Summer 2013 Examination

EC202

Microeconomic Principles II

Suitable for ALL candidates

Instructions to candidates
Time allowed: 3 hours + 15 minutes reading time.
This paper contains seven questions in three sections. Answer question one (section A) and **THREE** other questions, at least **ONE** from section B and at least **ONE** from section C. Question one carries 40% of the total marks; the other questions each carry 20% of the total marks.

Calculators are NOT permitted

Section A

1. Answer any **FIVE** parts from the following eight parts, (a)-(h). Each part carries eight marks.

   (a) Suppose a single-output firm has a production function given by

$$\phi(z_1, z_2) = \begin{cases} \sqrt{[z_1 - a]\, z_2} & \text{if } z_1 > a \geq 0,\ z_2 > 0, \\ 0 & \text{otherwise,} \end{cases}$$

   where z_1, z_2 are quantities of two inputs.

   i. Find the cost function.
   ii. Sketch the average and marginal cost curves.
   iii. Does ϕ exhibit increasing, constant, or decreasing returns to scale?

   (b) Which, if any, of the following statements are true? In each case explain the reasoning behind your answer.

   i. A consumer's cost function (expenditure function) must be homogeneous of degree 0 in all prices.
   ii. The effect of an increase in the price of good i on a consumer's demand for good j must be the same as the effect of an increase in the price of good j on the demand for good i.
   iii. For all normal goods the compensating variation of a price fall must be less than or equal to the equivalent variation.

   (c) An individual has a utility function $U = \mathcal{E}u(x)$ where x is a random variable with a known distribution, $\mathcal{E}$ is the expectations operator, and u is given by

$$u(x) = 2ax - x^2,$$

   where a is a parameter such that $a > \max x$.

   i. Show that U can be written as a function of the mean and variance of the distribution.
   ii. Is absolute risk aversion increasing / constant / decreasing?
   iii. Is relative risk aversion increasing / constant / decreasing?

   (d) Suppose social welfare is defined as

$$W = \begin{cases} \frac{\int_0^\infty y^{1-\epsilon} f(y)dy - 1}{1-\epsilon} & \text{if } \epsilon \geq 0,\ \epsilon \neq 1, \\ \int_0^\infty \log(y) f(y) dy & \text{replacing the case } \epsilon = 1 \end{cases}$$

   where y is income and $f()$ is the density function of income.

   i. Draw the contours of the social welfare function for the cases $\epsilon = 1$, $\epsilon \to 0$, $\epsilon \to \infty$.
   ii. What is equally-distributed-equivalent income in each case?

(e) For any static game define the Pure Strategy Nash equilibrium (PNE) and the Dominant Strategy equilibrium (DSE) solution concepts. Argue why any DSE is a PNE.

(f) Four friends wish to arrange a doubles tennis match. Each one of them can choose whether to call the tennis club to make arrangements, or to count on others to call. At least one of them has to call the club for the match to take place. The value of playing the match is equal to 10 for a player who did not invest time in organizing the game, and is equal to 9 for a player who called. A player's utility is 0 if the match is not played.

i. Find the mixed strategy Nash equilibrium of this game.

ii. What is the probability of the match being played?

iii. What would happen if only two players had to arrange a singles tennis match?

(g) Consider the classical Bertrand competition model. There are two firms who produce a single output at a constant marginal cost equal to 10. Demand in the market satisfies $d(p) = 20 - p$. Find the monopoly price for this market. Next suppose that the firms play the game infinitely many times. Consider a trigger-strategy that requires firms: to sell at the monopoly price whenever no firm has deviated in the past; and to sell at marginal cost otherwise. Find the lowest value for the discount factor for which this strategy is Subgame Perfect.

(h) A challenger contests an incumbent firm. The challenger can be weak with probability p or strong with probability $1 - p$. The challenger knows its type, but the incumbent does not. The challenger can decide how much effort to devote to prepare for the fight. Effort is costly. The total cost of effort level e is $C_w(e) = e^2$ for a weak player, and $C_s(e) = e^2/8$ for a strong player. After observing the effort level of the challenger, the incumbent can choose whether to fight the entrant. The incumbent's payoff is equal: to 2 if he fights a weak challenger; to 0 if he fights a strong challenger; and to 1 otherwise. The challenger's payoff is equal: to 1 minus the cost of effort, if he is weak and the incumbent fights; 2 minus the cost of effort, if he is strong and the incumbent fights; and is equal to 3 otherwise. Find all the separating Perfect Bayesian Equilibria of this game.

Section B [Answer at least ONE and no more than TWO questions]

2. In a monopolistically competitive industry, firm i produces an amount q_i of output at cost $b + cq_i$, where $b, c > 0$. The inverse demand function for firm i is given by

$$p_i = \frac{q_i^{\alpha-1}}{\sum_{j=1}^{n} q_j^{\alpha}}$$

where $0 < \alpha \leq 1$ and n is the number of firms in the industry

(a) Interpret the parameter α.

