REVISION LECTURE

EC202
http://darp.lse.ac.uk/ec202

1st December 2016

Frank Cowell

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The exam paper

- Scope of exam material
  - what's covered in the lectures…
  - … is definitive for the exam
- Structure and format of paper
  - you need to answer all the questions
  - 8 short questions (each is a complete mini-question)
  - 2 long questions (typically a multipart question)
- Mark scheme
  - 5 marks for each short question
  - 30 marks for each long question
  - multipart questions: marks per part shown on the exam paper

2014 1(d)

Suppose that in the equilibrium of an exchange economy everyone has the same income. Will the equilibrium be a fair allocation? Explain your answer.

- Mainly a "principles" question
- Provide a definition of fairness
- Describe the result given in the lecture
- Don't forget the last three words of the question!

Explain: Consider household h that chooses $\mathbf{x}^h$ to max $U^h(\mathbf{x}^h)$ subject to $\sum p_i x_i^h \leq y$ where y is the same income for every household. Suppose h chooses $\mathbf{x}^{*h}$ and k chooses $\mathbf{x}^{*k}$ in equilibrium. By definition of optimisation $U^h(\mathbf{x}^{*h}) \geq U^h(\mathbf{x}^{*k})$ and so h would not envy k's bundle in equilibrium.

2012 Q4 (a)

A person with wealth y_0 is considering investing in a risky enterprise. If the enterprise succeeds the value of the investment will double; if it fails everything invested is lost.

(a) If the person invests x and the probability of success is $\pi > \frac{1}{2}$ what is

- ex-post wealth y in the case of success?
- ex-post wealth in the case of failure?
- expected ex-post wealth $\mathbb{E}y$?

In the case of success the investor would have $[y_0 - x] + 2x = y_0 + x$ as ex-post wealth. Therefore

$$y = \begin{cases} y_0 + x & \text{with probability} \quad \pi \\ y_0 - x & \text{with probability} \quad 1 - \pi \end{cases}.$$

Expected ex-post wealth is $\mathbb{E}y = \pi[y_0 + x] + [1 - \pi][y_0 - x] = y_0 + x[2\pi - 1]$

2015 Q7

A monopolist has the cost function $\alpha_0 + \alpha_1 q + \alpha_2 q^2$ (where the αs are positive parameters and q is output) and faces the inverse demand function $p = \beta_1 - \beta_2 q$ (where $\beta_1 > \alpha_1$ and $\beta_2 > 0$).

(a) Find the expressions for the firm's average revenue, marginal revenue, average cost and marginal cost in terms of output. [4 marks]

Average costs are $\alpha_0/q + \alpha_1 + \alpha_2 q$. and marginal costs are $\alpha_1 + 2\alpha_2 q$. Average revenue is simply $\beta_1 - \beta_2 q$; therefore total revenue at q is $\beta_1 q - \beta q^2$ and so, differentiating this with respect to q, marginal revenue is $\beta_1 - 2\beta_2 q$.

Tips

- Follow the leads
  - examiners may be on your side!
  - so if you're pointed in the right direction, follow it…
- Pictures
  - help you to see the solution
  - help you to explain your solution to examiner
- What *should* the answer be?
  - take a moment before each part of the question
  - check the "shape" of the problem
  - use your intuition
- Does it make sense?
  - again take a moment to check after each part
  - we *all* make silly slips

The exam paper

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Question style – three types

- 1 Principles
  - reason on standard results and arguments
  - can use verbal and/or mathematical reasoning
- 2 Model solving
  - a standard framework
  - you just turn the wheels
- 3 Model building
  - usually get guidance in the question
  - longer question sometimes easier?
- One type not necessarily "easier" or "harder" than another
  - short questions usually get you to do both types 1 and 2
  - type 3 is usually only in the long questions

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2015 3

- Straight "principles" in the form of T/F
- Be sure to read the question carefully
- Be sure to *give your reasons*

A competitive equilibrium is always Pareto Efficient.

