Welfare and distributional analysis

- We’ve seen welfare basis for redistribution as a policy objective
  - how to assess the impact and effectiveness of such policy?
  - need appropriate criteria for comparing distributions of income
  - issues of distributional analysis (Cowell 2008, 2011)

- Distributional analysis covers broad class of economic problems
  - inequality
  - social welfare
  - poverty

- Implementation: use income as a proxy for utility?
  - rules out utility interdependence
  - neglects risk aversion
  - method of achieving comparability between unlike people
Utility and income: comparability

• Adjust for needs using an *equivalence scale*: \( x = \chi (y, a) \)
  - \( a \): personal attributes (identity, needs, abilities…)
  - \( y \): conventionally defined real income
  - \( x \): equivalised income (money-metric utility)

• Special case – income-independent equivalence scale
  - \( x = y / \nu(a) \)
  - Where \( \nu \) is number of equivalent adults

• Where does the function \( \chi \) come from?
  - official government sources
  - bodies such as OECD
  - models of household budgets

• Example: adjusting for need
  - plot share of food in budget against income
  - a reference household type…
  - Engel Equivalence Scale

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**Diagram:**
- Engel Equivalence Scale for childless couple and couple with children.
Overview...

Basic principles of distributional comparisons
Social-welfare functions

• A standard approach to a method of assessment
  • assume individual utility can be measured by \( x \)
  • assume a fixed population of size \( n \)
  • income distributions in extended Irene-Janet notation \( \mathbf{x} := (x_1, x_2, \ldots, x_n) \)

• Basic tool is a *social welfare function* (SWF)
  • maps set of distributions into the real line
  • i.e. for each distribution we get one specific number
  • \( W = W(x_1, x_2, \ldots, x_n) = W(\mathbf{x}) \)

• Properties will depend on economic principles
  • principles on which SWF be based?
  • Use a simple framework to list some of the basic axioms
  • -- *Amiel-Cowell (1999)* Appendix A
SWF axioms

- **Anonymity.** Suppose $x'$ is a permutation of $x$. Then:
  $$W(x') = W(x)$$

- **Population principle.**
  $$W(x) \geq W(y) \Rightarrow W(x,x,...,x) \geq W(y,y,...,y)$$

- **Decomposability.** Suppose $x'$ is formed by joining $x$ with $z$ and $y'$ is formed by joining $y$ with $z$. Then:
  $$W(x) \geq W(y) \Rightarrow W(x') \geq W(y')$$

- **Monotonicity.**
  $$W(x_1,x_2..., x_i+\delta,..., x_n) > W(x_1,x_2..., x_i,..., x_n)$$

- **Transfer principle.** (Dalton 1920) Suppose $x_i < x_j$ then for small $\delta$:
  $$W(x_1,x_2..., x_i+\delta,..., x_j-\delta,..., x_n) > W(x_1,x_2..., x_i,..., x_n)$$

- **Scale invariance.**
  $$W(x) \geq W(y) \Rightarrow W(\lambda x) \geq W(\lambda y)$$
Use axioms to characterise important *classes* of SWF

**Anonymity** and **population** principle imply we can write SWF in either Irene-Janet form or in terms of distribution function

- most modern approaches use these assumptions
- but may need to standardise for needs etc

Introduce **decomposability** and get class of Additive SWFs $\mathcal{W}$:

- $W(x) = \sum_i u(x_i)$
- or equivalently $W(F) = \int u(x) \, dF(x)$
- think of $u(\bullet)$ as *social utility* or *social evaluation* function

The class $\mathcal{W}$ is of great importance

- But $\mathcal{W}$ excludes some well-known welfare criteria
Classes of SWFs (2)

• From $\mathcal{W}$ we get important subclasses

• If we impose monotonicity we get
  • $\mathcal{W}_1 \subseteq \mathcal{W} : u(\cdot)$ increasing
  • subclass where marginal social utility always positive

• If we further impose the transfer principle we get
  • $\mathcal{W}_2 \subseteq \mathcal{W}_1 : u(\cdot)$ increasing and concave
  • subclass where marginal social utility is positive and decreasing

• We often need to use these special subclasses

• Illustrate their properties with a simple example…
Overview...

