INCOME TAX COMPLIANCE IN A PRINCIPAL-AGENT FRAMEWORK

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Previous analyses have modeled income tax evasion as a 'portfolio problem', deriving the optimal consumption of the 'risky asset' (unreported income) assuming a fixed probability of detection. We compare an alternative audit policy to the standard random audit policy. We focus on an 'audit cutoff' policy, in which an agent triggers an audit if reported income is 'too low', and is not audited if reported income is 'sufficiently high'. We find that random audit rules are weakly dominated by audit cutoff rules. Given lump-sum taxes and fines, audit cutoff rules are the least-cost policies which induce truthful reporting of income.

1. Introduction

One of the most interesting features of modern systems of income taxation is their essentially voluntary nature. While an individual taxpayer always faces some chance of being audited by the Internal Revenue Service (IRS), to take the United States as an example, most individuals' actual tax assessments depend upon the income they choose to report, which is not necessarily the same as their true income. The purpose of this paper is to analyze income tax compliance (or evasion) in a way which incorporates this fundamental observation, yet differs substantially in other aspects from those models found in the extant economics literature.

There are three topic areas in that literature to which our model is related. The first is optimal taxation, of which a recent survey can be found in Sandmo (1976). The second related area is the work on 'crime and punishment' [e.g. Becker (1968), Stigler (1970), Becker and Stigler (1974)], and the third is the literature on tax evasion. While the latter two of these are closely related to each other, they are distinguished by the absence of a tax rate and an analog of 'reported income' in the work on crime and punishment. Research in both, however, typically treats noncompliance as a decision under uncertainty where the individual faces a given probability of detection and conviction, and given tax and penalty functions [e.g. Allingham and Sandmo (1972), Srinivasan (1973), Yitzhaki (1974)]. There has

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been some treatment of the optimal probability of detection and/or penalty for evasion, usually under the assumption that the objective of the government is to maximize some measure of social welfare [e.g. Kolm (1973), Singh (1973), Srinivasan (1973), Fishburn (1979), Kemp and Ng (1979), Polinsky and Shavell (1979), Christiansen (1980), Sandmo (1981)]. Without exception, these authors have restricted the probability of audit to be independent of a taxpayer's reported income, although in some cases this probability is made contingent upon other past information, such as whether the taxpayer has been previously convicted of tax evasion [e.g. Greenberg (1984), Landsberger and Meilijson (1982)]. All of these studies have ignored the possibility that a taxpayer's reported income may convey information regarding his true income to the IRS.

As mentioned earlier, the purpose of this paper is to examine some of these issues in income tax compliance starting from a different set of assumptions than those used in the extant literature. In particular, we wish to incorporate the information content of a taxpayer's report into the IRS' choice of an audit policy. Specifically, we will compare an alternative audit policy which uses the information conveyed by the taxpayer's report (albeit in a rather crude manner) to the standard random audit policy. Accordingly, we will use a highly simplified framework which yields sharp results suitable for comparison across models.

First, we assume that income is a random variable. Second, income is observed by the taxpayer costlessly, but can only be observed by the government if an audit cost is paid. Third, the objective of the tax-collecting agency is to maximize expected revenue net of audit costs; that is, the principal (the tax-collecting agency) desires to maximize net revenue but cannot observe the true income of the agent (the taxpayer) unless an audit is performed. The principal therefore asks the agent to report his or her income and then makes a decision whether to audit (this decision may be dependent on reported income and/or probabilistic). The agent responds to this system so as to maximize his or her own well-being.

There is also a question here of the optimal form and level of taxation. Initially we assume that taxation is lump-sum; that is, the agent is asked to pay a fixed amount if that is feasible; otherwise the taxpayer pays all of his or her income. We summarize our results under proportional taxation later in the paper.

Finally, there is the question of what fine should be imposed for noncompliance, or, in our model, under-reporting of income. This is also taken to be a choice variable of the principal.

In section 2 we set up the general model using a principal-agent framework. The probability of audit is a function of reported income, the tax in the event of no audit is a function of reported income, and the tax plus any applicable fine in the event of an audit is a function of reported and
actual income. Although independently arrived at and focused on a different problem, our formal model is closely related to that of Townsend (1979). Townsend analyzes a two-agent pure exchange economy in which agent 1’s endowment of the consumption good is fixed \(y_1\), while agent 2’s endowment is random \(y_2\). Agent 2 is strictly averse to risk, so the agents can enter into mutually beneficial contracts to transfer the consumption good. But verification of \(y_2\) is costly. Thus, the agents agree ex ante upon a contract \([g, S]\), where \(g\) is a transfer function and \(S\) is a verification region. Agent 2’s random endowment is realized, and if \(y_2 \in S\), then agent 2 asks for verification, and \(g(y_2)\) is transferred; if \(y_2 \in S'\) (the complement of \(S\)), then there is no verification and a pre-arranged amount is transferred. Analysis of this model and related extensions occupies most of Townsend’s paper, except for section 4, in which he introduces a two-state model which admits the possibility of stochastic verification, and which closely resembles the model we present in section 2 of this paper. He remarks that:

As to the nature of solutions to problems similar to 4.1, little has been determined. The difficulty is that the [self-selection] constraints seem quite messy analytically; examination of the necessary conditions for a maximum … has not yet provided much insight. In order to avoid putting measures on measures, a restriction to simple rather than continuous variables has been imposed. Yet this seems to make the characterization more difficult (p. 278).

