Measuring Inequality with Ordinal data

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Outline

Introduction

- Ordinal data issue widespread in inequality analysis
- Many applications proceed just as though cardinal:
  - life satisfaction / inequality of happiness: Oswald and Wu (2011), Stevenson and Wolfers (2008b), Yang (2008)
  - health status: Van Doorslaer and Jones (2003)
- Small literature that takes ordinal problem seriously
  - early approaches using 1st order dominance, the median
  - Abul Naga and Yalcin (2008,2010), Allison and Foster (2004), Zheng (2011)
  - but these have limitations
- Present approach based on Cowell and Flachaire (2014)

Income Inequality

- 3 ingredients:
  - **"income":** family income, earnings, wealth $x \in X \subseteq \mathbb{R}$.
  - **"income-receiving unit":** n persons
  - **method of aggregation:** function $X^n \rightarrow \mathbb{R}$
- Usually work with $X^n_\mu \subset \mathbb{R}$
- X^n_μ: Distributions obtainable from a given total income $n\mu$ using lump-sum transfers
- Obviously can't do that here: μ is undefined

Utility

Cardinalisation and inequality

- 3 ingredients:
  - **"income":** $u = U(x)$.
  - **"income-receiving unit":** n persons (as before)
  - **method of aggregation:** function $\mathbb{U}^n \to \mathbb{R}$
- Problem of cardinalisation
- But just assuming cardinal utility is no use
  - Already pointed out in Atkinson (1970)
  - Dalton (1920) suggested inequality of (cardinal) utility
  - But if, for all i, you multiply u_i by $\lambda \in (0,1)$ and add $\delta = \mu[1-\lambda]$...
  - ...this will automatically reduce measured inequality.
- Is this just a technicality?
- Can we proceed just as with regular income?

Categorical variable

Example: Access to Services

	Case 1	Case 2
	n_k	n_k
Both Gas and Electricity	25	0
Electricity only	25	50
Gas only	25	50
Neither	25	0

- Suppose we have no information about needs / usage
- It seems clear that Case 1 is more unequal than Case 2

Example self-reported health

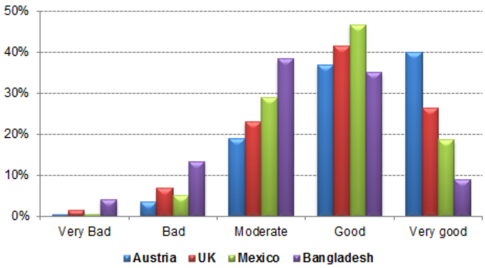
- World Health Survey (WHS)
  - a general population survey
  - developed by WHO
- Question: Health State Descriptions
  - overall health
  - including both physical and mental health
- In general, how would you rate your health today?
  - Very good
  - Good
  - Moderate
  - Bad
  - Very Bad
- Compare distributions across countries

SRH Results: four countries

	Austria	**UK**	**Mexico**	**Bangladesh**
	number of responses			
Very good	423	318	7193	494
Good	390	498	18112	1949
Moderate	200	278	11221	2132
Bad	36	82	2002	741
Very bad	4	17	218	228

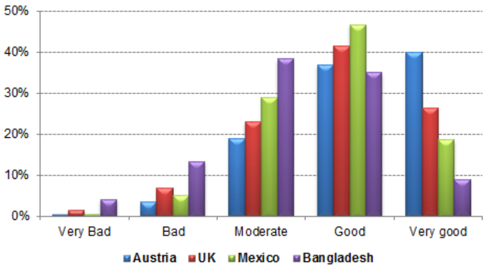
- For all countries: rank categories in order
- For each country: compute freq distributions across categories
- How to evaluate inequality?

