Summer 2014 examination

EC202

Microeconomic Principles 2

Suitable for ALL candidates

Instructions to candidates

Time allowed: **3 hours** + **15 minutes** reading time.

This paper contains seven questions in three sections. Answer **QUESTION ONE** of **Section A**, and **THREE** other questions, at least **ONE** question from **Section B** and at least **ONE** question from **Section C**. Question one carries 40% of the total marks; the other questions each carry 20% of the total marks.

Calculators are NOT allowed in this examination.

Section A

1. Answer any **FIVE** parts from the following eight parts, (a)-(h). Each part carries eight marks.

   (a) A single-output firm can use any of the following three production techniques

   $$\text{Technique \#1:} \quad q \le \min\left(\frac{1}{3}z_1, z_2\right)$$
   $$\text{Technique \#2:} \quad q \le \min\left(z_1, \frac{1}{3}z_2\right)$$
   $$\text{Technique \#3:} \quad q \le \min\left(\frac{2}{5}z_1, \frac{2}{5}z_2\right)$$

   where q denotes the quantity of output and $z_1,\ z_2$ the quantities of two inputs. The firm can also use combinations of techniques. Draw the isoquant for $q = 1$.

   (b) A market consists of N identical price-taking firms, where each firm i has the cost function $16 + q_i^2$. Market demand is given by $N\,[11 - p]$ where p is price. .

      i. What is the supply curve for firm i?

      ii. Assuming that N is a large number, carefully describe the equilibrium in this market

   (c) Mark each of the following true or false; in each case briefly explain your answer:

      i. If an exchange economy is replicated indefinitely the core of the economy shrinks to a single allocation.

      ii. By Walras' law, the sum over all goods of price times excess demand must equal zero, but only in the neighbourhood of equilibrium.

      iii. A general equilibrium will exist only if the weak axiom of revealed preference is satisfied by all excess demand functions.

      iv. In a general equilibrium it is not necessarily the case that excess demand equals zero in every market.

   (d) Suppose that in the equilibrium of an exchange economy everyone has the same income. Will the equilibrium be a fair allocation? Explain your answer.

(e) Consider the following *normal form game*:

$1\backslash 2$	L	R
T	$3,1$	$0,2$
M	$2,0$	$1,1$
B	$4,1$	$3,2$

Does any of the two players has a dominant strategy? Explain your answer.

(f) Consider a game where player 1 chooses between two strategies labelled U and D, while player 2 chooses between the two strategies labelled L and R. Assume that the strategy U is a dominant strategy for player 1 while player 2's payoffs are such that he is indifferent whatever the strategy choice of his opponent. What is the best reply of player 1? What is the best reply for player 2? What is the Nash equilibrium strategy profile in this game? Explain your answers

(g) Mark each of the following statement true or false; in each case briefly explain your answer:

i. Nash equilibrium strategies may be such each player is completely indifferent among all his available strategies.

ii. In a dynamic game Nash equilibrium strategies cannot be supported by *non-credible* threats on the part of the players.

iii. In a dynamic game Subgame Perfect equilibrium strategies cannot be supported by *non-credible* threats on the part of the players.

iv. It is possible to construct a mixed strategy equilibrium of a normal form game where one of the players randomizes with strictly positive probability on a strategy that is strictly dominated.

(h) What is the set of *feasible and individually rational payoffs* associated with the infinitely repeated game with stage game:

$1\backslash 2$	L	R
U	$1,1$	$0,0$
D	$0,0$	$4,4$

Section B

Answer at least **ONE** and no more than **TWO** questions.

2. Anne lives for three periods and her lifetime utility function is given as

$$\sum_{t=1}^{3} \log(x_t - a_t),$$

where x_t is Anne's consumption in period t and the a_t are parameters such that $a_2 > a_3 > a_1 > 0$. The price of consumption goods may be different in each period. Anne receives exogenous money income in each of the three periods and can borrow or lend freely at the per-period interest rate r.

(a) Solve Anne's optimisation problem and interpret the solution.

(b) Under what conditions will Anne save in period 1? If the price of period-2 goods were expected to rise, what effect would there be on period-1 savings?

(c) If the price of period-3 goods were expected to rise, what effect would there be on period-1 savings? Is this greater than the effect in part (b)?

(d) What would be the effect on period-1 savings of a rise in the price of period-1 goods? Explain why this effect is different from those analysed in parts (b) and (c).

3. Fred's current wealth y is given by y_0+y_1 where y_0 is the value of his financial assets and y_1 is the current value of his house. Unfortunately Fred's house was built in an area liable to flooding; if there is a flood his house will be worth nothing, but his financial assets will remain unaffected; if there is no flood the value of all his wealth remains unchanged; the probability of flood is π. A company offers insurance against flood: if Fred buys an amount x of insurance the insurance company will charge a premium κx where $\pi < \kappa < 1$ and $x \leq y_1$.

   (a) If Fred buys an amount x of insurance, find his ex-post wealth y in two cases: (i) where there is no flood (ii) where there is a flood.

   (b) Fred's utility is $\mathcal{E}u\left(y\right)$ where u is an increasing, concave function and $\mathcal{E}$ is the expectations operator. Is Fred risk averse? Will he take out full insurance on his house?

   (c) If $u(y) = \log\left(y\right)$ find x^*, the optimal value of x.

   (d) How will x^* change if (i) the value of Fred's financial assets increases and (ii) the value of Fred's house increases? Briefly comment on your answer.