Classes of SWF and distributional comparisons
Ranking and dominance

• Introduce two simple concepts
  • Illustrate them first using the Irene-Janet representation
  • take income vectors $x$ and $y$ for a given $n$

• First-order dominance:
  • $y_1 > x_1$, $y_2 > x_2$, $y_3 > x_3$
  • Each ordered income in $y$ larger than that in $x$

• Second-order dominance:
  • $y_1 > x_1$, $y_1 + y_2 > x_1 + x_2$, $y_1 + y_2 + \ldots + y_n > x_1 + x_2 + \ldots + x_n$
  • Each cumulated income sum in $y$ larger than that in $x$

• Need to generalise this a little
  • given anonymity+pop principle can represent distributions in $F$-form
  • $q$: population proportion ($0 \leq q \leq 1$)
  • $F(x)$: proportion of population with incomes $\leq x$
  • $\mu(F)$: mean of distribution $F$
1st-Order approach

• Basic tool is the quantile, expressed as
  \[ Q(F; q) := \inf \{ x \mid F(x) \geq q \} \]
  • “smallest income such that cumulative frequency is at least as great as \( q \)”

• Use this to derive a number of intuitive concepts
  • interquartile range, decile-ratios, semi-decile ratios
  • graph of \( Q \)

• Also to characterise 1st-order (quantile) dominance:
  • “\( G \) quantile-dominates \( F \)” means:
    for every \( q \), \( Q(G;q) \geq Q(F;q) \),
    for some \( q \), \( Q(G;q) > Q(F;q) \)

• A fundamental result:
  • \( G \) quantile-dominates \( F \) iff \( W(G) > W(F) \) for all \( W \in \mathcal{W}_1 \)
Parade and 1\textsuperscript{st}-order dominance

- Plot quantiles against proportion of population
- Parade for distribution $F$
- Parade for distribution $G$

- Here $G$ clearly quantile-dominates $F$
- But (as often happens) what if it doesn’t?
- Examine second-order method
2\textsuperscript{nd}-Order approach

- Basic tool is the \textit{income cumulant}, expressed as
  \[C(F; q) := \int Q(F; q) x \, dF(x)\]
  - “The sum of incomes in the Parade, up to and including position \(q\)”

- Use this to derive a number of intuitive concepts
  - the “shares” ranking, Gini coefficient
  - graph of \(C\) the \textit{generalised Lorenz curve}

- Also to characterise the idea of 2\textsuperscript{nd}-order (cumulant) dominance:
  - “\(G\) cumulant-dominates \(F\)” means:
    - for every \(q\), \(C(G; q) \geq C(F; q)\),
    - for some \(q\), \(C(G; q) > C(F; q)\)

- A fundamental result (Shorrocks 1983):
  - \(G\) cumulant-dominates \(F\) iff \(W(G) > W(F)\) for all \(W \in \mathcal{W}_2\)
GLC and 2nd-order dominance

- Plot cumulations against proportion of population
- GLC for distribution $F$
- GLC for distribution $G$

- Intercept on vertical axis is at mean income

Cumulative income

$C(.; q)$

$C(G; .)$

$C(F; .)$

$\mu(G)$

$\mu(F)$

$q$

0 1

Cumulative income
2nd-Order approach (continued)

- A useful tool: the *share* of the proportion $q$ of distribution $F$ is
  \[ L(F; q) := \frac{C(F; q)}{\mu(F)} \]
  - “income cumulation at divided $q$ by total income”

- Yields Lorenz dominance, or the “shares” ranking:
  - “$G$ Lorenz-dominates $F$” means:
    - for every $q$, $L(G; q) \geq L(F; q)$,
    - for some $q$, $L(G; q) > L(F; q)$

- Another fundamental result *(Atkinson 1970)*:
  - For given $\mu$, $G$ Lorenz-dominates $F$ iff $W(G) > W(F)$ for all $W \in \mathcal{M}_2$
Lorenz curve and ranking

- Plot shares against proportion of population
- Perfect equality
- Lorenz curve for distribution $F$
- Lorenz curve for distribution $G$

Here $G$ clearly Lorenz-dominates $F$

So $F$ displays more inequality than $G$

But what if $L(F)$ and $L(G)$ intersect?