False. If there are consumption or production externalities then a CE may be inefficient.

A competitive-equilibrium allocation can never be a fair allocation.

False. If incomes are equal then the competitive equilibrium allocation is fair in the sense that no agent envies the consumption of any other agent.

In the neighbourhood of a competitive equilibrium, social-welfare changes are proportional to changes in national income. [3 marks]

False. For changes in social welfare to be proportional to changes in national income we need to be at a global optimum where the social marginal value of a dollar of income is equated across all individuals.

2014 1(a)

A single-output firm can use any of the following three production techniques

$$\text{Technique \#1:} \quad q \le \min\left(\frac{1}{3}z_1, z_2\right)$$

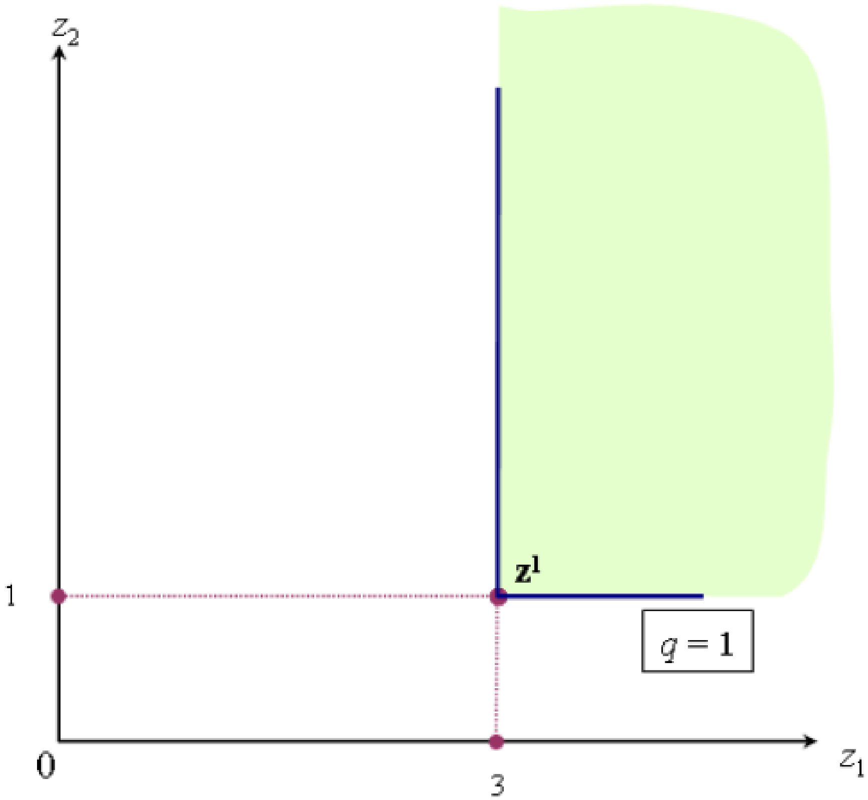
$$\text{Technique \#2:} \quad q \le \min\left(z_1, \frac{1}{3}z_2\right)$$

$$\text{Technique \#3:} \quad q \le \min\left(\frac{2}{5}z_1, \frac{2}{5}z_2\right)$$

