EC202 Classes 2018-19

September 2018

Guidelines

Before each class

Your class experience starts in the lecture:

- Make your own written notes in lectures.
- Use the course text to review the lectures and work through the mini problems. This will set the context for the class.
- Complete the **worksheets** (handed out in lectures) ready to discuss in class. Team up in small groups for this, if you like.
- Answer all questions in the **problem set** for the class and post a pdf of your answers on Moodle by midnight on Sunday. Do not team up for this: these solutions need to be your own work.

During each class

You are expected to be at every class, and to get involved:

- Make sure you come with a copy of your answers to that week's problem set.
- Join in the conversation on the parts marked **"Discuss"** in the problem sets.
- Make sure that you fully understand why each problem "works."

After each class

- Solutions to all questions will be posted on Moodle.
- Your class teacher will check your progress from your on-line submissions.
- Detailed marking will be carried out for some, but not all, questions. You will get feedback on this.

EC202, 2018-19. Problem set 1 (week 2)

1. Take the production function $\sqrt{z_1 z_2}$ where z_i is the quantity of input i.

   (a) Find the equation for the isoquants. Do the isoquants touch the axes? Explain.

   (b) Let m be the marginal rate of technical substitution $-\frac{dz_2}{dz_1}$ and let r be the input ratio $\frac{z_2}{z_1}$. Find m as a function of r.

2. Now take the production function $\frac{\mathbf{1}}{\mathbf{2}}z_1^2 + \frac{\mathbf{1}}{\mathbf{2}}z_2^2$.

   (a) Draw the isoquants.

   (b) For a given level of output identify the cost-minimising input combination(s) on the diagram.

   (c) **Discuss:** Why would the Lagrangian method be inappropriate for finding an optimum?

   (d) **Discuss:** How could you use your answer to (b) to find the cost function?

3. Finally, take the production function: $\left[\frac{\mathbf{1}}{\mathbf{2}}z_1^\beta + \frac{\mathbf{1}}{\mathbf{2}}z_2^\beta\right]^{\frac{1}{\beta}}$ where β is a parameter that may take any value from $-\infty$ to 1.

   (a) Answer part (a) of Q1 for this production function.

   (b) Answer part (b) of Q1 for this production function.

   (c) Show that the elasticity of r with respect to m is $\sigma = \frac{1}{1-\beta}$.

   (d) **Discuss:** How is the production function of Q1 related to this production function?

Follow-up:

- Exercise 2.6 (fairly easy)
- Exercise 2.11 (tougher)

EC202, 2018-19. Problem set 2 (week 3)

1. A price-taking firm has a fixed cost F_0 and marginal costs $c = a + bq$. where q is output.
   (a) What is the lowest price $\underline{p}$ at which it will produce a positive output?
   (b) If the price is above $\underline{p}$, find the firm's optimal output q^*.

2. Consider a monopolist with the same cost structure as Q1 and with the inverse demand function $p = A - \frac{1}{2}Bq$ (where $A > a$ and $B > 0$) .
   (a) Find the expression for the monopolist's marginal revenue in terms of output.
   (b) Illustrate the optimum in a diagram
   (c) Find the amount of output q^{**} that the monopolist would produce.
   (d) What is the price charged p^{**} and the marginal cost c^{**} at output level q^{**}?

3. For the price taking firm in Q1 and the monopolist in Q2:
   (a) Will the monopolist produce more/less than the competitive firm? Why?
   (b) **Discuss:** What might happen if a regulatory agency introduced a price ceiling $\overline{p}$ on the monopolist? Would it behave like the firm in Q2?

Follow-up:

- Exercise 3.2 (A more interesting cost function than Q1, Q2)
- Exercise 3.5 (Addresses the "Discuss" point in Q3)

EC202, 2018-19. Problem set 3 (week 4)

1. Consider the following four types of preferences:

$$\begin{aligned} \text{Type A} &: \alpha \log x_1 + [1-\alpha]\log x_2 \\ \text{Type B} &: \beta x_1 + x_2 \\ \text{Type C} &: \gamma [x_1]^2 + [x_2]^2 \\ \text{Type D} &: \min\{\delta x_1, x_2\} \end{aligned}$$

   where x_1, x_2 denote respectively consumption of goods 1 and 2 and $\alpha, \beta, \gamma, \delta$ are strictly positive parameters with $\alpha < 1$.

   (a) Draw the indifference curves for each case

   (b) Find the cost function if the consumer's preferences are of type A.

