_{\ell=1}^{n} CV_{\ell}$ then appropriate criterion seems to be $S > 0$.
  - gainers could compensate losers
  - but $S$ is unobservable (information on preferences is private)
- So, use instead the announced $CV$, $\hat{CV}_i$, and define
  $$\hat{S} := \sum_{\ell=1}^{n} \hat{CV}_{\ell}, \quad \hat{S}_i := \hat{S} - \hat{CV}_i, \quad i = 1, \ldots, n$$
- If $\hat{S}$ and $\hat{S}_i$ have opposite signs then person $i$ is pivotal.
- Now consider the following criterion
  1. Approve (reject) the project if $\hat{S} \geq 0$ ($\hat{S} < 0$)
  2. If $i$ is pivotal, then impose a penalty of $\hat{S}_i$ on person $i$.
- Mechanism guarantees that truth-telling is a dominant strategy.
Payoff to $i$ under the mechanism

<table>
<thead>
<tr>
<th>$\hat{S}_i &lt; 0$ ((\theta^\circ) chosen)</th>
<th>$\hat{S} \geq 0$ ((\theta') chosen)</th>
</tr>
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<tbody>
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<td>$\hat{S}_i &lt; 0$</td>
<td>$\hat{S}_i \geq 0$</td>
</tr>
<tr>
<td>$\hat{S}_i$</td>
<td>$\hat{S}_i$</td>
</tr>
</tbody>
</table>

- Person $i$’s true valuations are $v_i(\theta^\circ), v_i(\theta')$
- Person $i$ announces $\hat{v}_i(\theta^\circ), \hat{v}_i(\theta')$

**Limitations:**
- the amounts $\hat{S}_i$ have to be computed for all $n$ persons
- mechanism applies only to binary projects
A binary choice

- Tax/subsidy to persuade people to reciprocate? (Gradstein 1998)

- All have same ziff utility, endowed with 1 unit of private good
  - $i$’s contribution choice is represented as $z_i \in \{0, 1\}$
  - $i$ has unobservable cost of contribution $c_i$
  - Utility: $\psi(g) + 1 - z_i c_i$

- Public good production: $g = \phi(q)$
  - where $q$ is proportion of contributors

- For an efficient outcome want low-cost agents to contribute
  - for some cut-off value $\bar{c}$,
    \[
      z_i = \begin{cases} 
        1 & \text{if } c_i \leq \bar{c}, \\
        0 & \text{otherwise}
    \end{cases}
    \]

- If provision is left to private action there will be underprovision
Binary choice: tax/subsidy

- Government knows the distribution $F(\cdot)$ of contribution costs
  - can condition on the threshold value $\bar{c}$
- Design tax/subsidy scheme based on observables:
  - subsidy $s > 0$ if you contribute
  - tax $t > 0$ if you don’t contribute
- person with critical cost $\bar{c}$ gets utility:
  - $\psi(g) + 1 - \bar{c} + s = \psi(g) + 1 - t$
  - implies $s + t = \bar{c}$
- Those with costs $c_i < \bar{c}$ will contribute; those with $c_i > \bar{c}$ will not
- Breakeven achieved if $sF(\bar{c}) = t[1 - F(\bar{c})]$
  - the necessary tax to achieve this is $t = \bar{c}F(\bar{c})$
Provision-point mechanism

- Voluntary contribution *plus* target value \( z^* \) *plus* refund scheme
  - target \( z^* \) exceeded: a rebate in proportion to your contribution
  - if \( z^* \) is not reached, all contributions refunded

- If total contributions are \( \bar{z} \) and agent \( i \)'s share is \( \pi_i \), utility is

\[
\psi(\phi(z^*)) + \pi_i[\bar{z} - z^*] + y_i - z_i, \quad \text{if } \bar{z} \geq z^*
\]
\[
y_i \quad \text{otherwise}
\]

- Each agent *appears* to have an incentive to report truthfully

- Issues arising:
  - \( z^* \) must be exogenous
  - but how is \( z^* \) determined?
  - better than voluntarism in practice? (Rondeau et al. 2005)
Lottery mechanism

- If it is a fair lottery with fixed prize $P$ then amount of public good is $g = \phi(\bar{z} - P)$
  - probability of winning is $\pi_i = z_i / \bar{z}$
  - expected utility is $\psi_i(g) + \pi_i P + y_i - z_i$

- Again $i$ makes the Cournot assumption when maximising
  - FOC gives $\psi'_i(g) = \beta(P)/(z_i - P)$ where $\beta(P) := 1 - \bar{z}P/z_i^2 < 1$
  - A higher $P$ results in more public good being provided

- Fixed-prize lottery introduces an offsetting externality
  - each time you buy a lottery ticket you affect others’ chances of winning (Morgan 2000, Morgan and Sefton 2000)
Summary

• Design principles associated with social-choice problem
  • Arrow and Gibbard-Satterthwaite theorems connected
  • associated with an imperfect-information problem

• Public goods combine special properties
  • more than one “cause for market failure”
  • easy to solve the characterisation problem
  • implementation problems are much harder

• Mechanism design depends on:
  • the type of public good
  • the economic environment (Morgan 2000, Rondeau et al. 2005)
Coming up...

- Much more on design principles and application
  - apply principles to taxation
    - [lecture 3]
- Introduce “hidden-action” problems
  - moral hazard social insurance
    - [lecture 6]
- More on “hidden-characteristics” problem
  - adverse selection and health insurance
    - [lecture 7]
- Develop analysis of externalities
  - key component of public goods
    - but also wider importance
    - [lecture 9]