We, too, find the characterization of optimal policies excessively complicated; accordingly, we restrict attention to two special, but interesting, cases. In particular, we assume lump-sum taxes and fines, and consider two simple forms of audit policy.

In section 3 we consider the standard case in which audits are random, occurring with a probability which is independent of the reported level of income. In this case the principal sets a lump-sum tax, a probability of audit and a fine for under-reporting.

In section 4 we analyze another form of audit policy. The principal still sets a lump-sum tax and a fine for under-reporting, but now sets a cutoff level on reported income such that taxpayers who report income less than the cutoff are audited with probability one and taxpayers who report income greater than or equal to the cutoff are never audited. We find that the optimal audit cutoff policy weakly dominates the optimal random audit policy for any given distribution of income, and strictly dominates it for some distributions. Despite its simplicity, the audit cutoff rule has some desirable properties. First, it induces taxpayers to report their incomes truthfully. Moreover, given lump-sum taxes and fines, the only audit policies which induce truthful reporting are those which require the principal to audit
with probability one any agent who reports income less than the desired tax. Finally, the audit cutoff policy is shown to be the audit policy which induces truthful reporting at least cost to the principal.

In section 5 we present some simple examples in which the audit cutoff policy strictly dominates the random audit policy, and consider some comparative static effects. In section 6 we summarize our results under lump-sum taxation and report the results of reconsidering the analysis under proportional taxation. In this framework, there is complete equivalence between the equilibrium consequences of lump-sum and proportional taxation. We also outline several variations and extensions of the basic model.

2. The general model

The following assumptions describe the most general form of the problem we wish to consider. We will subsequently make simplifying assumptions as needed.

Assumption 1. The income of each taxpayer is a random variable, independently and identically distributed according to $G(I)$, where $g(I) = G'(I) > 0$ for all $I \in [0, \infty)$.

Assumption 2. The taxpayer observes $I$ costlessly, the principal observes $I$ only if it audits the taxpayer, at a cost per audit of $c \geq 0$. The taxpayer makes a report of his or her income, $x$, to the principal.

Assumption 3. If the taxpayer is not audited, his or her transfer to the principal is given by the function $t(x)$, which represents the tax owed.

Assumption 4. If an audit is performed, the taxpayer is assessed an amount given by the function $f(x, I)$, which represents the tax owed plus any applicable fines.

Let $r(x, I)$ denote expected net income to the agent if income $I$ is observed and $x$ is reported. Then

$$r(x, I) = [1 - p(x)][I - t(x)] + p(x)[I - f(x, I)].$$

Assumption 5. The agent chooses his or her report so as to maximize $r(x, I)$, given $t(x)$, $f(x, I)$, and $p(x)$.

We denote an optimal report for an agent with income $I$ by $\phi(I)$. There need not be a unique optimal report; that is, $\phi(\cdot)$ may be a correspondence. In this event, we allow the principal to select its most-preferred alternative.
from the set \( \phi(I) \). This corresponds to the assumption (which is standard in the principal–agent literature) that whenever the agent is indifferent it takes that action which is most preferred by the principal. Let \( R(x, I) \) denote expected net revenue to the principal from an individual with income \( I \) who reports \( x \). Then

\[
R(x, I) = [1 - p(x)]t(x) + p(x)[f(x, I) - c].
\]

**Assumption 6.** Given \( \phi(\cdot) \), the principal selects transfer functions \( t(\cdot) \), \( f(\cdot, I) \), an audit probability function \( p(\cdot) \) and a report \( x \in \phi(I) \) so as to maximize \( ER(x, I) \), where the expectation is taken with respect to the distribution of income \( G(\cdot) \).

**Definition.** A given policy \( (p(\cdot), t(\cdot), f(\cdot, I)) \) induces truthful reporting if, given that policy, \( I \in \phi(I) \) for all \( I \).

Let \( P \equiv \{p(\cdot), t(\cdot), f(\cdot, I)\} \) denote the set of feasible policies for the principal. The Revelation Principal [see, for example, Myerson (1979) or Harris and Townsend (1981)] in this context is as follows: given any audit/tax/fine policy in the set \( P \), there exists another feasible policy which induces taxpayers to report their incomes truthfully and leaves both the principal and the agent at least as well off as the original policy. Thus, without loss of generality, one can restrict attention to policies which induce truthful reporting.\(^1\)

Although this result helps to provide a partial characterization of an optimal policy for the IRS, the task of fully characterizing it remains formidable. Rather than attempting this task, we will consider some special, but interesting, cases. In particular, we will impose lump-sum taxes and fines.

