



Summer 2015 examination

EC202

Microeconomic Principles 2

Suitable for ALL candidates

Instructions to candidates

Time allowed: **3 hours + 15 minutes** reading time.

This paper contains six short questions worth 8 marks each (**Section A**) and six long questions each worth 20 marks (**Section B** and **Section C**). Answer **FIVE** short questions from **Section A**, and **THREE** long questions, at least **ONE** question from **Section B** and at least **ONE** question from **Section C**.

Calculators are NOT allowed in this examination.

Section A

Answer **FIVE** of questions 1-6. Each question carries 8 marks

1. A single-output firm has a production function given by:

$$\phi(z_1, z_2) = \begin{cases} [z_1 - 1]^{1/4}[z_2 - 1]^{1/4} & \text{if } z_1, z_2 > 1, \\ 0 & \text{otherwise.} \end{cases}$$

where z_1, z_2 are quantities of two inputs.

(a) Derive the firm's cost function. [5 marks]

(b) Derive the firm's supply curve in a competitive market. [3 marks]

2. In an uncertain situation with two possible states of the world $i = 0, 1$ a person's preferences are given by

$$-\frac{1}{y_0} - \frac{1}{y_1}$$

where y_i is the payoff in state i .

(a) Show that the person is risk averse. [3 marks]

(b) Is the person's absolute risk aversion increasing, constant or decreasing? Explain. [2 marks]

(c) If the payoffs are $y_0 = \$500$ and $y_1 = \$1500$ find the certainty-equivalent income and the risk premium. [3 marks]

3. State whether each of the of the following is true or false. Briefly explain your answers.

(a) A competitive equilibrium is always Pareto Efficient. [2 marks]

(b) A competitive-equilibrium allocation can never be a fair allocation. [3 marks]

(c) In the neighbourhood of a competitive equilibrium, social-welfare changes are proportional to changes in national income. [3 marks]

4. Consider the following *normal form game*:

| | | | |
|----------|----------|----------|----------|
| 1\2 | <i>L</i> | <i>C</i> | <i>R</i> |
| <i>T</i> | 1, 2 | 0, 0 | 0, 1 |
| <i>M</i> | 5, 4 | 4, 3 | 3, 2 |
| <i>B</i> | 0, 5 | 0, 4 | 1, 3 |

- (a) Do either of the two players have a dominant strategy? [4 marks]
 (b) What is the *Dominant Strategy equilibrium* of this normal form game? [4 marks]

5. Consider a simultaneous-move normal form game where player 1 chooses from three actions, labelled *U*, *M* and *D*, while player 2 chooses from three actions, labelled *L*, *C* and *R*. Assume that player 1's payoffs are such that he is indifferent whatever the strategy choice of player 2. Player 2, on the other hand, has a strictly dominant strategy *L*.

- (a) What are the best replies of player 1? [2 marks]
 (b) What are the best replies of player 2? [2 marks]
 (c) What are the *pure strategy Nash equilibria* of this normal form game? [4 marks]

6. State whether each of the of the following is true or false. Briefly explain your answers.

- (a) There does not exist a non-degenerate mixed strategy Nash equilibrium of a normal form game where a player chooses a strictly dominated strategy with positive probability. [2 marks]
 (b) In a perfect information extensive form game *Nash equilibria* can only be supported by *credible threats* off the equilibrium path. [3 marks]
 (c) A *Pooling equilibrium* of a signalling game is such that the equilibrium strategy choice of the informed player signals his true type to the uninformed player. [3 marks]

Section B

Answer at least **ONE** and no more than **TWO** questions.

7. A monopolist has the cost function $\alpha_0 + \alpha_1 q + \alpha_2 q^2$ (where the α s are positive parameters and q is output) and faces the inverse demand function $p = \beta_1 - \beta_2 q$ (where $\beta_1 > \alpha_1$ and $\beta_2 > 0$).

