EC202 Classes
2017-18

September 26, 2017

Guidelines

Before the class

Your class experience starts in the lecture:

- Make your own written notes in lectures. Don't just rely on the on-line resources.
- Use the course text to review the lectures. Read all of each assigned chapter in order to get the context for the class.
- Work through the mini problems in the course text. Like the classes, they are designed to help you with some of the steps involved in the reasoning.
- Complete the Example Worksheets (handed out in lectures) ready to discuss in class

During each class

You are expected to attend every class and participate:

- Teachers assign specific students to prepare presentations of the material in the schedule below.
- But *all* students need to make a good attempt in advance of the class.
- Team up in small groups if you find this helpful.
- Be ready to get involved in the discussion sessions (they are marked in bold, "**Discuss**")
- Make sure that you personally understand why each exercise "works."

Hand-in work

In most weeks there are assignments for you to hand in: see the separate schedule. These hand-ins are to be *your own work* only: don't prepare them with class-mates or friends.

EC202, 2017-18. Class, week 2

1. Complete the example sheets for Lectures 1 and 2.
2. Draw the isoquants for the following production functions:

$$\alpha_1 z_1 + \alpha_2 z_2 \tag{1}$$

$$\min\left\{\frac{z_1}{\alpha_1}, \frac{z_2}{\alpha_2}\right\} \tag{2}$$

$$\alpha_1 z_1^2 + \alpha_2 z_2^2 \tag{3}$$

3. For a given level of output identify the cost-minimising input combination(s) on the diagram.
4. **Discuss:**
   - Why would the Lagrangian method be inappropriate for these production functions?
   - How can you use the answer to Q3 to find the cost function in each case?
5. For each of these three cases:
   - Write down the cost function, using the answers to Q3 and Q4
   - Check the properties of the cost function highlighted in the lecture

EC202, 2017-18. Class, week 3

1. Complete the example sheets for Lectures 3 and 4.
2. A price-taking firm has a fixed cost F_0 and marginal costs $c = a + bq$. where q is output.
   (a) What is the lowest price $\underline{p}$ at which it will produce a positive output?
   (b) If the price is above $\underline{p}$, find the firm's optimal output q^*.
3. Consider a monopolist with the same cost structure as Q2 and with the inverse demand function $p = A - \frac{1}{2}Bq$ (where $A > a$ and $B > 0$) .
   (a) Find the expression for the monopolist's marginal revenue in terms of output.
   (b) Illustrate the optimum in a diagram
   (c) Find the amount of output q^{**} that the monopolist would produce.
   (d) What is the price charged p^{**} and the marginal cost c^{**} at output level q^{**}?
4. **Discuss:**
   - Will the monopolist produce more/less than the competitive firm? Why?
   - What might happen if a regulatory agency introduced a price ceiling $\overline{p}$ on the monopolist? Would it behave like the firm in Q2?

EC202, 2017-18. Class, week 4

1. Complete the example sheets for Lectures 5 and 6.
2. Draw the indifference curves for the following four types of preferences:

$$\begin{aligned} \text{Type A} &: \alpha \log x_1 + [1-\alpha] \log x_2 \\ \text{Type B} &: \beta x_1 + x_2 \\ \text{Type C} &: \gamma [x_1]^2 + [x_2]^2 \\ \text{Type D} &: \min\{\delta x_1, x_2\} \end{aligned}$$

   where x_1, x_2 denote respectively consumption of goods 1 and 2 and $\alpha, \beta, \gamma, \delta$ are strictly positive parameters with $\alpha < 1$.

3. Find the cost function if the consumer's preferences are of type A.
4. **Discuss:**
   - How does Q3 relate to the lecture examples?
   - Why would the Lagrangian method be inappropriate for B, C, D?
   - What can we usefully "borrow" from week 2's class here?
   - What must the cost function be for each of cases B, C, D?
   - What is distinctive about consumer demands in each of cases A,...,D?
5. You are planning to carry out an empirical study of consumer demand using the following specification for consumer expenditure on each good i:

$$e_i = \alpha_i p_i + \sum_{j=1}^{n} \beta_{ij} p_j + \gamma_i y + \delta_i, \ i = 1, ..., n$$

   where n is the number of goods, p_i is the price of good i, e_i is the consumer's expenditure on good i, y is income and the α_i, β_{ij}, γ_i, δ_i terms are parameters. You are concerned that the specification be consistent with standard consumer theory and you discuss this issue with three friends. They reply with the following three opinions:
   (1) "the equation will work for any values of the parameters;"
   (2) "you need to impose the restrictions $\beta_{ij} = 0$, $\sum_{i=1}^{n} \delta_i = 1$;"
   (3) "you need to impose restrictions such as $\beta_{ij} = -\gamma_i \alpha_j$, $\gamma_i \geq 0$, $\sum_{i=1}^{n} \gamma_i = 1$, $\delta_i = 0$."

   Which statement is correct and why?