**Assumption 7.** Taxes and fines are lump-sum. That is, if the agent is not audited, the principal asks the agent to pay \( t(x) = \min\{T, x\} \), where \( x \) is reported income. If the agent is audited and has under-reported, the principal exacts a payment of \( f(x, I) = \min\{T + F, I\} \). \( T \) and \( F \) may be interpreted as the desired tax and the fine for under-reporting, respectively. No fine is ever paid if income is reported truthfully [that is, \( f(I, I) = \min\{T, I\} \)].

Under Assumption 7, the payoff to the principal is

\[
R(x, I) = \min\{T, x\}[1 - p(x)] + \begin{cases} 
[\min\{I, T + F\} - c]p(x) & \text{if } x < I, \\
[\min\{I, T\} - c]p(x) & \text{if } x \geq I.
\end{cases}
\] (1)

\(^1\)Without imposing any additional constraints upon the set \( P \), it is clear that the Revelation Principle applies. If the policy \( (p(\cdot), t(\cdot), f(\cdot, I)) \) results in the optimal reporting function \( \phi(I) \), then the policy given by \( (p(\phi(\cdot)), t(\phi(\cdot)), f(\phi(\cdot), I)) \) has an optimal reporting function which is identically \( I \). Moreover, the payoffs to both the principal and the agent are unaffected by this substitution.
Eq. (1) allows for the possibility of over-reporting, i.e. for $x > I$. Although it turns out to make no difference in the optimal policies, we rule this out since it makes the analysis which follows less tedious. That is, we henceforth assume that $\phi(I) \leq I$ for all $I$.

For a taxpayer who reports an income of $x < I$, net income is

$$r(x, I) = \begin{cases} 
(1 - p(x)) [I - \min\{x, T\}] + p(x) [I - T - F] & \text{if } I \geq T + F, \\
(1 - p(x)) [I - \min\{x, T\}] & \text{if } I < T + F.
\end{cases}$$

(2)

If an individual under-reports and is not audited, then he or she pays only the minimum of reported income or the tax; if audited, the individual must pay the fine $F$ as well, whenever that is possible. For taxpayers who report truthfully $(x - I)$, net income is

$$r(x, I) = \begin{cases} 
(1 - p(x)) [I - T] + p(x) [I - T] & \text{if } I \geq T, \\
0 & \text{if } I < T.
\end{cases}$$

(3)

Under the restrictions imposed by Assumption 7, we will compare two important types of audit policies. We first characterize optimal random audits, in which the probability of audit is independent of reported income. Next we characterize optimal audit cutoff policies, under which an agent triggers an audit by reporting too little income. Despite its simplicity, this audit policy does induce taxpayers to report their incomes truthfully; moreover, it is the least-cost policy (given lump-sum taxes and fines) which does so.

3. Random audits

In this section we consider the optimal strategy for the principal when the audit probability function $p(\cdot)$ is independent of the level of reported income. As mentioned earlier, this is the random audit formulation used in the previous literature on this subject.

Assumption 8. The audit probability function $p(\cdot)$ is independent of $x$.

Denote the probability of an audit by $p$. Then from (2) and (3), if $x < I$,

$$r_x(x, I) = \begin{cases} 
-(1 - p) & \text{if } x < T, \\
0 & \text{if } x \geq T,
\end{cases}$$

(4)

and if $x = I$,

$$r(I, I) = 0$$

(5)
where the subscript is used to denote a partial derivative. The implication of (4) and (5) is that if an agent under-reports his or her income, \( \phi(I) = 0 \) is optimal. Thus, \( \phi(I) \) is either 0 (given under-reporting) or 1 (given truthful reporting). Comparing these choices using (2) and (3) yields the following lemma.

**Lemma 1.** Under assumptions 1–8, for any given \( p \), \( T \) and \( F \), the agent’s optimal reporting rule is

\[
\phi(I) = \begin{cases} 
0 & \text{if } p < T/(T+F) \\
I & \text{if } p \geq T/(T+F) \\
0 & \text{if } p < T/I \\
I & \text{if } p \geq T/I \\
0 & \text{for } I < T.
\end{cases}
\]

**Proof.** If \( I \geq T+F \), \( r(0,I) = I - p(T+F) \) and \( r(I,I) = I - T \). Thus, \( \phi(I) = 0 \) if and only if \( p < T/(T+F) \).

If \( T \leq I < T+F \), \( r(0,I) = (1-p)I \) and \( r(I,I) = I - T \). Thus, \( \phi(I) = 0 \) if and only if \( p < T/I \).