- (a) Find the expressions for the firm's average revenue, marginal revenue, average cost and marginal cost in terms of output. [4 marks]
- (b) Show that the monopolist will choose to produce

$$q^* := \frac{\beta_1 - \alpha_1}{2[\alpha_2 + \beta_2]}$$

[3 marks].

- (c) Show that at q^* the price charged p^* exceeds the marginal cost c^* . [4 marks]
- (d) A regulator now imposes a ceiling on the price charged, so that the monopolist now faces the additional constraint $p \leq \bar{p}$. Find the new expressions for average revenue and marginal revenue in terms of output. [4 marks]
- (e) Explain how the price ceiling affects the monopolist's output if $c^* \leq \bar{p} \leq p^*$, where the values c^* and p^* are the values of marginal cost and price found in the unregulated problem of part (c). [5 marks]

8. A person lives for two periods. She receives an exogenously given income stream (y_1, y_2) over the two periods but, by investing an amount z in education during period 1, she can augment income in period 2 by an amount

$$\tau [1 - e^{-z}]$$

where $\tau > 0$ represents her innate talent.

- (a) Assuming that the person faces a perfect market for borrowing and lending at a uniform rate r find the person's optimal investment in education, z^* . Under what circumstances would $z^* = 0$? [7 marks]
- (b) If $z^* > 0$ how is it affected by the interest rate r and talent τ ? Provide an intuitive explanation. [4 marks]
- (c) Suppose the consumer's utility function is given by

$$\alpha \log x_1 + [1 - \alpha] \log x_2$$

where x_i is the amount of consumption in period i and the parameter α lies between 0 and 1. What is her optimal amount of borrowing in period 1? [5 marks]

- (d) How is borrowing affected by the interest rate r and talent τ ? [4 marks]

9. In a two-good exchange economy there are n agents of type a with utility function $-\frac{1}{2} [x_1^a]^{-2} - \frac{1}{2} [x_2^a]^{-2}$ and n agents of type b with utility function $\ln(x_1^b) + \ln(x_2^b)$, where x_i^h means consumption of good i by an agent of type h . Each a -type has an endowment $(10,0)$ of the two goods and each b -type has an endowment $(0,32)$.

- (a) Find the excess demand functions for each of the two goods. [8 marks]
- (b) Show that there is a competitive equilibrium with price ratio $p_1/p_2 = 8$. [4 marks]
- (c) Is this equilibrium stable? Is it unique? [5 marks]
- (d) What is the core allocation if n is large? [3 marks]

Section C

Answer at least **ONE** and no more than **TWO** questions.

10. Two players, labelled $i \in \{1, 2\}$ play the following game. Both players choose among their alternative actions simultaneously and independently.

Player 1 chooses between actions $\{U, M, D\}$ while player 2 chooses between actions $\{L, R\}$. When the action profile chosen is (U, L) the vector of the players' payoffs is $(5, 5)$, where the first number is player 1's payoff while the second number is player 2's payoff. When the action profile chosen is (U, R) the vector of the players' payoffs is $(3, 9)$. When the action profile chosen is (M, R) the vector of the players' payoffs is $(5, 5)$, while when the action profile chosen is (M, L) the vector of the players' payoffs is $(9, 5)$. Finally, when the action profile chosen is (D, L) the vector of the players' payoffs is $(3, 6)$, while when the action profile chosen is (D, R) the vector of the players' payoffs is $(0, 1)$.

- (a) Formulate the strategic situation described above as *a normal form game*. What are *the strategies* for the two players? Do any of the players have a *strictly dominant strategy*? Do any of the players have a *weakly dominant strategy*? [2 marks]
- (b) Identify the set of *pure strategy Nash equilibria* of this game. [4 marks]
- (c) Identify the set of *mixed strategy Nash equilibria* of this game. [4 marks]

Assume now that player 1 moves first and chooses from actions $\{U, M, D\}$. Player 2 observes player 1's action choice and only then chooses his actions that can be contingent on player 1's action choice. The payoffs are the same as described above.