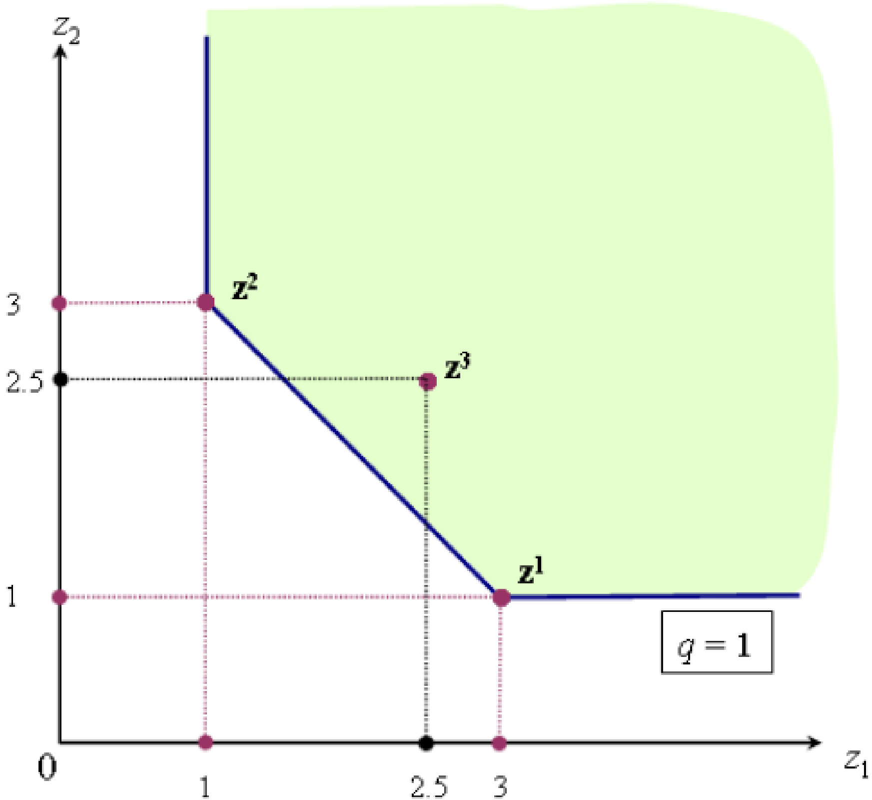
where q denotes the quantity of output and z_1, z_2 the quantities of two inputs. The firm can also use combinations of techniques. Draw the isoquant for $q = 1$.

- A simple model
- Be sure to draw the diagram carefully…
- …and **think**

2014 1(a)



z_2
1
0
3
z_1
$\mathbf{z}^1$
$q = 1$



z_2
3
2.5
1
0
1
2.5
3
z_1
$\mathbf{z}^2$
$\mathbf{z}^3$
$\mathbf{z}^1$
$q = 1$

2013 1(c)

- Principles and model-solving
- Write down the principle
- Write down the basics of the model and solve

An individual has a utility function $U = \mathcal{E}u(x)$ where x is a random variable with a known distribution, $\mathcal{E}$ is the expectations operator, and u is given by

$$u(x) = 2ax - x^2,$$

where a is a parameter such that $a > \max x$.

i. Show that U can be written as a function of the mean and variance of the distribution. $U = \mathcal{E}u(x) = 2a\mathcal{E}x - \mathcal{E}(x^2) = 2a\mathcal{E}x - (\mathcal{E}x)^2 - var(x)$

ii. Is absolute risk aversion increasing / constant / decreasing?

iii. Is relative risk aversion increasing / constant / decreasing?

- Use standard definition of expected utility and variance
- Define ARA $-\dfrac{u_{xx}(x)}{u_x(x)} = \dfrac{1}{a-x}$
- Differentiate *u* $u_x(x) = 2a - 2x$ $\quad u_{xx}(x) = -2$
- Evaluate ARA
- Multiply by *x* to get RRA

2012 Q4 (a)

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$$y = \begin{cases} y_0 + x & \text{with probability} \quad \pi \\ \\ y_0 - x & \text{with probability} \quad 1 - \pi \end{cases} .$$

Expected ex-post wealth is $\mathbb{E}y = \pi\,[y_0 + x] + [1 - \pi]\,[y_0 - x] = y_0 + x\,[2\pi - 1]$

2012 Q4 (b)

If the person's utility is given by $\mathbb{E}\log(y)$, find the optimal size of investment x.