SRH Inequality: Gini



	At	UK	Mx	BD	
(1,2,3,4,5)	0.111	0.130	0.116	0.154	(BD,UK,Mx,At)
(1,2,3,4,1000)	0.593	0.725	0.800	0.884	(BD,Mx,UK,At)
(-1000,2,3,4,5)	0.608	0.821	0.856	2.377	(BD,Mx,UK,At)

SRH Inequality: Coeff of Variation



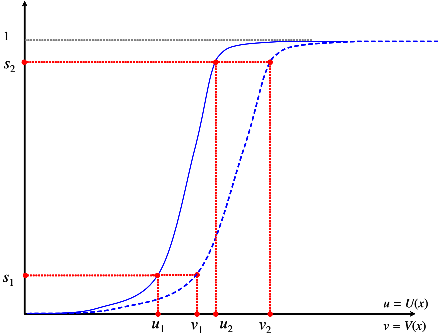
	At	UK	Mx	BD	
(1,2,3,4,5)	0.209	0.244	0.219	0.287	(BD,UK,Mx,At)
(1,2,3,4,1000)	1.210	1.638	2.056	3.088	(BD,Mx,UK,At)
(-1000,2,3,4,5)	187.5	11.43	40.45	5.264	(At,Mx,UK,BD)

Status and Information

- Step 1 is to define status
  - depends on the purpose of inequality analysis
  - depends on structure of information
  - conventional inequality approach only works in narrowly defined information structure
- In some cases a person's status is self-defining
  - income
  - wealth
- In some cases defined given additional distribution-free information
  - example: if it is known that utility is $\log(x)$
- In some cases requires information on distribution
  - GRE, TOEFL
  - "opportunity" (de Barros et al. 2008)

Status and Distribution (1)

- i's status uniquely defined for a given distribution of u



- disposes of the problem of cardinalisation
  - U and $V = \varphi(U)$ two cardinalisations of the utility of x
  - for each i:u_i and v_i map into s_i

Status and distribution (2)

- This approach works for categorical data
  - we just have an ordered arrangement of categories $1, 2, ..., k, ..., K$
  - and the numbers in each category $n_1, n_2, ..., n_k, ..., n_K$
- Merger principle
  - merge two adjacent categories that are irrelevant for i
  - then this should leave i's status unaltered
- Principle implies that status should be additive in the n_k
  - downward-looking status: $\sum_{\ell=1}^{k(i)} n_\ell$
  - upward-looking status: $\sum_{\ell=k(i)}^{K} n_\ell$
  - see also Yitzhaki (1979)

Elements of the Model

- Individual's status is given by $s \in S \subseteq \mathbb{R}$
  - status determined from utility?
- Vector of status in a population of size n : $\mathbf{s} \in S^n$
- $e \in S$: an equality-reference point
  - could be specified exogenously
  - could also depend on status vector $e = \eta(\mathbf{s})$
  - η need not be increasing in each component of $\mathbf{s}$
- Inequality: aggregate distance from e
  - don't need an explicit distance function
  - implicitly define through inequality ordering $\succeq$

Basic Axioms

- **[Continuity]** $\succeq$ is continuous on S^{n+1}
- **[Monotonicity]** If $\mathbf{s}, \mathbf{s}' \in S^n$ differ only in their ith component then (a) if $s_i' \geq e$:$s_i > s_i' \Longleftrightarrow (\mathbf{s}, e) \succ (\mathbf{s}', e)$; (b) if $s_i' \leq e$: $\Longleftrightarrow (\mathbf{s}, e) \succ (\mathbf{s}', e)$
- **[Independence]** If $\mathbf{s}(\varsigma, i), \mathbf{s}'(\varsigma, i) \in S^n$ satisfy$(\mathbf{s}(\varsigma, i), e) \sim (\mathbf{s}'(\varsigma, i), e)$ for some ς then $(\mathbf{s}(\varsigma, i), e) \sim (\mathbf{s}'(\varsigma, i), e)$ for all ς
- **[Anonymity]** For all $\mathbf{s} \in S^n$ and permutation matrix Π: $(\Pi\mathbf{s}, e) \sim (\mathbf{s}, e)$

Standard result

Theorem

Continuity, Monotonicity, Independence, Anonymity jointly imply $\succeq$ is representable by the continuous function $I : S_e^n \to \mathbb{R}$ where $I(\mathbf{s}; e) = \Phi\left(\sum_{i=1}^{n} d(s_i, e), e\right)$, where $d : S \to \mathbb{R}$ is a continuous function that is strictly increasing (decreasing) in its first argument if $s_i > e$ ($s_i < e$).