Finally, if \( I = T \), \( r(0,I) = (1-p)I \) and \( r(I,I) = 0 \). Thus, \( \phi(I) = 0 \) for all \( I < T \). Q.E.D.

**Remark 1.** If \( p < T/(T+F) \) and \( I < T+F \), then \( p < T/I \). Thus, if \( p < T/(T+F) \), then the agent will always report no income. However, if \( p \geq T/(T+F) \), an agent with income less than \( T/p \) will evade (i.e. report income equal to zero) while those with incomes greater than or equal to \( T/p \) will report truthfully.

Using lemma 1 and remark 1, we can write the principal’s expected revenue as

\[
ER(\phi(I), I) = \begin{cases} 
-cp + \int_0^{T+F} pI \,dG(I) + \int_{T+F}^{\infty} p(T+F) \,dG(I) & \text{if } p < T/(T+F), \\
-cp + \int_0^{T/p} pI \,dG(I) + \int_{T/p}^{\infty} T \,dG(I) & \text{if } p \geq T/(T+F).
\end{cases}
\]

Notice that \( ER(\cdot) \) is continuous at \( p = T/(T+F) \). We are concerned with maximizing (6) with respect to \( T \), \( p \) and \( F \). Consider \( T \) first. Again using
subscripts to denote partial derivatives, we have

\[ ER_T(\phi(I), I) = \begin{cases} 
  p[1 - G(T + F)] & \text{if } p < T/(T + F), \\
  [1 - G(T/p)] & \text{if } p \geq T/(T + F).
\end{cases} \tag{7} \]

Using (7) we have immediately that if \( p > 0 \), then the optimal tax is \( T^* = \infty \). If \( p = 0 \), then \( T \) is irrelevant.

Next consider the choice of \( p \). Here

\[ ER_p = \begin{cases} 
  -c + \int_0^{T/F} I \ dG(I) + \int_{T/F}^{\infty} (T + F) \ dG(I) & \text{if } p < T/(T + F), \\
  -c + \int_0^{T/F} I \ dG(I) & \text{if } p \geq T/(T + F).
\end{cases} \tag{8} \]

When \( T = \infty \), eq. (8) becomes:

\[ ER_p = -c + \int_0^{\infty} I \ dG(I), \tag{9} \]

since \( T/(T + F) \to 1 \) as \( T \to \infty \). In this case, \( p^* > 0 \) if and only if (9) is positive, and then \( p^* = 1 \). Otherwise, \( p^* = 0 \) and \( T \) is irrelevant. In either case \( F \) is irrelevant to the optimal \((p^*, T^*)\) combination, so long as it is non-negative. Thus, we have the following theorem.

**Theorem 1.** Under assumptions 1–8, \( F \) is irrelevant. Either \( p^* = 0 \) and \( T \) is also irrelevant or \( p^* = 1 \) and \( T^* = \infty \). Moreover, \( p^* = 0 \) if and only if average income is no greater than the audit cost \( c \), and \( p^* = 1 \) if and only if average income exceeds the audit cost \( c \). That is,

\[ p^* = 0 \quad \text{if and only if} \quad \int_0^{\infty} I \ dG(I) \leq c, \]

and

\[ p^* = 1 \quad \text{if and only if} \quad \int_0^{\infty} I \ dG(I) > c. \]

The intuition behind theorem 1 is clear. Since random audits do not distinguish between agents on the basis of their reported income, either \( p^* = 0 \) or \( p^* = 1 \) must be optimal. In the former case, \( T \) and \( F \) are clearly irrelevant. In the latter, the principal might as well take everything. Thus, \( F \) is again irrelevant and the decision whether to audit at all depends on comparing average income to the audit cost.
4. Audit cutoffs

In this section, we modify assumption 8 in the following way.

**Assumption 9.** The audit probability function must take the form

\[ p(x) = \begin{cases} 1 & \text{if } x < i, \\ 0 & \text{if } x \geq i, \end{cases} \]

where \( i \in [0, \infty) \).

In other words, the principal always audits the agent if income less than \( i \) is reported. Otherwise an audit never takes place. This audit policy has the feature that an agent can trigger an audit by reporting income which is too low. In this case, revenue to the principal (net of audit costs) from an agent with income \( I \) who reports income \( x \) is

\[
R(x, I) = \begin{cases} \min\{x, T\} & \text{if } i \leq x \leq I, \\ \min\{I, T\} - c & \text{if } i > x = I, \\ \min\{I, T + F\} - c & \text{if } x < \min\{i, I\}. \end{cases}
\]

The logic behind (11) is as follows. If the agent reports an income greater than the cutoff, no audit takes place and either the reported income or the tax is paid, whichever is smaller. If the agent reports truthfully, but less than the cutoff, an audit takes place but no fine is imposed. Finally, if the agent reports less than the cutoff but lies, the tax plus the fine is imposed, whenever that is possible.