The value of x is chosen to maximise

$$\pi \log(y_0 + x) + [1 - \pi] \log(y_0 - x).$$

Assume that there is an interior solution. The first-order condition for a maximum is

$$\frac{\pi}{y_0 + x} + \frac{\pi - 1}{y_0 - x} = 0 \tag{8}$$

which implies

$$x^* = [2\pi - 1] \, y_0 \tag{9}$$

which is positive given the condition $\pi > \frac{1}{2}$ specified in the question.

2012 Q4 (c)…

The government proposes to tax any *gain* from the investment at a rate t but without a loss-offset provision: the payoff in the case of success would be $[1-t]x$ but the outcome in the case of failure would be just as before. Find the optimal x.

Without loss offset final wealth would be

$$y = \begin{cases} y_0 + x\,[1-t] & \text{with probability} \quad \pi \\ y_0 - x & \text{with probability} \quad 1-\pi \end{cases} .$$

…2012 Q4 (c)

The government proposes to tax any *gain* from the investment at a rate t but without a loss-offset provision: the payoff in the case of success would be $[1-t]x$ but the outcome in the case of failure would be just as before. Find the optimal x.

So the value of x is chosen to maximise

$$\pi \log(y_0 + x[1-t]) + [1-\pi]\log(y_0 - x).$$

Again assume that there is an interior solution. The first-order condition for a maximum is now

$$\frac{\pi[1-t]}{y_0 + x[1-t]} + \frac{\pi - 1}{y_0 - x} = 0.$$

Solving for x we find

$$x^{**} = \pi y_0 - \frac{1-\pi}{1-t} y_0 \qquad (10)$$

Clearly this only makes sense if $x^{**} \geq 0$ which requires

$$t \leq \frac{2\pi - 1}{\pi}$$

2012 Q4 (d)

Suppose that the government were to modify this tax and allow full loss offset, so that in the case of failure one only loses $[1-t]x$ rather than x. Again find the optimal x.

With full loss offset the final wealth is

$$y = \begin{cases} y_0 + x[1-t] & \text{with probability} \quad \pi \\ y_0 - x[1-t] & \text{with probability} \quad 1-\pi \end{cases}.$$

The first-order condition for an interior maximum is

$$\frac{\pi - 1}{y_0 - x[1-t]} + \frac{\pi}{y_0 + x[1-t]} = 0$$

and so we have

$$x^{***} = \frac{2\pi - 1}{1-t} y_0.$$

2012 Q4 (e)

Suppose the government abandoned the proposed tax on gains and replaced it with a tax on *ex-post wealth*. Show that the investment decision would be exactly the same as in part (b). Why is this?

Now

$$y = \begin{cases} [y_0 + x]\,[1-t] & \text{with probability} \quad \pi \\ \\ [y_0 - x]\,[1-t] & \text{with probability} \quad 1-\pi \end{cases}$$

Following the same argument as before, the FOC is

$$\frac{\pi - 1}{[y_0 - x]\,[1-t]} + \frac{\pi}{[y_0 + x]\,[1-t]} = 0$$

but this is exactly the same as in (8) so the solution must be the same and is given by (9). The reason for this is that preferences exhibit constant relative risk aversion – homothetic contours in state space. For any x taxing ex-post wealth just moves the outcome point in along a ray through the origin.

2012 Q4 (f)

Show that a rise in the tax rate would reduce investment under tax scheme (c), increase it under tax scheme (d) and leave it unchanged under tax scheme (e).