No clear statement about inequality (or welfare) is possible without further information
Overview...

Quantifying social values

- Equity, Social Welfare, Taxation
  - Welfare axioms
  - Rankings
  - Equity and social welfare
  - Taxation and sacrifice
Equity and social welfare

• So far we have just general principles
  • may lead to ambiguous results
  • need a tighter description of social-welfare?
• Consider “reduced form” of social welfare
  • \( W = \Omega (\mu, I) \)
  • \( \mu = \mu(F) \) is mean of distribution \( F \)
  • \( I = I(F) \) is inequality of distribution \( F \)
  • \( \Omega \) embodies trade-off between objectives
• \( I \) can be taken as an “equity” criterion
  • from same roots as SWF?
  • or independently determined?
• Consider a “natural” definition of inequality
Gini coefficient

- Redraw Lorenz diagram
- A “natural” inequality measure…?
  \[ 1 - 2 \int_0^1 L(F; q) dq \]
  - normalised area above Lorenz curve
- Also represented as normalised difference between income pairs:
  - In $F$-form:
    \[ \frac{1}{2\mu(F)} \int \int |x - x'| dF(x)dF(x') \]
  - In Irene-Janet terms:
    \[ \frac{1}{2n^2\mu(x)} \sum_{i=1}^{n} \sum_{j=1}^{n} |x_i - x_j| \]
- Intuition is neat, but
  - inequality index is clearly arbitrary
  - can we find one based on welfare criteria?
SWF and inequality

- The Irene & Janet diagram
- A given distribution
- Distributions with same mean
- Contours of the SWF
- Construct an equal distribution $E$ such that $W(E) = W(F)$
- Equally-Distributed Equivalent income
- Social waste from inequality

- contour: $x$ values such that $W(x) = \text{const}$
- curvature of contour indicates society’s willingness to tolerate “efficiency loss” in pursuit of greater equality
An important family of SWF

- Take the $\mathcal{W}_2$ subclass and impose **scale invariance**.
- Get the family of SWFs where $u$ is iso-elastic:

$$u(x) = \frac{x^{1-\varepsilon} - 1}{1 - \varepsilon}, \quad \varepsilon \geq 0$$

- has same form as CRRA utility function
- **Parameter $\varepsilon$ captures society’s inequality aversion.**
  - Similar interpretation to individual risk aversion
  - See Atkinson (1970)
Welfare-based inequality

• From the concept of social waste Atkinson (1970) suggested an inequality measure:

\[ I(F) = 1 - \frac{\xi(F)}{\mu(F)} \]

• Atkinson further assumed
  • additive SWF, \( W(F) = \int u(x) \, dF(x) \)
  • isoelastic \( u \)

• So inequality takes the form

\[ I^\xi_A(F') := 1 - \frac{1}{\mu(F')} \left[ \int x^{1-\varepsilon} dF(x) \right]^{\frac{1}{1-\varepsilon}} \]

• But what value of inequality aversion parameter \( \varepsilon \)?
Values: the issues

- SWF is central to public policy making
  - Practical example in *HM Treasury (2011)* pp 93-94
  - We need to focus on two questions…

**First:** do people care about distribution?
- Experiments suggest they do – *Carlsson et al (2005)*
- Do social and economic factors make a difference?

**Second:** What is the shape of $u$? (*Cowell-Gardiner 2000*)
- Direct estimates of inequality aversion
- Estimates of risk aversion as proxy for inequality aversion
- Indirect estimates of risk aversion
- Indirect estimates of inequality aversion from choices made by government
Preferences, happiness and welfare?