Similarly, the agent’s residual income is

\[
r(x, I) = \begin{cases} I - \min\{x, T\} & \text{if } i \leq x \leq I, \\ I - \min\{T, I\} & \text{if } i > x = I, \\ I - \min\{T + F, I\} & \text{if } x < \min\{i, I\}. \end{cases}
\]

Following as in section 3, we have an analogous result to lemma 1.

We have also examined audit policies which combine the basic features of random audits and audit cutoffs. We call these mixed policies and they are of the form:

\[
p(x) = \begin{cases} p_1 & \text{if } x < i, \\ p_2 & \text{if } x \geq i, \end{cases}
\]

where \( 0 \leq p_j \leq 1 \) for \( j = 1, 2 \) and \( i \in [0, \infty) \). Mixed policies allow for audit probabilities which may increase or decrease with reported income. Although the proof is too tedious to include here, it can be shown that the optimal mixed policy reduces to the optimal audit cutoff policy.
Lemma 2. Under assumptions 1–7 and 9, for any given i, T and F:

\[
\phi(I) = \begin{cases} 
[0, I] & \text{if } I < i \\
i & \text{if } I \geq i \\
[0, I] & \text{if } I < T \\
[i, I] & \text{if } I \geq T.
\end{cases}
\]

Proof. The level of reported income only affects \(r(x, I)\) on the first branch of (12), and for that case \(r_x \leq 0\). Hence, \(\phi(I) = i\) on the first branch. On the second branch we set \(\phi(I) = I\) and on the third \(\phi(I) = [0, \min\{i, I\}]\). Hence, (12) implies:

\[
\max_x r(x, I) = \begin{cases} 
I - \min\{i, T\} & \text{for } x \geq i, \\
\begin{cases} 
1 & \text{if } I \geq T \\
0 & \text{if } I < T \end{cases} & \text{for } i > x = I, \\
\begin{cases} 
I - T - F & \text{if } I \geq T + F \\
0 & \text{if } I < T + F \end{cases} & \text{for } x < \min\{i, I\}.
\end{cases}
\] (13)

To determine optimal reports, we need only compare reports of \(i, I\) and 0 (since any report \(x \in [0, \min\{i, I\}]\) yields the same outcome as a report of 0). It turns out that there are but two relevant cases.

Case 1. \(i < T\). When \(I < i\) in this case, a report of \(i\) is inadmissible by assumption. A report of \(I\) implies \(r(I, I) = 0\), as does a report of 0, \(r(0, I) = 0\). Hence, \(\phi(I) \in [0, I]\).

When \(i \leq I < T\), \(r(i, I) = I - i, r(I, I) = 0\) and \(r(0, I) = 0\). Thus \(\phi(I) = i\).

For \(T \leq I < T + F\), \(r(i, I) = I - i, r(I, I) = I - T\), and \(r(0, I) = 0\). Hence, again \(\phi(I) = i\).

Finally, when \(I \geq T + F\), \(r(i, I) = I - i, r(I, I) = T\) and \(r(0, I) = I - T - F\). Hence \(\phi(I) = i\).

To summarize this case, when \(i < T\), any income level in \([0, I]\) may be reported, an audit occurs regardless, and the agent keeps nothing. If \(I \geq i\), the agent always reports \(i\), pays \(i\), and is never audited. The fine \(F\) is irrelevant.

Case 2. \(i \geq T\). If \(I < T\), a report of \(i\) is inadmissible. Furthermore, \(r(I, I) = 0 = r(0, I)\). Hence, \(\phi(I) \in [0, I]\).

If \(T \leq I < i, i\) is still inadmissible as a report. But \(r(I, I) = I - T\) and \(r(0, I) = \max\{I - T - F, 0\}\). Hence, \(\phi(I) = I\).

If \(I \geq i\), then \(r(i, I) = I - T, r(I, I) = I - T\), and \(r(0, I) = \max\{0, I - T - F\}\). Hence, \(\phi(I) \in [i, I]\) (since any report in \([i, I]\) results in a payoff of \(I - T\) so long as \(i \geq T\)).
To summarize, if $i \geq T$, the agent may report any $x \in [0, I]$ when $I < T$ and may report any $x \in [i, I]$ when $I \geq T$ (whenever $i$ is admissible). Q.E.D.

Assuming the agent is truthful whenever he or she and the principal are otherwise indifferent, lemma 2 implies:

$$\phi(I) = \begin{cases} 
I & \text{if } I < i \\
i & \text{if } I \geq i \\
 & \text{for } i < T, \\
 & \text{if } i \geq T.
\end{cases}$$  \hspace{1cm} (14)

It is interesting to note that under the audit cutoff rule there may be universal truthful reporting; when there is some evasion, it is the relatively high income individuals who evade. This is in direct contrast to the results in the random audit case. In that case there might be universal evasion; when there is some compliance, it is the relatively high income agents who comply.