Differentiating (9), (10) and (11) we find

$$\begin{aligned} \frac{\partial x^*}{\partial t} &= 0 \\ \frac{\partial x^{**}}{\partial t} &= -\frac{1-\pi}{[1-t]^2} y_0 < 0 \\ \frac{\partial x^{***}}{\partial t} &= \frac{x^{***}}{1-t} > 0 \end{aligned}$$

2012 Q4 – assessment

- It seems like a long question
  - broken into digestible pieces
  - makes model-building easier
  - shows you exactly what to do, stage by stage
  - important message….
- "long" ≠ "difficult"
- Core of problem involves elementary things
  - carefully specify budget constraint in each question part
  - plug into simple utility function
  - use FOC to get result
  - applies to parts (b)—(e)
- Finishing off
  - final part just involves simple differentiation of the solution
  - don't forget the "explain why" in part (e)
  - see also Ex 8.12

2015 Q7

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(a) Find the expressions for the firm's average revenue, marginal revenue, average cost and marginal cost in terms of output. [4 marks]

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2015 7(b)

Show that the monopolist will choose to produce

$$q^* := \frac{\beta_1 - \alpha_1}{2\left[\alpha_2 + \beta_2\right]}$$

The first-order condition for the monopolist is given by MC=MR so that

$$\alpha_1 + 2\alpha_2 q = \beta_1 - 2\beta_2 q$$

from which the solution q^* follows.

2015 7(c)

Show that at q^* the price charged p^* exceeds the marginal cost c^*.

Substituting for q^* we also get

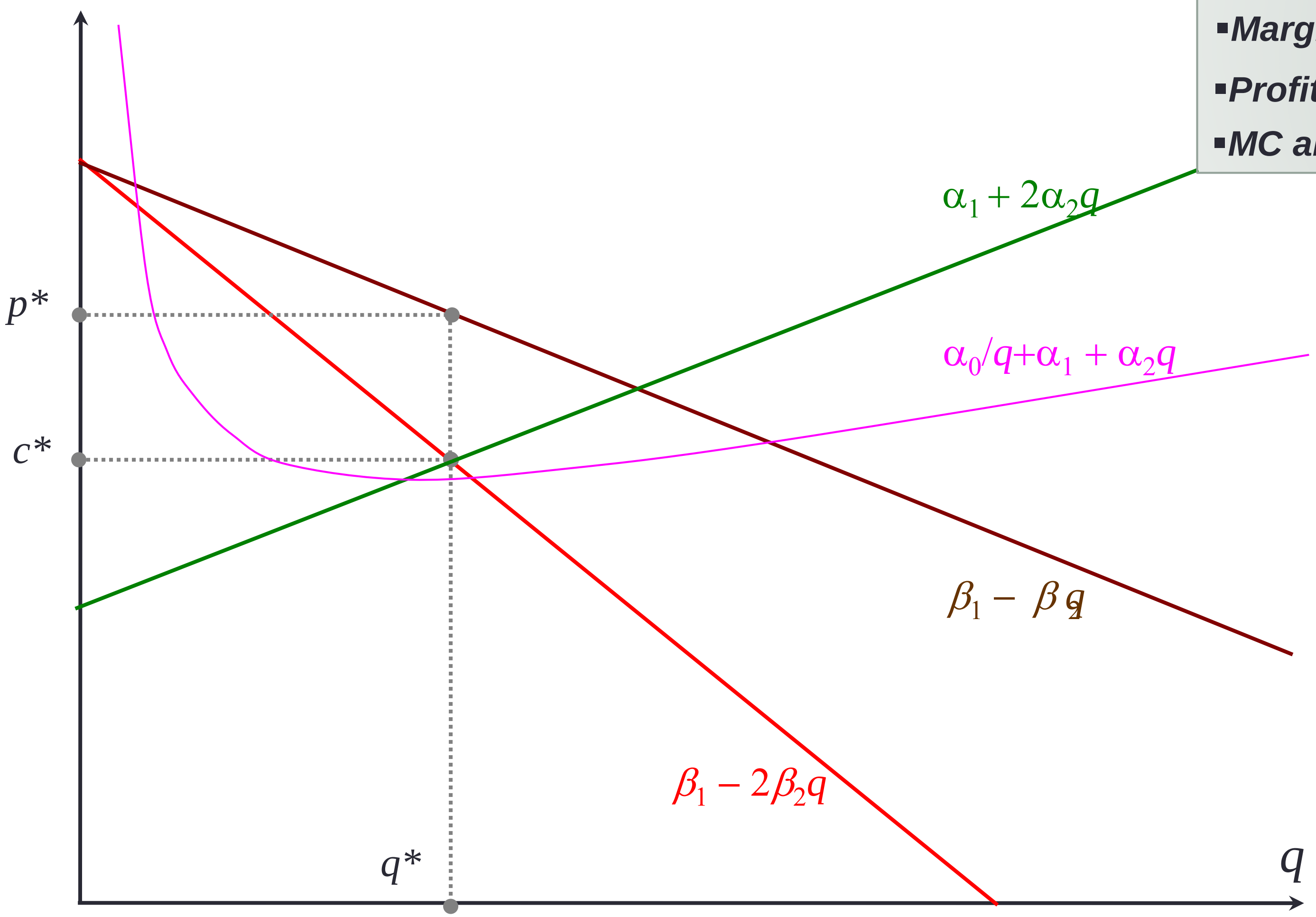
$$c^* = \beta_1 - 2\beta_2 q^* = \frac{\alpha_2\beta_1 + \alpha_1\beta_2}{\beta_2 + \alpha_2}$$