4. In a two-commodity exchange economy there are two groups of people: type a have the utility function $2\log(x_1^a)+\log(x_2^a)$ and an endowment of ℓ units of commodity 1 and k units of commodity 2; type b have the utility function $\log(x_1^b)+2\log(x_2^b)$ and an endowment of $3-\ell$ units of commodity 1 and $7-k$ units of commodity 2, where $0 \leq \ell \leq 3$ and $0 \leq k \leq 7$.

   (a) Show that the equilibrium price of good 1 in terms of good 2 is $\frac{7+k}{6-\ell}$.

   (b) What are the individuals' incomes (y^a, y^b) in equilibrium as a function of k and ℓ? Assume that it is possible to carry out lump-sum transfers of commodity 2 (k can be varied), but impossible to transfer commodity 1 (ℓ is fixed). Draw a diagram of the set of attainable income distributions

      i. in the case where $\ell = 0$;

      ii. in the case where $\ell = 3$.

   The government seeks to maximise the welfare function $y^a + y^b$.

   (c) Is the government inequality-averse?

   (d) What would be the optimal distribution of income in the case $\ell = 0$? Comment on the result.

   (e) What would be the optimal distribution of income in the case $\ell = 3$? Comment on the result.

Section C

Answer at least **ONE** and no more than **TWO** questions.

5. Two players, labelled $i \in \{1, 2\}$ play the following game. Both players choose between two alternative actions simultaneously and independently.

   Player 1 chooses between actions $\{U, D\}$ while player 2 chooses between actions $\{L, R\}$. When the action profile chosen is (U, L) the vector of the players' payoffs is $(6, 6)$, where the first number is player 1's payoff while the second number is player 2's payoff. When the action profile chosen is (U, R) the vector of the players' payoffs is $(2, 8)$. When the action profile chosen is (D, L) the vector of the players' payoffs is $(8, 4)$ and finally when the action profile chosen is (D, R) the vector of the players' payoffs is $(4, 4)$.

   (a) Formulate the strategic situation described above as *a normal form game*. What are *the strategies* for the two players?

   (b) Identify the set of *pure strategy Nash Equilibria* of this game.

   (c) Identify the set of *mixed strategy Nash equilibria* of this game.

   Assume now that player 1 moves first and chooses between actions $\{U, D\}$. Player 2 observes player 1's action choice and only then chooses between actions $\{L, R\}$. The payoffs are the same described above.

   (d) Formulate this new strategic situation as *an extensive form game*. What are *the strategies* for the two players?

   (e) What is the *normal form associated with the extensive form* of this dynamic game?

   (f) Identify the set of *pure strategy Nash Equilibria* of this dynamic game.

   (g) Identify the set of *Subgame Perfect equilibria* of this dynamic game.

6. Consider the following Cournot duopoly game.

Two firms labelled $i \in \{1, 2\}$ simultaneously and independently choose their output level q_i so as to maximize their profits.

Both firms have the same *constant returns to scale* technology and their *constant marginal cost*

$$c = 1. \tag{1}$$

The inverse market demand is

$$p = 2 - q_1 - q_2. \tag{2}$$

(a) Represent the Cournot competition described above as a *normal form game*. What are the *strategies* of the two firms?

(b) Compute the *best replies strategies* of firm 1 and firm 2.

(c) Identify the set of *Nash equilibria* of this game and the associated *equilibrium strategies and profits* for both firms.

Consider now the following change in the timing of competition.

Firm 1 is a Stackelberg leader and as such chooses its output q_1 first. Firm 2 observes the output choice of firm 1 and only then decides how much to produce. Firms' technology and the market demand are the same as in (1) and (2) above.

(d) Represent the Stachelberg competition described above as an *extensive form game*. What are the *strategies* of the two firms?

(e) Compute the *best replies strategies* of firm 1 and firm 2 in this dynamic game.

(f) Identify the set of *Subgame Perfect equilibria* of this game and the associated *equilibrium profits and strategies* for both firms.

(g) Compare the Nash equilibrium quantities and profits you derived in (c) above with the quantities and profits associated with the Subgame Perfect equilibria you identified in (f).

7. Consider the following normal form game:

$1\backslash 2$	L	C	R
T	$1,1$	$0,0$	$1,0$
M	$0,5$	$4,4$	$2,1$
B	$0,0$	$5,0$	$3,3$

(a) Identify the *set of pure strategy Nash equilibria* of this game.

(b) Identify the *mixed strategy Nash equilibria* of this game.

Assume now that this game is played in two consecutive periods. Each player discount the future at the same rate δ. The average discounted payoff of the players is:

$$\Pi_i = \frac{1}{1+\delta}\left[g_i(a_i^1, a_{-i}^1) + \delta g_i(a_i^2, a_{-i}^2)\right] \tag{3}$$

where $g_i(a_i^t, a_{-i}^t)$ is the stage game payoff of player i if the strategy profile chosen by both players in period $t \in \{1,2\}$ is (a_i^t, a_{-i}^t): $a_1^t \in \{T, M, B\}$ and $a_2^t \in \{L, C, R\}$.

(c) Construct strategies for the two-periods repeated game that support the payoff $(3,3)$ in each period of the game as a *Subgame Perfect equilibrium*. For what values of the discount factor δ are these strategies subgame perfect.

(d) Construct strategies for the two-periods repeated game that support the payoff $(4,4)$ in period $t=1$ and the payoff $(3,3)$ in period $t=2$ for both players as a *Subgame Perfect equilibrium*. For what values of the discount factor δ are these strategies subgame perfect.