   (c) **Discuss:** Would the Lagrangian method be appropriate for B, C, D?

   (d) **Discuss:** What could we "borrow" from Problem set 1 here?

2. A person with fixed money income has the following utility function

$$U(x_1, x_2) = \begin{cases} x_1^{\alpha}[x_2 - \beta]^{1-\alpha} & \text{if } x_1 > 0, x_2 > \beta, \\ 0 & \text{otherwise,} \end{cases}$$

   where the parameters α, β satisfy $0 < \alpha < 1$ and $\beta > 0$.

   (a) Draw the indifference curves and interpret the parameters α and β.

   (b) Find the cost function, the ordinary demand and compensated demand for the two goods.

3. You plan to use the following specification for an empirical study:

$$e_i = \alpha_i p_i + \sum_{j=1}^{n} \beta_{ij} p_j + \gamma_i y + \delta_i,\ i = 1, ..., n$$

   where e_i is the consumer's expenditure on good i, p_i is the price of good i, y is income and the α_i, β_{ij}, γ_i, δ_i terms are parameters. You want the specification be consistent with standard consumer theory and three friends offer the following opinions: (1) "the equation will work for any values of the parameters;" (2) "you need to impose the restrictions $\beta_{ij} = 0$, $\sum_{i=1}^{n} \delta_i = 1$;" (3) "you need to impose restrictions such as $\beta_{ij} = -\gamma_i \alpha_j$, $\gamma_i \geq 0$, $\sum_{i=1}^{n} \gamma_i = 1$, $\delta_i = 0$." Carefully explain which of these opinions is correct.

Follow-up:

- Exercise 4.6 (a work-out on elasticities)
- Exercise 4.8 (a follow-through on Q3)

EC202, 2018-19. Problem set 4 (week 5)

1. A person has preferences represented by the utility function $U(\mathbf{x}) = \sum_{i=1}^{n} \alpha_i \log(x_i)$ where $\alpha_i > 0$ and $\sum_{i=1}^{n} \alpha_i = 1$.
   (a) Find the demand for good i in terms of income y and prices $\mathbf{p}$.
   (b) Show that the cost function is $C(\mathbf{p}, \upsilon) = Ae^{\upsilon} p_1^{\alpha_1} p_2^{\alpha_2} ... p_n^{\alpha_n}$.
   (c) If the utility function had been expressed as $\hat{U}(\mathbf{x}) = x_1^{\alpha_1} x_2^{\alpha_2} ... x_n^{\alpha_n}$ how, if at all, would the answers to parts (a) and (b) change?
   (d) **Discuss:** How could this be used as a model of a person's lifetime decisions?

2. A consumer with given income y buys quantities x_1, x_2 of two goods at prices p_1, p_2. The utility function is $\alpha\sqrt{x_1} + x_2$, where $\alpha \geq 0$ is a parameter.
   (a) Find the demand functions for the two goods. Will both goods be consumed in positive quantities at the optimum?
   (b) Find the indirect utility function and the cost function.
   (c) Assuming that both goods are consumed in positive amounts, find the compensating variation for a change in the price of good 1.

3. Let the demand by household 1 for good 1 be given by

$$x_1^1 = \left\{ \begin{array}{ll} \frac{y}{4p_1} & \text{if } p_1 > a \\ \frac{y}{2p_1} & \text{if } p_1 < a \\ \frac{y}{4a} \text{ or } \frac{y}{2a} & \text{if } p_1 = a \end{array} \right\},$$

   where $a > 0$.
   (a) Draw the demand curve for household 1 and sketch an indifference map that would yield it.
   (b) Suppose there is a second, identical household: show that at $p_1 = a$ there are three possible values of $\frac{1}{2}[x_1^1 + x_1^2]$.
   (c) Extend the argument to n_h identical consumers. Show that as $n_h \to \infty$ the possible values of the consumption of good 1 per household becomes the entire segment $\left[\frac{y}{4a}, \frac{y}{2a}\right]$.
   (d) **Discuss:** What kind of good might be represented by this model?

Follow-up:

- Exercise 4.14 (an application of consumer welfare)
- Exercise 5.9 (application to labour supply)

EC202, 2018-19. Problem set 5 (week 6)

1. A person lives for two periods. Her utility function is given by $\alpha \log x_1 + [1 - \alpha] \log x_2$ (where x_i is the amount of consumption in period i and $0 < \alpha < 1$) and receives an exogenous income (y_1, y_2) in the two periods. Income in both periods is fixed and there is a perfect market for borrowing and lending at a uniform rate r

   (a) What is the interpretation of the parameter α?