EC202, 2017-18. Class, week 5

1. Complete the example sheets for Lectures 7 and 8.

2. A person has preferences represented by the utility function $\sum_{i=1}^{n} \alpha_i \log(x_i)$ where $\alpha_i > 0$ and $\sum_{i=1}^{n} \alpha_i = 1$. Show that:

   - for given income y and prices $\mathbf{p}$, demand for commodity i is $x_i^* = \frac{\alpha_i y}{p_i}$,
   - the cost function is $C(\mathbf{p}, \upsilon) = Ae^{\upsilon} p_1^{\alpha_1} p_2^{\alpha_2} ... p_n^{\alpha_n}$.

3. **Discuss:**

   - Take the preferences in Q2 for the case $n = 2$. How could this be used as a simple model of a person's lifetime decisions?
   - In this case write $\alpha_1 = \alpha$ and $\alpha_2 = 1 - \alpha$. What is the interpretation of α here?

4. Take a two-period model of the lifetime, as discussed in Q3. The consumer's has an amount A in period 1, which is partly spent on consumption in period 1 and partly invested in an asset paying a rate of interest r.

   - Obtain the optimal allocation of (x_1, x_2)
   - Explain how consumption varies with A, r and α.

5. In the Kingdom of Brexonia everyone has the preferences specified in Q2.

   - Brexonia is about to leave the European Union. As a consequence the price of wine will double but the price of milk will fall to one-eighth of its previous value. Use compensating-variation analysis to evaluate the impact on a typical Brexonian consumer's welfare of these price changes.
   - Brexonian experts have estimated that a typical Brexonian spends more than three times as much on wine as on milk. The experts conclude that leaving the European Union is against the interests of Brexonia. Are they right?
   - **Discuss:** What additional assumptions do you need to make in order to get a clear-cut answer here?

EC202, 2017-18. Class, week 6

1. Complete the example sheet for Lecture 9.

2. A person lives for two periods. Her utility function is given by $\alpha \log x_1 + [1-\alpha] \log x_2$ (where x_i is the amount of consumption in period i) and receives an exogenous income (y_1, y_2) in the two periods; by investing an amount z in education during period 1 she can augment income in period 2 by an amount $\tau [1 - e^{-z}]$. The parameters satisfy $0 < \alpha < 1$ and $\tau > 0$.

    - Assuming that there is a perfect market for borrowing and lending at a uniform rate r, find her optimal investment in education z^*. Under what circumstances would $z^* = 0$?
    - If $z^* > 0$ how is it affected by (a) r and (b) τ?
    - What is the person's optimal amount of borrowing in period 1?

3. **Discuss:**

    - What is the interpretion of the parameters α and τ in Q2?
    - What is the role of the separation theorem in Q2?

EC202, 2017-18. Class, week 7

1. Complete the example sheets for Lectures 10 and 11.

2. Suppose the 3-good economy in Lecture 10's example is modified so the production constraint is $[q_1]^2 + [q_2]^2 + Aq_3 \leq 0$, where A is a positive constant. Production is organised to maximise profits at given prices. Let p_1 and $p_2$denote the prices of goods 1 and 2 expressed in terms of good 3. Show that

   - profit-maximising net outputs are $q_i = {}^1\!/\!{}_2 A p_i, \quad i = 1, 2,$
   - maximised profits are $\Pi = {}^1\!/\!{}_4 A \left[p_1^2 + p_2^2\right]$.

3. **Discuss:**

   - What would happen to the transformation curve if the constant A had a larger value?
   - Give a simple interpretation of the constant A.

EC202, 2017-18. Class, week 8

1. Complete the example sheets for Lectures 12 and 13.

2. **Discuss:** Which of the following statements is right? If a statement is wrong, how is it wrong?

   - If an exchange economy is replicated indefinitely the core of the economy shrinks to a single allocation.
   - By Walras' law, the sum over all goods of price times excess demand must equal zero, but only in the neighbourhood of equilibrium.
   - A general equilibrium will exist only if the weak axiom of revealed preference is satisfied by all excess demand functions.
   - In a general equilibrium it is not necessarily the case that excess demand equals zero in every market..

3. In a two-good economy the excess demand functions for goods 1 and 2 are, respectively:

$$7 - 12\rho + 6\rho^2 - \rho^3$$
$$\rho^4 - 6\rho^3 + 12\rho^2 - 7\rho.$$

   where ρ is the price of good 1 in terms of good 2.

   - Show how to check Walras' Law for this economy.
   - How many equilibria does the economy have? Explain your reasoning.
   - Is the system stable under tatonnement? Explain.

4. In an uncertain situation where there are exactly two possible states of the world $i = 0, 1$ a person has preferences represented by the function $-\alpha y_0^{-\gamma} - \beta y_1^{-\delta}$, where y_i is the payoff in state i and $\alpha, \beta, \gamma, \delta$ are parameters.

   - What restriction on parameters is required to ensure that these preferences can be represented by a von-Neumann-Morgenstern utility function?
   - If the parameters have the values $\alpha = \beta = \gamma = \delta = 1$ and the payoffs are $y_0 = 1$ and $y_1 = 3$ find the certainty equivalent

EC202, 2017-18. Class, week 9

1. Complete the example sheets for Lectures 14 and 15.

2. **Discuss:** Use the week 8, Q4 model for *two* people:

   - How could we depict differing degrees of risk aversion between the two people?
   - How could we depict the possibility that the two people have different subjective probabilities for states 0 and 1?