Corollary

Inequality is total "distance" from equality. Distance d is continuous. $d(s, e)$ is increasing in status if you move away from the reference point.

Structure Theorem

- We need more structure on the problem
- **[Scale invariance 1]** For all $\lambda \in \mathbb{R}_+$: if $\mathbf{s}, \mathbf{s}', \lambda\mathbf{s}, \lambda\mathbf{s}' \in S^n$ and $e, e' \in S$ then $(\mathbf{s}, e) \sim (\mathbf{s}', e') \Rightarrow (\lambda\mathbf{s}, e) \sim (\lambda\mathbf{s}', e')$.
- **[Scale invariance 2]** For all $\lambda \in \mathbb{R}_+$: if $\mathbf{s}, \mathbf{s}', \lambda\mathbf{s}, \lambda\mathbf{s}' \in S^n$ and $e, e', \lambda e, \lambda e' \in S$ then $(\mathbf{s}, e) \sim (\mathbf{s}', e') \Rightarrow (\lambda\mathbf{s}, \lambda e) \sim (\lambda\mathbf{s}', \lambda e')$

Theorem

Impose also Scale irrelevance 1. Then $d(s,e) = A(e)\, s^{\alpha(e)}$

Theorem

Impose instead Scale Invariance 2. Then $d(s,e) = e^{\beta}\phi\left(\frac{s}{e}\right)$ *.where* β *is a constant and* ϕ *is arbitrary*

Corollary

Inequality represented as $I_\alpha(\mathbf{s}; e) := \frac{1}{\alpha[\alpha-1]}\left[\frac{1}{n}\sum_{i=1}^{n} s_i^{\alpha} - e^{\alpha}\right]$

A usable inequality index?

- A *class* of functions available as inequality measures:
  - $\Phi(I_\alpha(\mathbf{s};e), e)$
  - $e = \eta(\mathbf{s})$, the reference point
  - $I_\alpha(\mathbf{s};e) := \frac{1}{\alpha[\alpha-1]}\left[\frac{1}{n}\sum_{i=1}^{n} s_i^\alpha - e^\alpha\right]$
- Do functions $\Phi(I_\alpha(\mathbf{s};e), e)$ "look like" inequality measures?
  - transfer principle?
  - reference point?
  - sensitivity to parameters
- What is the appropriate form for Φ?
  - may depend on the reference status e
  - may depend on interpretation

Four distributional scenarios (1)

	Case 0		Case 1		Case 2		Case 3	
	n_k	s_i	n_k	s_i	n_k	s_i	n_k	s_i
B	0		25	1	0		25	1
E	50	1	25	$3/4$	50	1	25	$3/4$
G	25	$1/2$	25	$1/2$	50	$1/2$	50	$1/2$
N	25	$1/4$	25	$1/4$	0		0	
$\mu(\mathbf{s})$		$11/16$		$5/8$		$3/4$		$11/16$

- n_k is # persons in category $k \in \{\text{B}, \text{E}, \text{G}, \text{N}\}$
- $s_i = \frac{1}{n}\sum_{\ell=1}^{k(i)} n_\ell$ – *downward*-looking status

Four distributional scenarios (1′)

	Case 0		Case 1		Case 2		Case 3	
	n_k	s'_i	n_k	s'_i	n_k	s'_i	n_k	s'_i
B	0		25	1/4	0		25	1/4
E	50	1/2	25	1/2	50	1/2	25	1/2
G	25	3/4	25	3/4	50	1	50	1
N	25	1	25	1	0		0	
$\mu(\mathbf{s})$		11/16		5/8		3/4		11/16

- n_k is # persons in category $k \in \{\mathrm{B,E,G,N}\}$
- $s'_i = \frac{1}{n}\sum_{\ell=k(i)}^{K} n_\ell$ – *upward*-looking status

Four distributional scenarios (2)

	Case 0		Case 1		Case 2		Case 3	
	n_k	s_i	n_k	s_i	n_k	s_i	n_k	s_i
B	0		25	1	0		25	1
E	50	1	25	$3/4$	50	1	25	$3/4$
G	25	$1/2$	25	$1/2$	50	$1/2$	50	$1/2$
N	25	$1/4$	25	$1/4$	0		0	
$\mu(\mathbf{s})$		$11/16$		$5/8$		$3/4$		$11/16$