$$p^* = \beta_1 - \beta_2 q^* = c^* + \frac{1}{2}\beta_2 \frac{\beta_1 - \alpha_1}{\alpha_2 + \beta_2}$$

Given the assumption $\beta_1 > \alpha_1$ we have $p^* > c^*$.

2015 7(c)

- *AC and MC curves*
- *Demand (average revenue)*
- *Marginal revenue*
- *Profit-maximising output*
- *MC and price at q**



2015 7 (d)

A regulator now imposes a ceiling on the price charged, so that the monopolist now faces the additional constraint $p \leq \bar{p}$. Find the new expressions for average revenue and marginal revenue in terms of output. [4 marks]

- *Demand (average revenue)*
- *Marginal revenue*

$$\mathrm{AR}(q) = \left\{ \begin{array}{l} \bar{p} \text{ if } q \leq \bar{q} \\ \beta_1 - \beta_2 q \text{ if } q \geq \bar{q} \end{array} \right\}$$

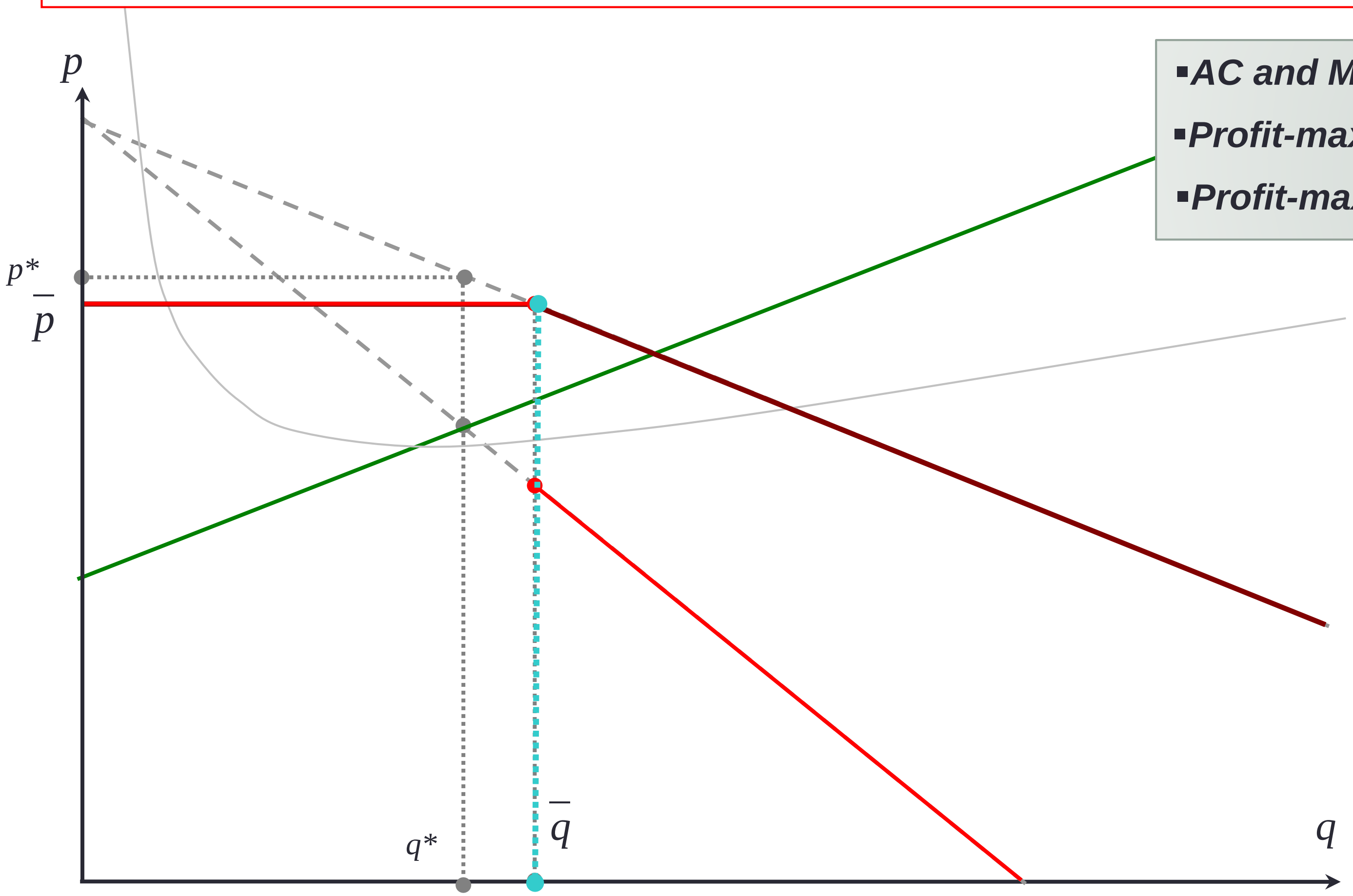
$\bar{q} := [\beta_1 - \bar{p}] / \beta_2$:

$$\mathrm{MR}(q) = \left\{ \begin{array}{l} \bar{p} \text{ if } q < \bar{q} \\ \bar{p} \text{ or } \beta_1 - 2\beta_2 \bar{q} \text{ if } q = \bar{q} \\ \beta_1 - 2\beta_2 q \text{ if } q > \bar{q} \end{array} \right\}$$

2015 Q7 (e)

Explain how the price ceiling affects the monopolist's output if $c^* \leq \bar{p} \leq p^*$, where the values c^* and p^* are the values of marginal cost and price found in the unregulated problem of part (c). [5 marks]

- ***AC and MC curves***
- ***Profit-max q and p (unregulated)***
- ***Profit-max q and p (regulated)***



2015 Q7 – assessment

- Make good use of a diagram to “see” the problem
- Re-use the solutions
  - one part of the problem…
  - …helps to build the next.
- Don’t be fazed by the presence of a discontinuity
  - everything is nice and regular either side of it.

Tips

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  - so if you're pointed in the right direction, follow it…
- Pictures
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- What *should* the answer be?
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  - use your intuition
- Does it make sense?
  - again take a moment to check after each part
  - we *all* make silly slips

Take away

- What's the point?
  - take a moment or two: make notes to yourself
  - what is the *main point* of the question?
  - and the subpoints?
- See the big picture
  - balance out the answer
  - if pressed for time, don't rush to put in extra detail
- Be an economist with your own time
  - don't solve things twice
  - reuse results
  - answer all the questions!
- Keep calm – and do well!