



Summer 2015 examination

# **EC202**

## **Microeconomic Principles 2**

Suitable for ALL candidates

### **Instructions to candidates**

Time allowed: **3 hours + 15 minutes** reading time.

This paper contains six short questions worth 8 marks each (**Section A**) and six long questions each worth 20 marks (**Section B** and **Section C**). Answer **FIVE** short questions from **Section A**, and **THREE** long questions, at least **ONE** question from **Section B** and at least **ONE** question from **Section C**.

Calculators are NOT allowed in this examination.

## Section A

Answer **FIVE** of questions 1-6. Each question carries 8 marks

1. A single-output firm has a production function given by:

$$\phi(z_1, z_2) = \begin{cases} [z_1 - 1]^{1/4}[z_2 - 1]^{1/4} & \text{if } z_1, z_2 > 1, \\ 0 & \text{otherwise.} \end{cases}$$

where  $z_1, z_2$  are quantities of two inputs.

- (a) Derive the firm's cost function. [5 marks]
- (b) Derive the firm's supply curve in a competitive market. [3 marks]
2. In an uncertain situation with two possible states of the world  $i = 0, 1$  a person's preferences are given by

$$-\frac{1}{y_0} - \frac{1}{y_1}$$

where  $y_i$  is the payoff in state  $i$ .

- (a) Show that the person is risk averse. [3 marks]
- (b) Is the person's absolute risk aversion increasing, constant or decreasing? Explain. [2 marks]
- (c) If the payoffs are  $y_0 = \$500$  and  $y_1 = \$1500$  find the certainty-equivalent income and the risk premium. [3 marks]
3. State whether each of the following is true or false. Briefly explain your answers.

- (a) A competitive equilibrium is always Pareto Efficient. [2 marks]
- (b) A competitive-equilibrium allocation can never be a fair allocation. [3 marks]
- (c) In the neighbourhood of a competitive equilibrium, social-welfare changes are proportional to changes in national income. [3 marks]

4. Consider the following *normal form game*:

1\2	L	C	R
T	1, 2	0, 0	0, 1
M	5, 4	4, 3	3, 2
B	0, 5	0, 4	1, 3

- (a) Do either of the two players have a dominant strategy? [4 marks]
- (b) What is the *Dominant Strategy equilibrium* of this normal form game? [4 marks]
5. Consider a simultaneous-move normal form game where player 1 chooses from three actions, labelled *U*, *M* and *D*, while player 2 chooses from three actions, labelled *L*, *C* and *R*. Assume that player 1's payoffs are such that he is indifferent whatever the strategy choice of player 2. Player 2, on the other hand, has a strictly dominant strategy *L*.
- (a) What are the best replies of player 1? [2 marks]
- (b) What are the best replies of player 2? [2 marks]
- (c) What are the *pure strategy Nash equilibria* of this normal form game? [4 marks]
6. State whether each of the following is true or false. Briefly explain your answers.
- (a) There does not exist a non-degenerate mixed strategy Nash equilibrium of a normal form game where a player chooses a strictly dominated strategy with positive probability. [2 marks]
- (b) In a perfect information extensive form game *Nash equilibria* can only be supported by *credible threats* off the equilibrium path. [3 marks]
- (c) A *Pooling equilibrium* of a signalling game is such that the equilibrium strategy choice of the informed player signals his true type to the uninformed player. [3 marks]

## Section B

Answer at least **ONE** and no more than **TWO** questions.

7. A monopolist has the cost function  $\alpha_0 + \alpha_1 q + \alpha_2 q^2$  (where the  $\alpha$ s are positive parameters and  $q$  is output) and faces the inverse demand function  $p = \beta_1 - \beta_2 q$  (where  $\beta_1 > \alpha_1$  and  $\beta_2 > 0$ ).

- (a) Find the expressions for the firm's average revenue, marginal revenue, average cost and marginal cost in terms of output. [4 marks]
- (b) Show that the monopolist will choose to produce

$$q^* := \frac{\beta_1 - \alpha_1}{2 [\alpha_2 + \beta_2]}$$

[3 marks].