- Case 0 to Case 1:
  - 25 people promoted from E to B
  - if e equals to any of values taken by $\mu(\mathbf{s})$
  - then inequality increases

Four distributional scenarios (3)

	Case 0		Case 1		Case 2		Case 3	
	n_k	s_i	n_k	s_i	n_k	s_i	n_k	s_i
B	0		25	1	0		25	1
E	50	1	25	$3/4$	50	1	25	$3/4$
G	25	$1/2$	25	$1/2$	50	$1/2$	50	$1/2$
N	25	$1/4$	25	$1/4$	0		0	
$\mu(\mathbf{s})$		$11/16$		$5/8$		$3/4$		$11/16$

- Case 0 to Case 2:
  - 25 people promoted from N to G
  - if e equals to any of values taken by $\mu(\mathbf{s})$
  - then inequality decreases

"Transfer Principle"?

	Case 0		Case 1		Case 2		Case 3	
	n_k	s_i	n_k	s_i	n_k	s_i	n_k	s_i
B	0		25	1	0		25	1
E	50	1	25	$3/4$	50	1	25	$3/4$
G	25	$1/2$	25	$1/2$	50	$1/2$	50	$1/2$
N	25	$1/4$	25	$1/4$	0		0	
$\mu(\mathbf{s})$		$11/16$		$5/8$		$3/4$		$11/16$

- Case 0 to Case 1: inequality increases
- Case 0 to Case 2: inequality decreases
- Case 0 to Case 3: combination results in ambiguous change

Reference point

- **Mean status:** $e = \eta(\mathbf{s}) = \mu(\mathbf{s})$
  - for continuous distributions will equal 0.5
  - for categorical data, there is no counterpart to fixed-mean assumption in income-inequality analysis
- **Median status**: $e = \eta(\mathbf{s}) = \text{med}(\mathbf{s})$
  - not well-defined: any value in interval $M(\mathbf{s})$
  - $M(\mathbf{s}) = [1/2, 1)$ in cases 0 and 2
  - $M(\mathbf{s}) = [1/2, 3/4)$ in cases 1 and 3
- **Max status**: $e = 1$
  - for constant e this is only value that makes sense
- **Min status**: $e = 0$
  - counterpart for peer-exclusive case

Sensitivity

- α captures the sensitivity of measured inequality
- If α is high $I_\alpha\left(\mathbf{s};e\right) = \frac{1}{\alpha[\alpha-1]}\left[\frac{1}{n}\sum_{i=1}^{n} s_i^\alpha - e^\alpha\right]$, sensitive to high status-inequality
- If $\alpha = 0$ then $I_0\left(\mathbf{s};e\right) = -\frac{1}{n}\sum_{i=1}^{n}\log s_i + \log e$,
- If $e = \mu\left(\mathbf{s}\right)$ and $\alpha = 1$ then $\frac{1}{n}\sum_{i=1}^{n} s_i \log s_i - e\log e$

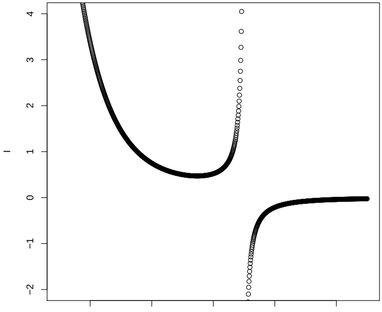
Behaviour of $I_0(\mathbf{s}; e)$

	Case 0	Case 1	Case 2	Case 3
$\mu(\mathbf{s})$	$^{11}/_{16}$	$^{5}/_{8}$	$^{3}/_{4}$	$^{11}/_{16}$
$\text{med}_1(\mathbf{s})$	$^{3}/_{4}$	$^{5}/_{8}$	$^{3}/_{4}$	$^{5}/_{8}$
$\text{med}_2(\mathbf{s})$	$^{1}/_{2}$	$^{1}/_{2}$	$^{1}/_{2}$	$^{1}/_{2}$
$I_0(\mathbf{s};\ \mu(\mathbf{s}))$	0.1451	0.1217	0.0588	0.0438
$I_0(\mathbf{s};\ \text{med}_1(\mathbf{s}))$	0.2321	0.1217	0.0588	-0.0515
$I_0(\mathbf{s};\ \text{med}_2(\mathbf{s}))$	-0.1732	-0.1013	-0.3465	-0.2746
$I_0(\mathbf{s};\ 1)$	0.5198	0.5917	0.3465	0.4184