From (14), then,

$$ER(\phi(I), I) = \begin{cases} 
\int_0^i (I - c) dG(I) + \int_i^\infty i dG(I) & \text{if } i < T, \\
\int_0^T (I - c) dG(I) + \int_i^T (T - c) dG(I) + \int_T^\infty T dG(I) & \text{if } i \geq T.
\end{cases}$$  \hspace{1cm} (15)

It is clear from (14) and (15) that $F$ is irrelevant to both the optimal reporting rule of the agent and the optimal policy of the principal. Furthermore,

$$ER_i(\phi(I), I) = \begin{cases} 
-cg(i) + 1 - G(i) & \text{if } i < T, \\
-cg(i) & \text{if } i > T.
\end{cases}$$  \hspace{1cm} (16)

Note that $ER(\phi(I), I)$ is continuous at $i = T$. It is decreasing in $i$ for $i > T$, so clearly $i \leq T$. However, in this case only the cutoff $i$ matters; $T$ is irrelevant. Thus, we can set $i = T$ and use either branch of (15) to solve for $T^*$. Substituting $i = T$ and differentiating with respect to $T$ implies:

$$ER_T(I, \phi(I)) = -cg(T) + 1 - G(T).$$  \hspace{1cm} (17)

**Theorem 2.** Under assumptions 1–7 and 9, $F$ is irrelevant. Furthermore, $i^* = T^*$ always. Beyond this, three cases are possible:

1. $i^* = T^* = 0$, 

(2) $i^* = T^* = \infty$,

(3) $i^* = T^* = \tilde{T}$, where $\tilde{T}$ solves $-cg(\tilde{T}) + 1 - G(\tilde{T}) = 0$.

The optimal tax/audit policy includes three possibilities: audit no one; audit everyone and take all their income; or audit those with reported income less than $\tilde{T}$ and take all their money, while taking $\tilde{T}$ from each agent who reports income greater than this amount.

It is easy to see why theorem 2 holds once one recognizes that $i^* = T^*$; the optimal tax and cutoff level must be identical. The key is that expected net revenue declines in $i$ for $i > T$ since audit costs increase in $i$ but expected gross revenue is unchanged. But with $i^* = T^*$, only taxpayers with income less than or equal to $T^*$ are audited, so the fine is again irrelevant. The possibility of an interior solution for $i^*$ and $T^*$ arises because it is now possible to extract information from taxpayer reports and thus trade off audit costs against gross revenues.

**Remark 2.** Note that for $\tilde{T}$ to provide a maximum, we must have

$$-cg'(\tilde{T}) - g(\tilde{T}) < 0.$$  \hspace{1cm} (18)

Denoting $h(I) = g(I)/(1 - G(I))$ as the hazard rate, $\tilde{T}$ is defined by $h(\tilde{T}) = 1/c$ and (18) is equivalent to $h'(\tilde{T}) > 0$, i.e. the hazard rate must be increasing at $\tilde{T}$. Actually, we can say more than this. Suppose $h'(I) > 0$ for all $I$. Then there is at most one interior critical point and it is a local maximum of $ER$. If this value exists it must also provide a global maximum of $ER$. For otherwise there would exist an interior local minimum as well, which is impossible. Thus, if $h'(I) > 0$ for all $I$ and there exists $T \in (0, \infty)$ such that $h(T) = 1/c$, then $\tilde{T}$ is optimal.

Cases (1) and (2) of Theorem 2 correspond to the two possibilities under random audits — either audit no one or audit everyone and take whatever income they have. The third possibility is the interesting one; it requires the principal to audit those with incomes less than some finite positive amount. Taxpayers with incomes below this level are always audited and those with incomes above this level are never audited. Taxpayers with incomes above the cutoff are indifferent about reporting their true incomes or the cutoff level; the principal is also indifferent between these two reports, so one could observe some evasion among these higher income individuals. Notice that only truthful taxpayers are audited. Thus, no fines are ever collected and audit costs are, in a sense, wasted. This highlights the nature of the audit as an incentive device; actual audit costs are the price paid for inducing those with higher incomes to report at least $T^*$.

Since an audit cutoff policy could generate the optimal random audit
outcomes but also admits an interior solution, it weakly dominates the random audit policy. In addition to raising at least as much revenue as a random audit policy, an audit cutoff policy has several other desirable features.

Stiglitz (1976) and Weiss (1976) have suggested that randomness in the tax or audit rates may have beneficial incentive effects when labor supply decisions are endogenous and individuals are risk averse. When viewed from an ex ante perspective (i.e. before income is realized), an audit cutoff policy looks like a random audit policy with \( p = G(T^*) \). Thus, if labor supply decisions are made before the realization of (say) a random wage rate, then whatever benefits might be derived from using a (purely) random audit policy still apply when one uses an audit cutoff policy.

