Summer 2012 Examination

EC202

Microeconomic Principles II

Suitable for ALL candidates

Instructions to candidates
Time allowed: 3 hours + 10 minutes reading time.
This paper contains seven questions in three sections. Answer question one (section A) and **THREE** other questions, at least **ONE** from section B and at least **ONE** from section C. Question one carries 40% of the total marks; the other questions each carry 20% of the total marks.

Calculators are NOT permitted

Section A

1. Answer any **five** questions from (a)-(h). Each question carries eight marks.

   (a) State whether each of the following statements is true or false. Briefly explain your answers

   - "Long-run marginal cost must be less than or equal to short-run marginal cost."
   - "If an exchange economy is replicated a large number of times then the core of the economy shrinks to a single allocation."
   - "Increasing relative risk aversion implies increasing absolute risk aversion."

   (b) It is often claimed that the price system enables economic decision-making to be decentralised. Explain what this means and the conditions under which the price mechanism performs this role.

   (c) State Walras' law. Explain it in terms of the properties of individual agents' demand and supply functions.

   (d) Under what circumstances is it possible to infer changes in social welfare from changes in national income?

   (e) Consider two competing lobbyists trying to influence a politician with donations. The politician has to choose between one of two policies $\{A, B\}$. The revenue of the first lobbyist is $x_A > 0$ if policy A is chosen and zero otherwise; while the revenue of the second lobbyist is $x_B > x_A$ if policy B is chosen and zero otherwise. Both lobbyists can decide how much spend to influence the politician's decision. The politician chooses the policy A whenever the first lobbyist spends strictly more than the second lobbyist, and policy B otherwise.

   i. Set up the game played by the two lobbyists as a game of complete information. Derive the best response of one of the two lobbyists.

   ii. Does the game possess a Pure Strategy Nash Equilibrium? Explain.

(f) Three hungry lions are members of a hierarchical group, and face a prey. If lion 1 does not eat the prey, the prey escapes, and the game ends. If lion 1 eats the prey, it becomes slow, and lion 2 can eat it. If lion 2 does not eat lion 1, the game ends; if it eats lion 1, it may then be eaten by lion 3. Each lion prefers to eat than to be hungry, but prefers to be hungry than to be eaten.

i. Draw the extensive form game. How could you map such a dynamic game to a static game of complete information?

ii. Find all the Pure Strategy Nash Equilibria of the game and the unique Subgame Perfect Equilibrium of the game.

iii. What would be Subgame Perfect Equilibrium of the game if the group were composed by a finite number n of lions?

(g) Consider an economy with two goods: money m and consumption x. A single firm sells the consumption good in the market. The firm uses money to produce consumption. In particular, it costs x^2 units of money to produce x units of consumption. All the consumers are identical. Every one of them is endowed with M units of money, but with no consumption. The preferences of a consumer with x units of consumption and m units of money satisfy:

$$U(x, m) = x^{1/2} + 3m$$

i. Find the profit maximising price that the monopolist would set, if he is bound to choose among linear pricing schedules, $P(x) = px$.

ii. Find the profit maximising price schedule that the monopolist would set, if he is bound to choose among two-part tariffs, $P(x) = p_0 + p_1 x$.

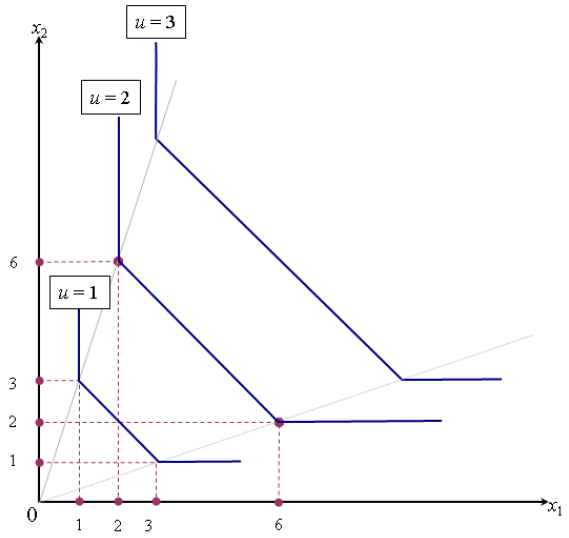
(h) Consider Spence's signalling model. A worker's type is $t \in \{1, 4\}$. The probability that any worker is of type $t = 1$ is equal to $1/4$, while the probability that $t = 4$ is equal to $3/4$. The productivity of a worker in a job is $t^{1/2}$. Each worker chooses a level of education $e \geq 0$. The total cost of obtaining education level e is $C(e|t) = e^2/t$. The worker's wage is equal to his expected productivity.