- $I_0(\mathbf{s};\ \mu(\mathbf{s}))$, $I_0(\mathbf{s};\ \text{med}_1(\mathbf{s}))$: inequality *decreases* for
  - Case 0 to 1, or Case 2 to 3
  - movement changes both the $\mu(\mathbf{s})$ and $\text{med}_1(\mathbf{s})$ ref points
- $I_0(\mathbf{s};\ \text{med}_2(\mathbf{s})) < 0$ for *all* cases in example!
- But $I_0(\mathbf{s}; 1)$ seems sensible

Inequality measure

- For ordinal data, peer-inclusive status
- $I_\alpha(\mathbf{s},1) = \begin{cases} \frac{1}{\alpha(\alpha-1)}\left[\frac{1}{n}\sum_{i=1}^{n} s_i^{\alpha} - 1\right], & \text{if } \alpha \neq 0,\ \alpha<1 \\ -\frac{1}{n}\sum_{i=1}^{n}\log s_i. & \text{if } \alpha=0 \end{cases}$



Implementation

- Description of sample

$$x_i = \begin{cases} 1 & \text{with sample proportion } p_1 \\ 2 & \text{with sample proportion } p_2 \\ \dots & \\ K & \text{with sample proportion } p_K \end{cases},$$

- Point estimate of the index:
  - $$I_\alpha = \begin{cases} \frac{1}{\alpha(\alpha-1)} \left[\sum_{i=1}^{K} p_i \left[\sum_{j=1}^{i} p_j \right]^{\alpha} - 1 \right] & \text{if } \alpha \neq 0,1 \\ -\sum_{i=1}^{K} p_i \log \left[\sum_{j=1}^{i} p_j \right] & \text{if } \alpha = 0 \end{cases}$$
  - function of K parameter estimates $(p_1, p_2, \dots, p_K)$ following a multinomial

Asymptotics

- From the CLT I_α is asymptotically Normally distributed
- Estimator of cov matrix of $(p_1, p_2, \ldots, p_k)$ is

$$\Sigma = \frac{1}{n} \begin{bmatrix} p_1(1-p_1) & -p_1 p_2 & \ldots & -p_1 p_K \\ -p_2 p_1 & p_2(1-p_2) & \ldots & -p_2 p_K \\ \vdots & \vdots & \vdots & \vdots \\ -p_K p_1 & -p_K p_2 & \ldots & p_K(1-p_K) \end{bmatrix}$$

- $\widehat{\text{Var}}(I_\alpha) = D \Sigma D^\top$ with $D = \left[\frac{\partial I_\alpha}{\partial p_1} ; \frac{\partial I_\alpha}{\partial p_2} ; \ldots ; \frac{\partial I_\alpha}{\partial p_K} \right]$
- $\frac{\partial I_\alpha}{\partial p_l} = \frac{1}{\alpha(\alpha-1)} \left(\left[\sum_{i=1}^{l} p_i \right]^\alpha + \alpha \sum_{i=l}^{K-1} p_i \left[\sum_{j=1}^{i} p_j \right]^{\alpha-1} \right), \alpha \neq 0$
- $\frac{\partial I_0}{\partial p_l} = -\log \left[\sum_{j=1}^{l} p_j \right] - \sum_{i=l}^{K-1} p_i \left[\sum_{j=1}^{i} p_j \right]^{-1}$