   (b) Find the amount that she borrows/saves in period 1.

2. There is a banking crisis and the person finds that she is unable to borrow in period 1. Use a diagram to show the possible outcomes to the optimisation problem now.

3. Take the model of Q1 but assume now that by investing an amount z in education during period 1 the person can augment income in period 2 by an amount $\tau [1 - e^{-z}]$ where $\tau > 0$.

   (a) What is the interpretation of the parameter τ ?

   (b) Find her optimal investment in education z^*. Under what circumstances would $z^* = 0$?

   (c) If $z^* > 0$ how is it affected by (a) r and (b) τ?

   (d) What is the person's optimal amount of borrowing in period 1?

   (e) **Discuss:** What is the role of the separation theorem here?

Follow-up:

- Exercise 5.5 (A three-period savings model)
- Exercise 6.5 (A more general model with investment)

EC202, 2018-19. Problem set 6 (week 7)

1. Suppose the 3-good economy in Lecture 10's example is modified so the production constraint is $[q_1]^2 + [q_2]^2 + Aq_3 \leq 0$, where A is a positive constant. Production is organised to maximise profits at given prices. Let p_1 and p_2 denote the prices of goods 1 and 2 expressed in terms of good 3.

   (a) Show that profit-maximising net outputs are $q_i = {}^1\!/_2 A p_i, \quad i = 1, 2,$ and maximised profits are $\Pi = {}^1\!/_4 A \left[p_1^2 + p_2^2\right]$.

   (b) **Discuss:** Give an interpretation of the constant A. What would happen to the transformation curve if A had a larger value?

2. In a two-good exchange economy there are 1 000 units of commodity 1 and 2 000 units of commodity 2 and equal numbers of two types of trader a and b. Suppose each a-type is endowed with an equal share of commodity 1 and has preferences given by $x_1^a x_2^a$; each b type is endowed with an equal share of commodity 2 and has preferences given by $\min\left\{x_1^b, x_2^b\right\}$ where x_i^h denotes consumption by trader h of commodity i. What is the contract curve in this case? Is there a competitive equilibrium? If so find the equilibrium allocation and prices.

3. Consider an economy that is similar to that of question 2 but that where there are equal numbers of two types of trader a and c: an a type's endowment and preferences are exactly as in question 2; each c type has an equal share of commodity 2 and has preferences given by $3x_1^c + x_2^c$ where again x_i^c denotes consumption by trader c of commodity i. What is the contract curve in this case? Find the competitive equilibrium allocation and prices.

Follow-up:

- Exercise 7.3 (More practice with the contract curve)
- Exercise 7.8 (Equilibrium analysis with a trickier set of preferences)

EC202, 2018-19. Problem set 7 (week 8)

1. **Discuss:** Which of the following statements is right? If a statement is wrong, how is it wrong?

   - If an exchange economy is replicated indefinitely the core of the economy shrinks to a single allocation.
   - By Walras' law, the sum over all goods of price times excess demand must equal zero, but only in the neighbourhood of equilibrium.
   - A general equilibrium will exist only if the weak axiom of revealed preference is satisfied by all excess demand functions.
   - In a general equilibrium it is not necessarily the case that excess demand equals zero in every market..

2. In a two-good economy the excess demand functions for goods 1 and 2 are, respectively:

$$7 - 12\rho + 6\rho^2 - \rho^3$$
$$\rho^4 - 6\rho^3 + 12\rho^2 - 7\rho.$$

   where ρ is the price of good 1 in terms of good 2.

   - Show how to check Walras' Law for this economy.
   - How many equilibria does the economy have? Explain your reasoning.
   - Is the system stable under tatonnement? Explain.

3. In an uncertain situation where there are exactly two possible states of the world $i = 0, 1$ a person has preferences represented by the function $-\alpha y_0^{-\gamma} - \beta y_1^{-\delta}$, where y_i is the payoff in state i and $\alpha, \beta, \gamma, \delta$ are parameters.