There has also been much discussion of the desirability of horizontal equity in a tax system [e.g. Stiglitz (1976) and Rosen (1978)]. While a random audit policy is horizontally equitable in an ex ante sense (i.e. before anyone is audited they face an identical probability of audit), it is not horizontally equitable ex post (i.e. some individuals with the same income are audited while others are not). The audit cutoff policy, however, is horizontally equitable both ex ante and ex post. That is, before income is determined, each individual faces a probability of \( G(T^*) \) of being audited; after income is realized, all those with the same income make the same report and suffer the same consequences. Thus, there is no question of treating identical people differently.

Another desirable feature of audit cutoff policies is that they provide incentives for taxpayers to report their incomes truthfully. That is, the optimal audit cutoff policy of section 4,

\[
p(x) = \begin{cases} 
1 & \text{if } x < T, \\
0 & \text{if } x \geq T,
\end{cases}
\]

has \( I \) as a best report, as can be seen from eq. (14).3 Although compliance is not necessarily desirable on strictly economic grounds, legal scholars and policymakers have expressed concern that widespread noncompliance with the tax laws might result in a general decrease in law-abidingness.

**Theorem 3.** A necessary and sufficient condition for a policy \((p(\cdot), T, F)\) to induce truthful reporting is that \( p(x) = 1 \) for all \( x < T \).

**Proof.** Using eqs. (2) and (3), we see that for \( I < T \), \( r(I, I) \geq r(x, I) \) for all \( x < I \).

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3Note that a truthful report need not be the only best report. The optimal audit cutoff policy induces truthful reporting, but an agent with income greater than \( T \) also has \( \phi(I) = T \) as a best report.
if and only if
\[0 \geq (1 - p(x))(I - \min\{x, T\})\]

for all \(x < I < T\). This inequality holds if and only if \(p(x) \equiv 1\) for all \(x < T\).

For \(T \leq I < T + F\), \(r(I, I) \geq r(x, I)\) for all \(x < I\) if and only if
\[I - T \geq (1 - p(x))(I - \min\{x, T\}).\]

Note that for \(x < T\), \(p(x) \equiv 1\), so this inequality is satisfied for all \(x < T\). For \(x \geq T\), with \(x < I < T + F\), \(\min\{x, T\} = T\), so the above inequality reduces to
\[I - T \geq (1 - p(x))(I - T).\]

This is satisfied for any non-negative \(p(x)\).

Finally, for \(I \geq T + F\), \(r(I, I) \geq r(x, I)\) for all \(x < I\) if and only if
\[I - T \geq (1 - p(x))(I - \min\{x, T\}) + p(x)(I - T - F).\]

For \(x < T\), \(p(x) \equiv 1\) and \(I - T \geq I - T - F\) for all non-negative \(F\). For \(x \geq T\), \(\min\{x, T\} = T\), so the inequality reduces to
\[I - T \geq (1 - p(x))(I - T) + p(x)(I - T - F),\]

which is true for all non-negative \(p(x)\) and \(F\). Q.E.D.

**Corollary.** The audit policy given in eq. (19) is the least-cost policy of the form \((p(\cdot), T, F)\) which induces truthful reporting.

**Proof.** By theorem 3, the only restrictions imposed on \((p(\cdot), T, F)\) by the requirement of truthful reporting are that \(p(x) \equiv 1\) for all \(x < T\), \(p(x) \geq 0\) for all \(x\), and \(F \geq 0\). Any function \(p(x)\) for \(x \geq T\) will induce truthful reporting and \(p(x) \equiv 0\) does so at least audit cost to the principal. Q.E.D.

**Remark 3.** Given lump-sum taxes and fines, it can be shown that any optimal audit policy \(p(\cdot)\) must have \(p(x) \equiv 0\) for all \(x \geq I\). To see this, note that for agents with incomes \(I \geq T\), truthful reporting dominates any report \(x \in [T, I]\). That is, \(r(I, I) = I - T\), while
\[r(x, I) = (1 - p(x))(I - T) + p(x)(I - \min\{I, T + F\})\]

for all \(x \in [T, I]\). So long as \(p(x) \geq 0\) (> 0), it is at least as good (better) to report \(I\) as to report \(x \in [T, I]\). Thus, any report \(x \geq T\) will be truthful and it will not pay to audit
agents reporting \( x \geq T \) with positive probability. Therefore \( p(x) = 0 \) for all \( x \geq T \).