Confidence Intervals

- 3 variants of CIs: Asymptotic, Percentile Bootstrap, Studentized Bootstrap
- $CI_{asym} = [I_\alpha - c_{0.975}\widehat{\text{Var}}(I_\alpha)^{1/2}\,;\, I_\alpha + c_{0.975}\widehat{\text{Var}}(I_\alpha)^{1/2}]$
    - $c_{0.975}$ from the Student distribution $T(n-1)$
    - do not always perform well in finite samples
- Bootstraps: generate resamples, $b = 1, \ldots, B$
    - for each resample b compute the inequality index
    - obtain B bootstrap statistics, I_α^b
    - also B bootstrap t-statistics $t_\alpha^b = (I_\alpha^b - I_\alpha)/\widehat{\text{Var}}(I_\alpha^b)^{1/2}$
- $CI_{perc} = [c^b_{0.025}\,;\, c^b_{0.975}]$
    - $c^b_{0.025}$ and $c^b_{0.975}$ are from EDF of bootstrap statistics
- $CI_{stud} = [I_\alpha - c^*_{0.975}\widehat{\text{Var}}(I_\alpha)^{1/2}\,;\, I_\alpha - c^*_{0.025}\widehat{\text{Var}}(I_\alpha)^{1/2}]$
    - $c^*_{0.025}$ and $c^*_{0.975}$ are from EDF of the bootstrap t-statistics

Performance Test

- Take an example with 3 ordered categories ($K = 3$)
- Samples are drawn from a multinomial distribution with probabilities $\pi = (0.3, 0.5, 0.2)$
- Is asymptotic or bootstrap distribution a good approximation of the exact distribution of the statistic?
  - if we are using 95% CIs of I_α
  - coverage error rate should be close to nominal rate, 0.05
- Check coverage error rate of CIs as sample size increases
  - $\alpha = -1, 0, 0.5, 0.99$
  - 199 bootstraps
  - 10 000 replications to compute error rates
  - $n = 20, 50, 100, 200, 500, 1000$

Estimation Methods Compared

	α	-1	0	0.5	0.99
Asymptotic B	$n = 20$	0.0606	0.0417	0.0598	0.0491
	$n = 500$	0.0523	0.0492	0.0521	0.0523
	$n = 1000$	0.0485	0.0540	0.0552	0.0549
Percentile B	$n = 20$	0.0384	0.0981	0.0912	0.1023
	$n = 500$	0.0509	0.0513	0.0552	0.0554
	$n = 1000$	0.0482	0.0556	0.0547	0.0551
Studentized B	$n = 20$	0.1275	0.0843	0.1041	0.1377
	$n = 500$	0.0518	0.0478	0.0429	0.0465
	$n = 1000$	0.0473	0.0522	0.0493	0.0503

- Asymptotic CIs perform OK in finite sample
- Percentile bootstrap performs well for $n > 50$
- Studentized bootstrap does not do well for small samples
- Reliable results for $\alpha = 0.99$ (index is undefined for $\alpha = 1$)

World Values Survey

- Life satisfaction question:

All things considered, how satisfied are you with your life as a whole these days? Using this card on which 1 means you are "completely dissatisfied" and 10 means you are "completely satisfied" where would you put your satisfaction with your life as a whole? (code one number):

Completely dissatisfied – 1 2 3 4 5 6 7 8 9 10 – Completely satisfied

- Health question:

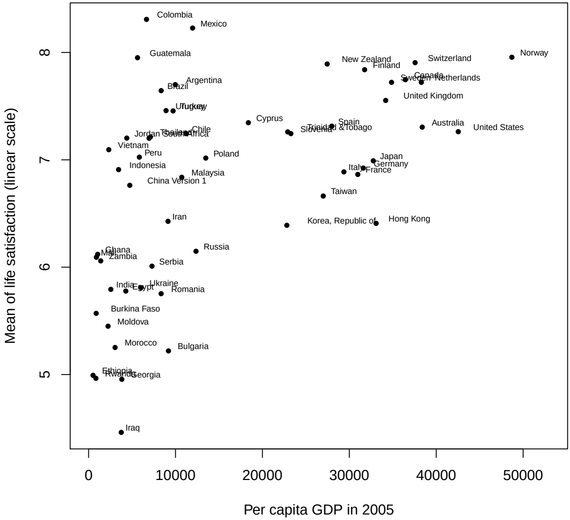
All in all, how would you describe your state of health these days? Would you say it is (read out):

