RELATIVE DEPRIVATION AND THE GINI COEFFICIENT*

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The aim of this note is to present an interpretation of the Gini coefficient that is consistent with a well-known theory of attitudes to social inequality, the theory of relative deprivation.[1] The essence of this theory is that the impact of deprivation resulting from not having X when others have it is an increasing function of the number of persons in the reference group who have X. In other words, the social evaluation of the deprivation inherent in a person's not having X is an increasing function of the proportion of those who do have it. By quantifying this statement, we shall show that one plausible concept of deprivation in a society can be represented by μG, where G is the Gini coefficient and μ is the income that each person would have in an egalitarian society (μ is average income). By analogy we show that $\mu(1 - G)$ is a measure of the satisfaction of the society.

We begin by defining relative deprivation and investigate the characteristics of a measure of it; we then aggregate the relative deprivation in the society.

RELATIVE DEPRIVATION

Runciman defines relative deprivation as follows:

> We can roughly say that [a person] is relatively deprived of X when (i) he does not have X, (ii) he sees some other person or persons, which may include himself at some previous or expected time, as having X (whether or not this is or will be in fact the case), (iii) he wants X, and (iv) he sees it as feasible that he should have X (*op. cit.*, p. 10).

Runciman rightly suggests that people compare themselves with some reference group within the society rather than with the whole society. Here, we assume one reference group; a decomposition of the Gini coefficient, such as Sen's, would allow us to use more than one, but this extension is beyond the scope of this note.[2]

We shall consider income as the object of relative deprivation: income should be considered as an index of the individual's ability

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to consume commodities; each unit of income represents a different bundle of commodities that he is able to consume. The range of possible deprivation for each person in the society is $(0, y^*)$, where y^* is the highest income existing in the society. For each person i, his own income y_i partitions the possible deprivation range into two segments (y_i, y^*), the range of income for which he is deprived, and $(0, y_i)$, the range of income for which he is satisfied. The total deprivation assigned to a person is the sum of the deprivation inherent in all units of income he is deprived of. Runciman defines the degree of deprivation inherent in not having X (the jth unit of income) as an increasing function of the proportion of persons in the society who have X.[3]

According to this definition, the degree of relative deprivation of the range $(y, y + dy)$ can be quantified by $1 - F(y)$, where $F(y) = \int_0^y f(z)dz$ is the cumulative income distribution, and $1 - F(y)$ is the relative frequency of persons with incomes above y.[4]

We define two functional relationships:

(1) $D(y_i) = \int_{y_i}^{y^*} [1 - F(z)]dz$ is the relative deprivation function of person i;

(2) $S(y_i) = \int_0^{y_i} [1 - F(z)]dz$ is the relative satisfaction function of person i.

Integrating (2),[5] we see that

$$S(y_i) = y_i[1 - F(y_i)] + \mu\phi(y_i), \tag{3}$$

where μ is average income and $\phi(y_i)$ is the value of the Lorenz curve (which shows the proportion of total income received by those whose income is less than or equal to y).

From (3) we get $S(0) = 0$, $S(y^*) = \mu$. That is, the satisfaction of the richest person in the society is μ, the average income, which is what he would receive if income were equally distributed. It is worth mentioning that $D(y_i) = \mu(1 - \phi_i) - y_i(1 - F_i)$ is the gap between the total income of those with more than y_i and their total income if they had y_i. In the case where $y_i = \mu$, we get $D(\mu) = \mu T$, where T is the relative mean deviation of the income distribution.[6]

The evaluation in terms of relative satisfaction of the marginal dollar, dS/dy, is unity at income level 0 and zero at income level y^* (the richest person in the society suffers no deprivation); and $d^2S/dy^2 = -f(y) \leq 0$. That is, marginal relative satisfaction is a nonincreasing function of income.