3. People hold wealth either in bonds or as cash, or a mixture of the two. The rate of return on bonds is a random variable with a known distribution: the expected rate of return is positive, but there is a positive probability that the rate of return will be below zero. Cash always yields a zero return.

   (a) If someone holds a mixed portfolio compute the expression for (pre-tax) wealth after the rate of return on bonds becomes known.

   (b) Compute disposable (post-tax) wealth if there is:

      i. a proportional income tax with full loss offset (i.e. losses and gains are treated symmetrically by the tax).

      ii. a proportional wealth tax.

   (c) Assume that the person chooses the composition of the portfolio so as to maximize the expected value of the utility of disposable wealth. Find the first-order condition which determines the optimal bond holding under each of the two taxation schemes.

   (d) The government's advisers point out that, if an income tax is used, an increase in the tax rate will increase the holding of bonds, but that a similar effect might not obtain if the tax were based on wealth. Are they right?

EC202, 2017-18. Class, week 10

1. Complete the example sheets for Lectures 16 and 17.

2. **Discuss:** Which of the following statements is right? If a statement is wrong, how is it wrong?

   - "In a given economy a Pereto-efficient allocation must be superior to a Pareto-inefficient allocation."
   - "A competitive equilibrium might not be Pareto Efficient."
   - "A competitive-equilibrium allocation can never be a fair allocation."
   - "In the absence of externalities you can always implement a Pareto-efficient allocation if firms maximise profits and consumers maximise utility."

3. In a two-commodity economy without production, there are two types of agent: each a-type agent has the utility function $\log(x_1^a) + \frac{1}{2}\log(x_2^a)$, each b-type has the utility function $\frac{1}{2}\log x_1^b + \log x_2^b$, where x_i^h denotes an h-type's consumption of good i. There are N agents of each type.

   (a) If the total quantity of goods available is given by $(3N, 12N)$ what is the set of Pareto-efficient allocations in this economy?

   (b) Which of the following allocations are efficient?

$$\left.\begin{array}{l}(x_1^a, x_2^a) = \left(\frac{4}{3}, 2\right)\\ \left(x_1^b, x_2^b\right) = \left(\frac{5}{3}, 10\right)\end{array}\right\} \tag{4}$$

$$\left.\begin{array}{l}(x_1^a, x_2^a) = (2, 4)\\ \left(x_1^b, x_2^b\right) = (1, 8)\end{array}\right\} \tag{5}$$

$$\left.\begin{array}{l}(x_1^a, x_2^a) = (2, 2)\\ \left(x_1^b, x_2^b\right) = (1, 9)\end{array}\right\} \tag{6}$$

$$\left.\begin{array}{l}(x_1^a, x_2^a) = (1, 3)\\ \left(x_1^b, x_2^b\right) = (2, 9)\end{array}\right\} \tag{7}$$

   (c) If each a-type has the endowment $(1, k)$ and each b-type has the endowment $(2, 12 - k)$, where $0 \leq k \leq 12$, show that the competitive equilibrium price of good 1 (in terms of good 2) is $[k + 12]/5$

   (d) For the case in part 3 find the equilibrium allocation as a function of k. If $k = 3$ show that the equilibrium allocation is (4) above.

   (e) Take the case in part 4 with $k = 3$. Suppose that endowments of good 2 can costlessly transferred between agents, but that endowments of good 1 cannot be transferred. A social planner wants to ensure allocation (5) as a competitive equilibrium outcome rather than (4) What transfer of endowment would ensure this? How would equilibrium prices change relative to the case of part (c)?

EC202, 2017-18. Class, week 11

1. Complete the example sheet for Lectures 18 and 19.

2. In a two-commodity exchange economy there are two groups of people: type a have the utility function $2\log(x_1^a)+\log(x_2^a)$ and an endowment of ℓ units of commodity 1 and k units of commodity 2; type b have the utility function $\log(x_1^b)+2\log(x_2^b)$ and an endowment of $3-\ell$ units of commodity 1 and $7-k$ units of commodity 2, where $0 \leq k \leq 7$.

   (a) Show that the equilibrium price of good 1 in terms of good 2 is $\frac{7+k}{6-\ell}$.

   (b) What are the individuals' incomes (y^a, y^b) in equilibrium as a function of k and ℓ? Assume that it is possible to carry out lump-sum transfers of commodity 2 (k can be varied), but impossible to transfer commodity 1 (ℓ is fixed). Draw a diagram of the set of attainable income distributions

      i. in the case where $\ell = 0$;

      ii. in the case where $\ell = 3$.

   (c) The government seeks to maximise the welfare function $y^a + y^b$.

      i. Is the government inequality-averse?

      ii. What would be the optimal distribution of income in the case $\ell = 0$? Comment on the result.

      iii. What would be the optimal distribution of income in the case $\ell = 3$? Comment on the result.