1 Very good, 2 Good, 3 Fair, 4 Poor.

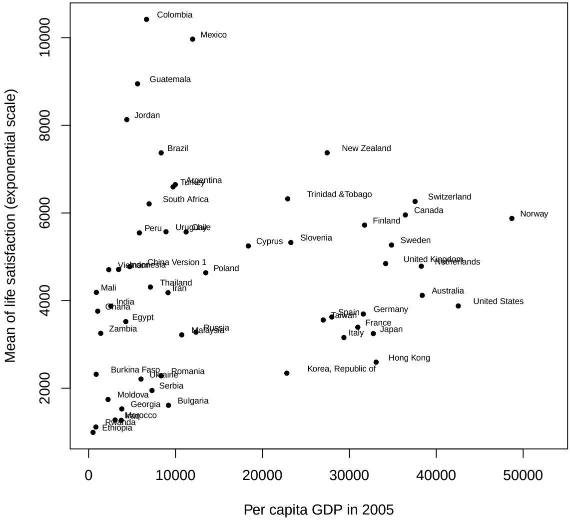
GDP and Life satisfaction

- Cross-country comparison of life satisfaction and GDP/head
  - happiness-income paradox (Easterlin 1974, Clark and Senik 2011)
  - weak relation happiness-income internationally? (Easterlin 1995, Easterlin et al. 2010)
  - or a strong relationship? (Hagerty and Veenhoven 2003, Deaton 2008, Stevenson and Wolfers 2008a, Inglehart et al. 2008)
- How should we quantify life satisfaction?
  - simple linearity of Likert scale? or exponential scale?
  - Ng (1997), Ferrer-i-Carbonell and Frijters (2004), Kristoffersen (2011)
- Is inequality of life satisfaction related to GDP/head?
  - Use I_0 and other members of the same family

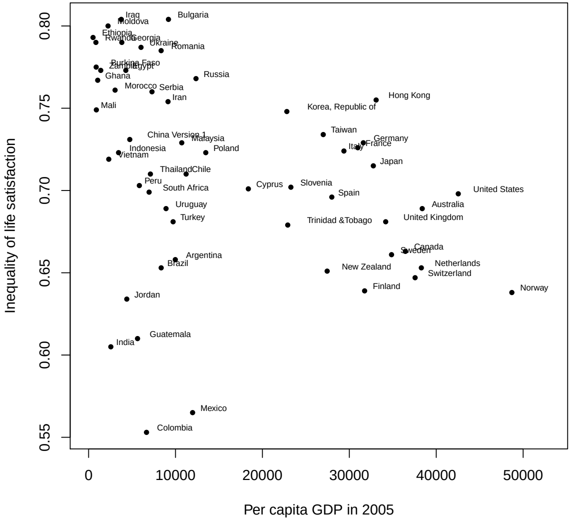
GDP and Life satisfaction (Linear)



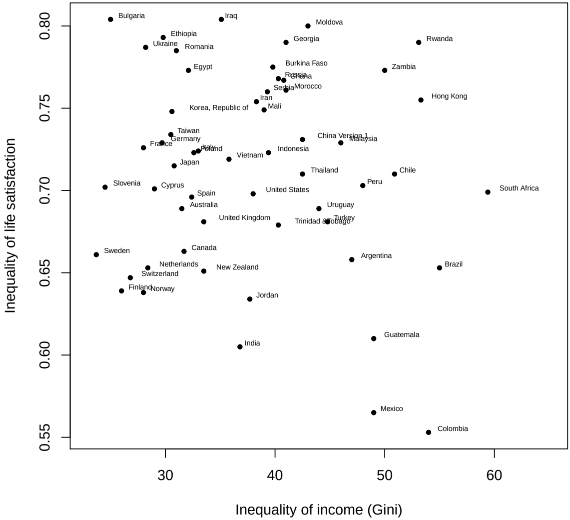
GDP and Life satisfaction (Exponential)