Using (2) and (3), we can rewrite the deprivation function (1) as[7]

$$D(y_i) = \mu - S(y_i). \quad (4)$$

Since $0 \leq S(y_i) \leq \mu$, it follows that $0 \leq D(y_i) \leq \mu$; that is, the degree of deprivation is the complement (to μ) of the degree of satisfaction; therefore, one can work with either $D(y_i)$ or $S(y_i)$ and get analogous results. We have chosen to work with $S(y_i)$.

The properties of $S(y_i)$ are

1. $\partial S/\partial y > 0$, that is, the higher the income, the higher the satisfaction; and $\partial^2 S/\partial y^2 \leq 0$, that is, marginal satisfaction is a non-increasing function of income.
2. The individual is indifferent to income transfers among those who are all poorer than he is ($\phi(y)$ and $F(y)$ do not change) or all richer than he is. Income transfers may change the demand functions for commodities and may therefore affect prices and incomes. This kind of general equilibrium approach is not considered here.
3. The individual's satisfaction increases when income is transferred from someone richer than he is to someone poorer ($\phi(y_i)$ increases), provided that his rank in the income distribution does not change.
4. An increase (decrease) in the income of someone richer than individual i will not change the latter's satisfaction, but it will increase (decrease) his deprivation ($\mu\phi(y)$ does not change, μ changes). Correspondingly, an increase in the income of someone poorer than individual i will increase the latter's satisfaction and will not change his deprivation.
5. Assume that the mean income is given, then the individual is indifferent to a small change in his rank that keeps his income constant:

$$\left.\frac{\partial S}{\partial F}\right|_{y,\mu} = -y + \mu\frac{\partial \phi}{\partial F} = 0.$$

The explanation of this outcome is that a rise in the individual's rank that is not accompanied by an increase in his income means that the income gap between him and those richer than he is must increase. The effect of the change in rank and the effect of the change in income gap cancel out.

Relative Deprivation in a Society

The degree of relative deprivation (satisfaction) in the society can be defined as average (or aggregate) deprivation (satisfaction).

Formally,

$$S = \int_0^{y^*} S(z)f(z)dz,$$

and using (3), we see that

$$S = \mu(1 - G),$$

where G is the Gini coefficient.[8]

Total relative deprivation is

$$D = \int_0^{y^*} D(z)f(z)dz = \mu G.$$

We have here proved that the Gini coefficient is a quantification of a well-known theory, the theory of relative deprivation. We have not tried to find the most general functional relationship between Runciman's approach and the Gini coefficient. It can easily be seen that one can quantify relative deprivation by $a[1 - F(y)]$, $a > 0$, instead of $1 - F(y)$. In that case, total satisfaction would be $a\mu(1 - G)$.

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NOTES

1. For a full review of the theory, see W. G. Runciman, *Relative Deprivation and Social Justice* (London: Routledge and Kegan Paul, 1966).
2. See Amartya Sen, "Poverty: An Ordinal Approach to Measurement," *Econometrica*, XLIV (March 1976), 219–31.
3. Runciman uses the example of promotions and writes: "The more people a man sees promoted when he is not promoted himself, the more people he may compare himself with in a situation where the comparison will make him feel relatively deprived" (*op. cit.*, p. 19).
4. That is, we assume that X is a binary variable representing the income range $(y, y + dy)$; i.e., people have X or they do not. Therefore, the scarcity of the agents who have X is identical with the scarcity of X.
5. According to Atkinson, $\mu\phi(y_i) = y_i F(y_i) - \int_0^{y_i} F(z)dz$ (equation (3), p. 246, in A. B. Atkinson, "On the Measurement of Inequality," *Journal of Economic Theory*, II (Sept. 1970), 244–63).
6. The index of relative mean deviation is $F(\mu) - \phi(\mu)$.
7. $$D(y_i) = \int_{y_i}^{y^*} [1 - F(z)]dz = \int_0^{y^*} [1 - F(z)]dz - \int_0^{y_i} [1 - F(z)]dz = S(y^*) - S(y_i).$$
8. According to Atkinson, $G = (1/\mu) \int_0^{y^*} [zF(z) - \mu\phi(z)]f(z)dz$ (*op. cit.*, p. 252).