   - What restriction on parameters is required to ensure that these preferences can be represented by a von-Neumann-Morgenstern utility function?
   - If the parameters have the values $\alpha = \beta = \gamma = \delta = 1$ and the payoffs are $y_0 = 1$ and $y_1 = 3$ find the certainty equivalent

Follow-up:

- Exercise 7.6 (More practice with excess demand functions)
- Exercise 8.9 (CARA instead of CRRA preference)

EC202, 2018-19. Problem set 8 (week 9)

1. **Discuss:** Use the week 8, Q3 model for *two* people:

   - How could we depict differing degrees of risk aversion between the two people?
   - How could we depict the possibility that the two people have different subjective probabilities for states 0 and 1?

2. Anne has an initial stock of wealth W and risks losing some of this wealth through fire. The probability of such a fire is known to be π and the loss if the fire occurs would be L (where $L < W$). Insurance cover against a fire is available at a premium κ, where $\kappa > \pi L$; it is also possible to take out partial cover on a pro-rata basis, so that an amount tL of the loss can be covered at cost $t\kappa$ where $0 < t < 1$.

   (a) Draw and explain a diagram depicting Anne's budget set.

   (b) Anne's preferences under uncertainty are given by a standard von Neumann-Morgenstern utility function. Explain why Anne will not choose full insurance.

   (c) Assuming that she is risk averse, find the conditions that will determine Anne's optimal value of t.

3. In the same economy as Q2 Beth has wealth greater than Anne's, but she faces the same possible loss through fire L with the same probability π; she can get insurance cover on exactly the same terms as Anne. Beth has the same preferences as Anne and these preferences exhibit decreasing absolute risk aversion. Use your answer to Q2 to show that the insurance cover Beth chooses is less than that chosen by Anne.

Follow-up:

- Exercise 8.13 (Practice with portfolio-choice model)
- Exercise 8.14 (An application to tax evasion)

EC202, 2018-19. Problem set 9 (week 10)

1. **Discuss:** Which of the following statements is right? If a statement is wrong, how is it wrong?

    - "In a given economy a Pareto-efficient allocation must be superior to a Pareto-inefficient allocation."
    - "A competitive equilibrium might not be Pareto Efficient."
    - "In the absence of externalities you can always implement a Pareto-efficient allocation if firms maximise profits and consumers maximise utility."

2. **Discuss:** Consider the private-sector "solution" to the efficiency problem discussed in lecture. Why might it be harder to apply to a consumption externality than a production externality?

3. In a two-commodity exchange economy there are two large equal-sized groups of traders. Each trader in group a has an endowment of 300 units of commodity 1; each person in group b has an endowment of 200 units of commodity 2. Each a-type person has preferences given by the utility function
$$U^a(\mathbf{x}^a) = x_1^a x_2^a$$
and each b-type person's utility can be written as
$$U^b(\mathbf{x}^b) = \frac{x_1^b x_2^b}{x_1^a}$$
where x_i^h means the consumption of good i by an h-type person.

    (a) Find the competitive equilibrium allocation

    (b) Explain why the competitive equilibrium is inefficient.

    (c) Suggest a means whereby a government could achieved an efficient allocation.

Follow-up:

- Exercise 9.1 (Spotting efficient allocations)
- Exercise 9.4 (Practice with welfare concepts)

EC202, 2018-19. Problem set 10 (week 11)

1. In a two-commodity exchange economy there are two groups of people: type a have the utility function $2\log(x_1^a)+\log(x_2^a)$ and an endowment of ℓ units of commodity 1 and k units of commodity 2; type b have the utility function $\log(x_1^b)+2\log(x_2^b)$ and an endowment of $3-\ell$ units of commodity 1 and $7-k$ units of commodity 2, where $0 \leq k \leq 7$.

   (a) Show that the equilibrium price of good 1 in terms of good 2 is $\frac{7+k}{6-\ell}$.

   (b) What are the individuals' incomes (y^a, y^b) in equilibrium as a function of k and ℓ?

2. In this economy, assume that it is possible to carry out lump-sum transfers of commodity 2 (k can be varied), but impossible to transfer commodity 1 (ℓ is fixed). Draw a diagram of the set of attainable income distributions

   (a) in the case where $\ell = 0$;

   (b) in the case where $\ell = 3$.

3. In this economy the government seeks to maximise the welfare function $y^a + y^b$.

   (a) **Discuss:** Is the government inequality-averse?

   (b) What would be the optimal distribution of income in the case $\ell = 0$? Comment on the result.

   (c) What would be the optimal distribution of income in the case $\ell = 3$? Comment on the result.

Follow-up:

- Exercise 9.7 (An extension of Q1-Q3)
- Exercise 9.10 (An application of consumer welfare in social decision making: see the follow-up to week 5)