GDP and Inequality of Life satisfaction



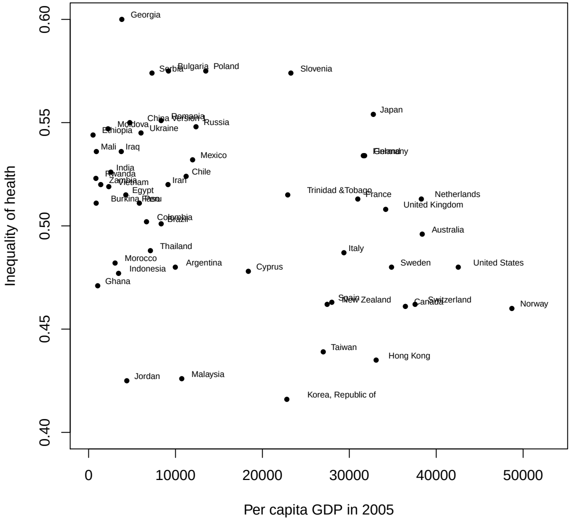
Income inequality and Inequality of Life satisfaction



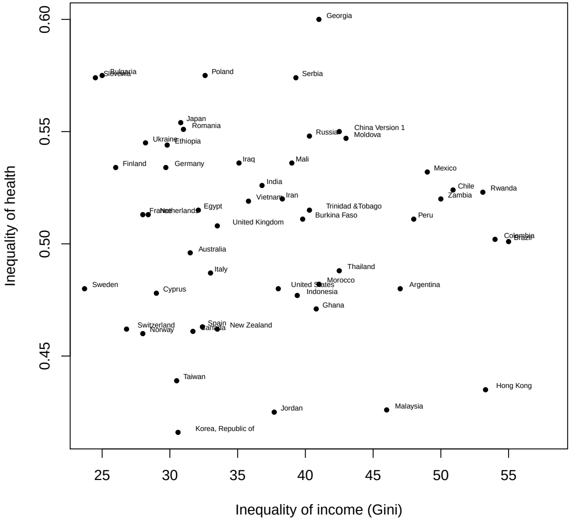
Health status

- Health is HRS
- Cross-country comparison of health and GDP
  - a significant positive relationship? (Deaton 2008)
- Cross-country comparison of inequality of health and Inequality of life satisfaction
  - use same inequality index as for life satisfaction

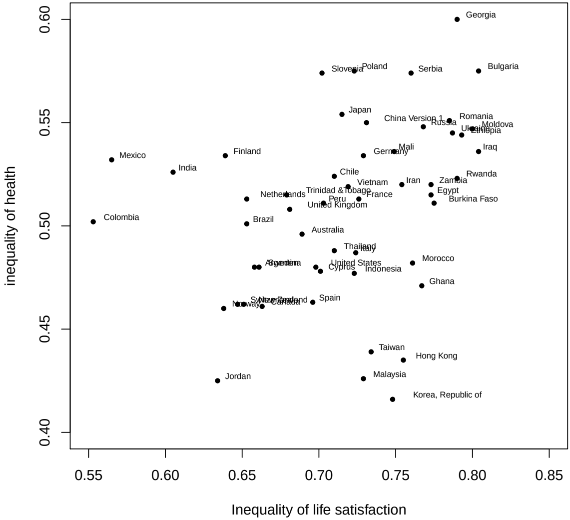
GDP and Inequality of health



Income inequality and health inequality



Inequality of life satisfaction and health inequality



Application: overview

- Satisfaction / GDP results sensitive to the cardinal interpretation of the answers
  - linear: positive relation below $15 000, flat after that (Layard 2003)
  - exponential: no relation
- OLS estimate of I_0(life satisfaction) on the GDP per capita small and negative
  - happiness-income relationship is weak in cross-country comparisons
- No clear relationship between I_0(health) on GDP per capita
- OLS estimate of I_0(health) on I_0(life satisfaction) produces a slope coefficient not significantly different from zero
  - health-life satisfaction relationship is not significant

Summary

- Inequality with ordinal data is a widespread phenomenon
- Conventional *I*-measures may make no sense
- Cowell and Flachaire (2014) approach:
  - separates out the issue of status from that of inequality-aggregation
  - allows you to choose “reference status”
  - gives a family of measures
- Nice properties empirically

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