• **Ebert, U. and Welsch, H. (2009)**
  - Evaluate subjective well being as a function of personal and environmental data using $\Omega$ form
  - Examine which inequality index seems to fit preferences best

• **Alesina et al (2004)**
  - Use data on happiness from social survey
  - Construct a model of the determinants of happiness
  - Use this to see if income inequality makes a difference

• Seems to be a difference in priorities between US and Europe

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>Continental Europe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of government in GDP</td>
<td>30%</td>
<td>45%</td>
</tr>
<tr>
<td>Share of transfers in GDP</td>
<td>11%</td>
<td>18%</td>
</tr>
</tbody>
</table>

• Do people in Europe care more about inequality?
The Alesina et al model

\[ \text{Happy}_{ist}^g = \alpha^g \text{Inequality}_{st} + \beta^g \text{MACRO}_{st} + \delta^g \text{MICRO}_{ist} + \eta^g_s + \mu^g_t + \epsilon^g_{ist} \]

- Ordered logit model
  - “Happy” is categorical: “not too happy”, “fairly happy”, “very happy”
  - individual, state, time, group.
  - Macro variables include inflation, unemployment rate
  - Micro variables include personal characteristics
  - \( \eta, \mu \) are state, time dummies

- Results
  - People declare lower happiness levels when inequality is high.
  - Strong negative effect of inequality on the European poor and leftists
  - No effects of inequality on happiness of US poor and left-wingers
  - Negative effect of inequality on happiness of US rich
  - No differences across the American right and the European right.
  - No differences between the American rich and the European rich
Inequality aversion and Elasticity of MU

• Consistent inequality preferences?
  • Preference reversals (Amiel et al 2008)

• What value for $\varepsilon$?
  • from happiness studies 1.0 to 1.5 (Layard et al 2008)
  • related to extent of inequality in the country? (Lambert et al 2003)
  • affected by way the question is put? (Pirttilä and Uusitalo 2010)

• Evidence on risk aversion is mixed
  • direct survey evidence suggests estimated relative risk-aversion 3.8 to 4.3 (Barsky et al 1997)
  • indirect evidence (from estimated life-cycle consumption model) suggests 0.4 to 1.4 (Blundell et al 1994)
  • in each case depends on how well-off people are
Does structure of taxation make sense in welfare terms?
Another application of ranking

• Tax and benefit system maps one distribution into another
  - \( c = y - T(y) \)
  - \( y \): pre-tax income \( c \): post-tax income

• Use ranking tools to assess the impact of this in welfare terms

• Typically this uses one or other concept of Lorenz dominance

• Linked to effective tax progression
  - \( T \) is progressive if \( c \) Lorenz-dominates \( y \)
  - see Jakobsson (1976)

• What Lorenz ranking would we expect to apply to these 5 concepts?

- original income
  + cash benefits
- gross income
  - direct taxes
- disposable income
  - indirect taxes
- post-tax income
  + non-cash benefits
- final income
Impact of Taxes and Benefits. UK 2006/7. Lorenz Curve

Proportion of Income

- Original Income
- Gross Income
- Disposable Income
- After Tax Income
- Final Income

(Equality Line)

- + cash benefits
- – direct taxes
- – indirect taxes
- + noncash benefits

- Big effect from benefits side
- Modest impact of taxes
- Direct and indirect taxes work in opposite directions
Implied tax rates in *Economic and Labour Market Review*

Impact of taxes and benefits: Brazil

Comportamento do índice de Gini e das rendas monetárias original, inicial, disponível, final – Brasil (2002-2003 e 2008-2009)

Source: Carlos Ribeiro et al. (2011)
Tax-benefit example: implications

- We might have guessed the outcome of example
- In most countries:
  - income tax progressive
  - so are public expenditures
  - but indirect tax is regressive
  - so Lorenz-dominance is not surprising
- In many countries greater reliance on indirect taxation
  - political convenience?
  - administrative simplicity?
- Structure of taxation should not be haphazard
- How should the \( y \rightarrow c \) mapping be determined?
Tax Criteria

- What broad principles should tax reflect?
  - “benefit received:” pay for benefits you get through public sector
  - “sacrifice:” some principle of equality applied across individuals
  - see Neill (2000) for a reconciliation of these two

- The sacrifice approach
  - Reflect views on equity in society.
  - Analyse in terms of income distribution
  - Compare distribution of $y$ with distribution of $y - T$?