- (c) Show that at  $q^*$  the price charged  $p^*$  exceeds the marginal cost  $c^*$ . [4 marks]
- (d) A regulator now imposes a ceiling on the price charged, so that the monopolist now faces the additional constraint  $p \leq \bar{p}$ . Find the new expressions for average revenue and marginal revenue in terms of output. [4 marks]
- (e) Explain how the price ceiling affects the monopolist's output if  $c^* \leq \bar{p} \leq p^*$ , where the values  $c^*$  and  $p^*$  are the values of marginal cost and price found in the unregulated problem of part (c). [5 marks]

8. A person lives for two periods. She receives an exogenously given income stream  $(y_1, y_2)$  over the two periods but, by investing an amount  $z$  in education during period 1, she can augment income in period 2 by an amount

$$\tau [1 - e^{-z}]$$

where  $\tau > 0$  represents her innate talent.

- (a) Assuming that the person faces a perfect market for borrowing and lending at a uniform rate  $r$  find the person's optimal investment in education,  $z^*$ . Under what circumstances would  $z^* = 0$ ? [7 marks]
- (b) If  $z^* > 0$  how is it affected by the interest rate  $r$  and talent  $\tau$ ? Provide an intuitive explanation. [4 marks]
- (c) Suppose the consumer's utility function is given by

$$\alpha \log x_1 + [1 - \alpha] \log x_2$$

where  $x_i$  is the amount of consumption in period  $i$  and the parameter  $\alpha$  lies between 0 and 1. What is her optimal amount of borrowing in period 1? [5 marks]

- (d) How is borrowing affected by the interest rate  $r$  and talent  $\tau$ ? [4 marks]

9. In a two-good exchange economy there are  $n$  agents of type  $a$  with utility function  $-\frac{1}{2}[x_1^a]^{-2} - \frac{1}{2}[x_2^a]^{-2}$  and  $n$  agents of type  $b$  with utility function  $\ln(x_1^b) + \ln(x_2^b)$ , where  $x_i^h$  means consumption of good  $i$  by an agent of type  $h$ . Each  $a$ -type has an endowment  $(10,0)$  of the two goods and each  $b$ -type has an endowment  $(0,32)$ .
- (a) Find the excess demand functions for each of the two goods. [8 marks]
  - (b) Show that there is a competitive equilibrium with price ratio  $p_1/p_2 = 8$ . [4 marks]
  - (c) Is this equilibrium stable? Is it unique? [5 marks]
  - (d) What is the core allocation if  $n$  is large? [3 marks]

## Section C

Answer at least **ONE** and no more than **TWO** questions.

10. Two players, labelled  $i \in \{1, 2\}$  play the following game. Both players choose among their alternative actions simultaneously and independently.

Player 1 chooses between actions  $\{U, M, D\}$  while player 2 chooses between actions  $\{L, R\}$ . When the action profile chosen is  $(U, L)$  the vector of the players' payoffs is  $(5, 5)$ , where the first number is player 1's payoff while the second number is player 2's payoff. When the action profile chosen is  $(U, R)$  the vector of the players' payoffs is  $(3, 9)$ . When the action profile chosen is  $(M, R)$  the vector of the players' payoffs is  $(5, 5)$ , while when the action profile chosen is  $(M, L)$  the vector of the players' payoffs is  $(9, 5)$ . Finally, when the action profile chosen is  $(D, L)$  the vector of the players' payoffs is  $(3, 6)$ , while when the action profile chosen is  $(D, R)$  the vector of the players' payoffs is  $(0, 1)$ .

- (a) Formulate the strategic situation described above as a *normal form game*. What are *the strategies* for the two players? Do any of the players have a *strictly dominant* strategy? Do any of the players have a *weakly dominant* strategy? [2 marks]
- (b) Identify the set of *pure strategy Nash equilibria* of this game. [4 marks]
- (c) Identify the set of *mixed strategy Nash equilibria* of this game. [4 marks]

Assume now that player 1 moves first and chooses from actions  $\{U, M, D\}$ . Player 2 observes player 1's action choice and only then chooses his actions that can be contingent on player 1's action choice. The payoffs are the same as described above.