(b) If each firm maximises profits while taking the output of all the other firms as given, find the first-order conditions yielding firm i's output conditional on the outputs of the other firms.

(c) Show that in equilibrium the optimal output for any firm must be

$$\frac{\alpha\,[n-1]}{n^2 c}$$

(d) Show that the price elasticity of demand for any firm is

$$\frac{n}{n - n\alpha + \alpha}.$$

(e) What is the intuitive meaning of the parameter α being 1? In this case show that the equilibrium number of firms in the industry is less than or equal to $1/\sqrt{b}$.

3. Suppose a person has non-labour income $\bar{y}$ and can earn additional income in the labour market. Income is spent on a single consumption good c.

(a) If she works ℓ hours in the labour market for hourly wage w and the price of the consumption good is 1, what is her budget constraint?

(b) If her preferences are given by the utility function

$$[T - \ell]^{\theta} + c^{\theta},$$

where T is the maximum number of hours available and $\theta < 1$, show that her optimal labour supply is given by

$$\ell = \max\left(\frac{Tw^{\sigma} - \bar{y}}{w^{\sigma} + w}, 0\right),$$

where $\sigma := \frac{1}{1-\theta}$. Under what conditions will she choose not to work?

(c) How will labour supply respond to changes in (i) $\bar{y}$, (ii) w, and (iii) T? Explain the intuition behind your results.

(d) Use your answer to part (c) to explain the possible labour-supply effects of (i) a change in cash benefits that are not linked to work, (ii) the introduction of a tax on wage income, (iii) the provision of free child care.

4. In a two-commodity exchange economy there are two groups of traders, each of size N. Each trader in group a has an endowment of 300 units of commodity 1; each person in group b has an endowment of 200 units of commodity 2. Each a-type person has preferences given by the utility function

$$U^a(\mathbf{x}^a) = \log x_1^a + \log x_2^a$$

and each b-type person's utility can be written as

$$U^b(\mathbf{x}^b) = \log x_1^b + \log x_2^b + k \log x_1^a$$

where x_i^h means the consumption of good i by an h-type person.

(a) If N is large find the equilibrium allocation.

(b) If $k > 0$ or $k < 0$ explain why the equilibrium in part (a) is not Pareto efficient.

(c) If $k = 0$ and $N = 1$, is the equilibrium found in part (a) still an equilibrium here? Explain what other equilibrium allocations might exist in this case.

Section C [Answer at least ONE and no more than TWO questions]

5. Consider two players engaged in an ultimatum game to share a surplus S. Player A can offer any share $\theta \in [0,1]$ of the surplus $S \in [0,10]$ to Player B. If B rejects the offer, no project is undertaken and both players receive a payoff of zero. If the offer is accepted, the payoff of player A is given by $S\theta(1-\theta)$, while the payoff of player B amounts to $S(\theta - S/10)$.

   (a) Find the Subgame-Perfect equilibrium of this game.

   (b) Find a Nash equilibrium of the game that is not Subgame Perfect.

   (c) Next suppose that, prior to the beginning of the game, Player B can choose the size of the surplus S provided that $S \in [0,10]$. Find the Subgame Perfect equilibrium of this game.

6. Consider the problem of a monopoly supplier of water in a council. Supplying water has a constant marginal cost of £6 per cubic metre. There are two types of households in the economy. The preferences of both types of households are separable in water and money. Wealthy households value x cubic metres of water according to the function $u_H(x) = 18\log(x+1)$, whereas low income households value water according to $u_L(x) = 12\log(x+1)$. Only one third of households is wealthy. The monopolist can offer contracts which specify the maximal water supply to the household, and the total price charged for this supply.

   (a) Suppose that the water supplier knows the income status of each household. Find the revenue-maximising contracts offered to high and low income households.

   (b) Next suppose that the income status of each household is unobservable. Find the revenue-maximising contracts offered by the monopolist.

   (c) How are equilibrium payoffs affected by the change in information structure. Quantify the information rents received by each household in part (b).

7. Consider a Principal-Agent problem with: two exogenous states of nature $\{A, B\}$; three effort levels $\{e_h, e_m, e_l\}$; and two output levels distributed as follows as a function of the state of nature and the effort level:

	A	B
Probability	80%	20%
Output Under e_h	25	5
Output Under e_m	5	25
Output Under e_l	5	5

The principal is risk neutral, while the agent has a utility function $w^{1/2}$, when receiving a wage w, minus the effort cost which is zero if e_l is chosen, 1 if e_m is chosen, and 2 if e_h is chosen. The agent's reservation utility is 0.

(a) Derive the optimal wage schedule set by the principal when both effort and output are observable.

(b) Derive the optimal wage schedule set by the principal when only output is observable.

(c) How much would the principal be willing to pay for a technology that reveals whether the agent chose e_h?