i. Find a pooling Perfect Bayesian Equilibrium.

ii. Find a separating Perfect Bayesian Equilibrium.

Section B [Answer at least ONE and no more than TWO questions]

2. Suppose a person's preferences are given by the indifference curves illustrated in the figure, where x_1, x_2 denote quantities of two goods and u denotes utility level.



(a) Do the preferences satisfy the axioms of (i) greed (ii) strict quasiconcavity (iii) smoothness? Briefly explain your answers.

(b) The person wants to find the minimum budget that will enable him to achieve a utility level of 8 when the prices of the goods are p_1 and p_2. Using the diagram describe the solution to this problem for the following cases: (i) $p_1 < p_2$, (ii) $p_1 > p_2$, (iii) $p_1 = p_2$.

(c) Provide a general definition of the consumer's cost function and find the cost function for these preferences.

(d) Provide a general definition of the indirect utility function and, using the answer to part (c), find the indirect utility function for these preferences.

(e) Suppose prices are $p_1 = 1$, $p_2 = 2$. The government introduces a sales tax on good 1 so that the price to consumers increases. It also wants to compensate some consumers (all of whom have these preferences) so that they are no worse off. An adviser suggests that this can be done by increasing these consumers' incomes so that they consume exactly the bundle that they were able to consume before the tax was introduced. Is this advice correct?

3. A firm has the cost function $a_0 + a_1 q + a_2 q^2$ where q is output and a_0, a_1, a_2 are positive parameters.

   (a) If the firm is a price-taker, find its supply function.

   (b) If, instead, the firm is a monopolist, find the expression for the firm's marginal revenue in terms of output, assuming that market price is given by $p = b_1 + b_2 q$, where $b_1 > a_1$ and $b_2 < 0$. Illustrate the optimum in a diagram and show that the firm will produce

   $$\hat{q} = \frac{b_1 - a_1}{2\,[a_2 - b_2]}.$$

   What is the price charged $\hat{p}$ and the marginal cost $\hat{c}$ at this output level?

   (c) Suppose this monopoly is regulated: the regulator can control the price by setting a ceiling $p_{\max}$. Plot the average and marginal revenue curves that would then face the monopolist.

   (d) Show that if the price ceiling is set so that $\hat{c} < p_{\max} < \hat{p}$ then the firm's output will rise above $\hat{q}$.

   (e) What will happen if $p_{\max}$ does not satisfy this condition?

4. A person with wealth y_0 is considering investing in a risky enterprise. If the enterprise succeeds the value of the investment will double; if it fails everything invested is lost.

   (a) If the person invests x and the probability of success is $\pi > \frac{1}{2}$ what is

   - ex-post wealth y in the case of success?
   - ex-post wealth in the case of failure?
   - expected ex-post wealth $\mathbb{E}y$?

   (b) If the person's utility is given by $\mathbb{E}\log(y)$, find the optimal size of investment x.

   (c) The government proposes to tax any *gain* from the investment at a rate t but without a loss-offset provision: the payoff in the case of success would be $[1-t]x$ but the outcome in the case of failure would be just as before. Find the optimal x.

   (d) Suppose that the government were to modify this tax and allow full loss offset, so that in the case of failure one only loses $[1-t]x$ rather than x. Again find the optimal x.

   (e) Suppose the government abandoned the proposed tax on gains and replaced it with a tax on *ex-post wealth*. Show that the investment decision would be exactly the same as in part (b). Why is this?

   (f) Show that a rise in the tax rate would reduce investment under tax scheme (c), increase it under tax scheme (d) and leave it unchanged under tax scheme (e).