- Three types of question:
  1. what’s a “fair” or “neutral” way to reduce incomes…?
  2. when will a tax system induce $L$-dominance?
  3. what is implication of imposing uniformity of sacrifice?
Amiel-Cowell (1999) approach

- The Irene & Janet diagram
- Initial distribution
- Possible directions for a “fair” tax
  - 1 Scale-independent iso-inequality
  - 2 Translation-independent iso-inequality
  - 3 “Intermediate” iso-inequality
  - 4 No iso-inequality direction

- An iso-inequality path?
  - 1 Proportionate reductions are “fair”
  - 2 Absolute reductions are “fair”
  - 3 Affine reductions are “fair”
  - 4 Amiel-Cowell: individuals perceive inequality comparisons this way.

- Based on Dalton (1920) conjecture
Tax and the Lorenz curve

• Assume a tax function $T(\cdot)$ based on income
  • a person with income $y$, pays an amount $T(y)$
  • disposable income given by $c := y - T(y)$

• Compare distribution of $y$ with that of $c$
  • will $c$ be more equally distributed than $y$?
  • will $T(\cdot)$ produce more equally distributed $c$ than $T^*(\cdot)$?
  • if so under what conditions?

• Define *residual progression* of $T$
  • $\left[ \frac{dc}{dy} \right][y/c] = \frac{[1 − T'(y)]}{[1 − T(y)/y]}$

• Disposable income under $T$ L-dominates that under $T^*$ iff
  • residual progression of $T$ is higher than that of $T^*$...
  • …for all $y$
  • see Jakobsson (1976)
A sacrifice approach

- Suppose utility function $U$ is continuous and increasing
- Definition of absolute sacrifice:
  - $(y, T)$ displays at least as much (absolute) sacrifice as $(y^*, T^*)$…
  - …if $U(y) - U(y-T) \geq U(y^*) - U(y^*-T^*)$
- Equal absolute sacrifice is scale invariant iff
  $$U(y) = \frac{y^{1-\varepsilon} - 1}{1 - \varepsilon}$$
  - …or a linear transformation of this
- (Young 1987)
- Apply sacrifice criterion to actual tax schedules $T(\cdot)$
  - Young (1990) does this for US tax system
  - Consider a UK application
Applying sacrifice approach

- Assume simple form of social welfare function
  - social welfare defined in terms of income
  - drop distinction between $U$ and $u$

- Use the equal absolute sacrifice principle
  - $u(y) - u(y - T(y)) = \text{const}$

- If insist on scale invariance $u$ is isoelastic
  - $y^{1-\varepsilon} - [y - T(y)]^{1-\varepsilon} = \text{const}$
  - $\varepsilon$ is again inequality aversion

- Differentiate and rearrange:
  - $y^{-\varepsilon} - [1 - T'(y)] [y - T(y)]^{-\varepsilon} = 0$
  - $\log[1 - T'(y)] = \varepsilon \log ([y - T(y)] / y)$

- Estimate $\varepsilon$ from actual structure $T(\cdot)$
  - get implied inequality aversion
  - For income tax (1999/2000): $\varepsilon = 1.414$
  - For IT and NIC (1999/2000): $\varepsilon = 1.214$
  - see Cowell-Gardiner (2000)
Conclusion

- Axiomatisation of welfare needs just a few basic principles
  - anonymity
  - population principle
  - decomposability
  - monotonicity
  - principle of transfers

- Ranking criteria can be used to provide broad judgments

- These may be indecisive, so specific SWFs could be used
  - scale invariant?
  - perhaps a value for $\varepsilon$ of around 0.7 – 2

- Welfare criteria can be used to provide taxation principles
References 1

References 2