- (d) Formulate this new strategic situation as *an extensive form game*. What are *the strategies* for the two players? What is the *normal form associated with the extensive form* of this dynamic game? [2 marks]
- (e) Identify the set of *pure strategy Nash equilibria* of this dynamic game. [4 marks]
- (f) Identify the set of (pure strategy) *Subgame Perfect equilibria* of this dynamic game. [4 marks]

11. Consider the following normal form game:

| | | | |
|------------------|------|------|------|
| $1 \backslash 2$ | L | C | R |
| U | 4, 4 | 3, 1 | 0, 2 |
| M | 1, 3 | 5, 5 | 0, 6 |
| D | 2, 0 | 6, 0 | 4, 4 |

- (a) Do any of the players have a *strictly dominated* strategy? [2 marks]
- (b) Identify the *set of pure strategy Nash equilibria* of this game. [3 marks]
- (c) Identify the *mixed strategy Nash equilibria* of this game. [3 marks]

Considering the original normal form game, assume now that the game is played in two consecutive periods. The two players have the same discount factor δ . The average discounted payoff of the players is:

$$\Pi_i = \frac{1}{1 + \delta} [g_i(a_i^1, a_{-i}^1) + \delta g_i(a_i^2, a_{-i}^2)]$$

where $g_i(a_i^t, a_{-i}^t)$ is the stage game payoff of player i if the strategy profile chosen by both players in period $t \in \{1, 2\}$ is (a_i^t, a_{-i}^t) : $a_1^t \in \{U, M, D\}$ and $a_2^t \in \{L, C, R\}$.

- (d) Construct strategies for the two-period repeated game that support the payoff (4, 4) in each period of the game as a *Subgame Perfect equilibrium*. For what values of the discount factor δ are these strategies subgame perfect? [5 marks]
- (e) Construct strategies for the two-period repeated game that support the payoff (5, 5) in period $t = 1$ and the payoff (4, 4) in period $t = 2$ for both players as a *Subgame Perfect equilibrium*. For what values of the discount factor δ are these strategies subgame perfect? [7 marks]

12. Consider the following Cournot duopoly game.

Two firms labelled $i \in \{1, 2\}$ simultaneously and independently choose their output level q_i so as to maximize their profits. Both firms have the same *constant returns to scale* technology and their *constant marginal cost* $c = 3$. The inverse market demand is $p = 4 - Q$, where $Q = q_1 + q_2$ denotes the aggregate quantity produced.

- (a) Represent the Cournot competition described above as a *normal form game*. What are the *strategies* of the two firms? [2 marks]
- (b) Compute the *best replies strategies* of firm 1 and firm 2 and identify the set of *Nash equilibria* of this game and the associated *equilibrium strategies and profits* for both firms. [5 marks]

Assume now that only one monopolist, endowed with the same technology of each of the two firms above, operates in this market.

- (c) Identify the optimal quantity choice of this monopolist and the profit such a monopolist will obtain in this market. [4 marks]
- (d) Why is it not a Nash equilibrium of the Cournot game above for firm 1 and 2 to produce a quantity equal to *half of the monopolist aggregate quantity* identified in (c) above? [2 marks]

Assume now that the simultaneous move Cournot duopoly game described above is repeated each period for an infinite number of periods. At the end of each period (period t) each firm observes the outcome of that period interaction, that is the pair of quantities chosen by both firms in that period (q_1^t, q_2^t) . Further assume that each firm discounts the future at the same rate δ and the expected present discounted payoff of these firms are:

$$\Pi_i = (1 - \delta) \sum_{t=0}^{\infty} \delta^t \pi_i(q_1^t, q_2^t) \quad (1)$$

where i takes either value 1 or 2 and $\pi_i(q_1^t, q_2^t)$ denotes the firm i 's profit in period t when the firms choose quantities (q_1^t, q_2^t) .

- (e) Construct *subgame perfect equilibrium strategies* for the two firms that in the infinitely repeated game support a quantity choice for each firm that equals half of the monopoly aggregate quantity identified in (c) above. For what values of the discount factor δ are these strategies subgame perfect? [7 marks]