Section C [Answer at least ONE and no more than TWO questions]

5. Consider three board members $\{1, 2, 3\}$ choosing which of three candidates $\{A, B, C\}$ is to be appointed as the new CEO of a firm. The preferences of the three board members over candidates respectively satisfy:

$$u_1(A) = 1 > u_1(B) = x > u_1(C) = 0$$
$$u_2(B) = 1 > u_2(C) = y > u_2(A) = 0$$
$$u_3(C) = 1 > u_3(B) = z > u_3(A) = 0$$

The preferences of each board member are known by all the other members.

(a) First consider a scenario in which board members can secretly cast one vote in favor of any of the three candidates, and in which the candidate appointed is the one who receives more votes. If there is no outright winner, and two or more candidates receive the largest number of votes, the CEO is chosen at random among these candidates. Model the environment as a game of complete information. Which candidates can be appointed outright (i.e. without a tie) in a Pure Strategy Nash Equilibrium of this game? Find the corresponding strategies for every equilibrium that you discuss. Which candidates, if any, cannot be appointed in any Pure Strategy Nash Equilibrium of this game?

(b) Next consider a scenario in which board members can secretly cast one vote against any of the three candidates, and in which the candidate appointed is the one who receives fewer votes. If two or more candidates receive the smallest number of votes, the CEO is chosen at random among these candidates. Model the environment as a game of complete information. Which candidates can be appointed outright in a Pure Strategy Nash Equilibrium of this game, when $x, y, z > 1/2$? Find the corresponding strategies for each of the equilibria you discuss. Which candidates, if any, cannot be elected in any Pure Strategy Nash Equilibrium of this game? Explain. How would your answer change were $x, y, z < 1/2$.

(c) Which of the two voting rules (described in parts (a) and (b)) would be chosen by a social planner that maximises the sum of utilities of the three board members in the least efficient Nash Equilibrium?

6. A group of neighbours are choosing whether to contribute a fixed amount towards the provision of a pure public good that will benefit them all equally. The good is provided if and only if at least three of them contribute; if it is not provided, contributions are not refunded. Let v denote the value of the public good to each of the neighbors, let f denote the fixed contribution, and assume that $v > f > 0$.

   (a) First assume that the group is composed of five neighbors. Find all pure strategy Nash equilibria of the game. Is there a Nash equilibrium in which more than 3 people contribute? One in which fewer than 3 contribute? Explain your reasoning carefully.

   (b) Next suppose that players face the same problem, but that they do not know whether there are three or five neighbors in the group. In particular, assume that, when choosing their contribution, players believe that the probability that there are only two more individuals is $q > 0$, while the probability that there are four more individuals is $1 - q > 0$. For which values of q does the game possess a Pure Strategy Bayes Nash equilibrium in which everyone contributes? For which values of q does the game possess a Pure Strategy Bayes Nash equilibrium in which nobody contributes?

   (c) Finally suppose that players face the same problem, but that only three neighbors $\{1, 2, 3\}$ are known to belong to the group, while the remaining two players $\{4, 5\}$ may or may not be part of the group. In particular, assume that the prior probability that there are only players $\{1, 2, 3\}$ in the group is $q > 0$, while the probability that all players are present is $1 - q > 0$. How would your answer to part (b) change in such an environment?

7. Consider a Principal-Agent problem with: three exogenous states of nature $\{H, M, L\}$; two effort levels $\{e_a, e_b\}$; and two output levels distributed as follows as a function of the state of nature and the effort level:

	H	M	L
Probability	20%	60%	20%
Output Under e_a	25	25	4
Output Under e_b	25	4	4

   The principal is risk neutral, while the agent has a utility function $w^{1/2}$, when receiving a wage w, minus the effort cost which is zero if e_b is chosen, and 1 otherwise. The agent's reservation utility is 0.

   (a) Derive the optimal wage schedule set by the principal when both effort and output are observable.

   (b) Derive the optimal wage schedule set by the principal when only output is observable.

   (c) If the principal cannot observe effort, how much would he be willing to pay for a technology that, prior to the beginning of the game, reveals when the state L is realized?