- (d) Formulate this new strategic situation as an *extensive form game*. What are *the strategies* for the two players? What is the *normal form associated with the extensive form* of this dynamic game? [2 marks]
- (e) Identify the set of *pure strategy Nash equilibria* of this dynamic game. [4 marks]
- (f) Identify the set of (pure strategy) *Subgame Perfect equilibria* of this dynamic game. [4 marks]

11. Consider the following normal form game:

$1 \setminus 2$	$L$	$C$	$R$
$U$	4, 4	3, 1	0, 2
$M$	1, 3	5, 5	0, 6
$D$	2, 0	6, 0	4, 4

- (a) Do any of the players have a *strictly dominated strategy*? [2 marks]
- (b) Identify the *set of pure strategy Nash equilibria* of this game. [3 marks]
- (c) Identify the *mixed strategy Nash equilibria* of this game. [3 marks]

Considering the original normal form game, assume now that the game is played in two consecutive periods. The two players have the same discount factor  $\delta$ . The average discounted payoff of the players is:

$$\Pi_i = \frac{1}{1 + \delta} [g_i(a_i^1, a_{-i}^1) + \delta g_i(a_i^2, a_{-i}^2)]$$

where  $g_i(a_i^t, a_{-i}^t)$  is the stage game payoff of player  $i$  if the strategy profile chosen by both players in period  $t \in \{1, 2\}$  is  $(a_i^t, a_{-i}^t)$ :  $a_1^t \in \{U, M, D\}$  and  $a_2^t \in \{L, C, R\}$ .

- (d) Construct strategies for the two-period repeated game that support the payoff (4, 4) in each period of the game as a *Subgame Perfect equilibrium*. For what values of the discount factor  $\delta$  are these strategies subgame perfect? [5 marks]
- (e) Construct strategies for the two-period repeated game that support the payoff (5, 5) in period  $t = 1$  and the payoff (4, 4) in period  $t = 2$  for both players as a *Subgame Perfect equilibrium*. For what values of the discount factor  $\delta$  are these strategies subgame perfect? [7 marks]

12. Consider the following Cournot duopoly game.

Two firms labelled  $i \in \{1, 2\}$  simultaneously and independently choose their output level  $q_i$  so as to maximize their profits. Both firms have the same *constant returns to scale* technology and their *constant marginal cost*  $c = 3$ . The inverse market demand is  $p = 4 - Q$ , where  $Q = q_1 + q_2$  denotes the aggregate quantity produced.

- (a) Represent the Cournot competition described above as a *normal form game*. What are the *strategies* of the two firms? [2 marks]
- (b) Compute the *best replies strategies* of firm 1 and firm 2 and identify the set of *Nash equilibria* of this game and the associated *equilibrium strategies and profits* for both firms. [5 marks]

Assume now that only one monopolist, endowed with the same technology of each of the two firms above, operates in this market.

- (c) Identify the optimal quantity choice of this monopolist and the profit such a monopolist will obtain in this market. [4 marks]
- (d) Why is it not a Nash equilibrium of the Cournot game above for firm 1 and 2 to produce a quantity equal to *half of the monopolist aggregate quantity* identified in (c) above? [2 marks]

Assume now that the simultaneous move Cournot duopoly game described above is repeated each period for an infinite number of periods. At the end of each period (period  $t$ ) each firm observes the outcome of that period interaction, that is the pair of quantities chosen by both firms in that period  $(q_1^t, q_2^t)$ . Further assume that each firm discounts the future at the same rate  $\delta$  and the expected present discounted payoff of these firms are:

$$\Pi_i = (1 - \delta) \sum_{t=0}^{\infty} \delta^t \pi_i(q_1^t, q_2^t) \quad (1)$$

where  $i$  takes either value 1 or 2 and  $\pi_i(q_1^t, q_2^t)$  denotes the firm  $i$ 's profit in period  $t$  when the firms choose quantities  $(q_1^t, q_2^t)$ .

- (e) Construct *subgame perfect equilibrium strategies* for the two firms that in the infinitely repeated game support a quantity choice for each firm that equals half of the monopoly aggregate quantity identified in (c) above. For what values of the discount factor  $\delta$  are these strategies subgame perfect? [7 marks]