5. Examples and comparative statics

Recall that for any given distribution of income \( G(\cdot) \), the audit cutoff rule weakly dominates the random audit policy. However, for some distributions this dominance may be strict. In particular, we know that this is true whenever a positive solution \( T^* \) exists for the equation

\[
h(T^*) = \frac{1}{c},
\]

and the hazard rate \( h(\cdot) \) is strictly increasing. Thus, the class of distributions with increasing hazard rate are of particular interest. Members of this class include the uniform distribution, truncated normal distributions, the Weibull distribution (with parameter \( \alpha > 1 \)), the gamma distribution (with parameter \( \beta > 1 \)), and distributions with linear hazard rates.

Whenever eq. (20) has an interior solution and the hazard rate is increasing, differentiation implies that \( \partial T^*/\partial c < 0 \). That is, an increase in audit costs lowers the optimal tax \( T^* \) (and the audit cutoff \( t^* \)). Consequently, fewer agents are audited and less revenue is collected.

We frequently think of taxing different classes of agents differently. For instance, there may be other observable characteristics of individuals which are correlated with income (e.g. education or location of residence). Then we could think of \( G(\cdot) \) as being parameterized by \( \rho \), where \( \rho \) yields some information about the likelihood of a type-\( \rho \) agent having income level \( I \). Denote this dependence by \( G(\cdot; \rho) \). To determine the comparative static effects of \( \rho \) on \( T^* = T^*(\rho) \), we would need to know the effect of a change in \( \rho \) upon \( h(\cdot; \rho) = g(\cdot; \rho)/[1 - G(\cdot; \rho)] \). Since this seems beyond intuition, we now compute \( T^* \) for some examples and consider several parametric variations of interest.

For the uniform distribution on \([a, b]\), \( G(I) = (I - a)/(b - a) \). Solving for \( T^* \) yields \( T^* = b - c \), which is interior so long as \( b - c > a \). The mean of \( I \) is \( \mu = (a + b)/2 \), while the variance is \( \sigma^2 = (b - a)^2/2 \). One might consider increasing the mean of the distribution, holding the variance constant, or vice versa. Computing \( a \) and \( b \) as functions of \( \mu \) and \( \sigma \) gives \( a = \mu - \sigma \sqrt{3} \) and \( b = \mu + \sigma \sqrt{3} \). Thus, \( T^* = \mu + \sigma \sqrt{3} - c \). Consequently, an increase in either \( \mu \) or \( \sigma \), holding the other fixed, increases \( T^* \). Agents facing uniform distributions of income with greater mean and the same variance (or greater variance and the same mean) should be required to pay higher taxes.

Another interesting parameter change is related to stochastic dominance. We say that \( G(\cdot; \rho_1) \) dominates \( G(\cdot; \rho_2) \) if \( G(I; \rho_1) < G(I; \rho_2) \) for all \( I \); that is, income is stochastically greater under \( \rho_1 \) than under \( \rho_2 \). For the uniform
distribution

\[ \frac{\partial G}{\partial a} = \frac{(1-b)(b-a)^2}{b-a} < 0 \]

and

\[ \frac{\partial G}{\partial b} = \frac{(a-I)(b-a)^2}{b-a} < 0. \]

Since \( T^* = b - c \) if \( b - c > a \) and \( T^* = a \) if \( b - c \leq a \), agents facing stochastically better uniform distributions of income should be taxed more (and should face a harsher audit cutoff policy).

For distributions with linear hazard rate, \( G(I) = 1 - e^{-(aI + \theta I^2/2)} \), for \( I \in [0, \infty) \) and \( a, \theta > 0 \). Solving for \( T^* \) yields \( T^* = \frac{(1 - ca)}{c\theta} \), which is interior for \( 1 > ca \). Since

\[ \frac{\partial G}{\partial \alpha} = I e^{-(aI + \theta I^2/2)} > 0 \]

and

\[ \frac{\partial G}{\partial \theta} = (1/2)I^2 e^{-(aI + \theta I^2/2)} > 0, \]

an increase in either \( \alpha \) or \( \theta \) results in a stochastically dominated distribution. Since \( \frac{\partial T^*}{\partial \alpha} = -1/\theta < 0 \), and \( \frac{\partial T^*}{\partial \theta} = -(1 - ca)/c\theta^2 < 0 \), again we should raise the optimal tax/cutoff \( T^* \) for distributions with stochastically higher income (i.e. those with lower values of \( \alpha \) and/or \( \theta \)).

6. Conclusions, variations and extensions

We have compared two interesting classes of audit policies under the assumption of lump-sum taxes and fines. We find that (purely) random audit rules are weakly dominated by audit cutoff rules. It can be shown (given lump-sum taxes and fines) that these audit cutoff rules are the least-cost policies which induce truthful reporting of income. Although space constraints do not permit us to include this analysis here, we have elsewhere [Reinganum and Wilde (1983)] analyzed the case of proportional taxation, and find that the dominance of audit cutoff rules over random audit rules holds for proportional as well as lump-sum taxation — in fact, the equilibrium consequences of the two are equivalent.

These results were established under fairly strong assumptions, however. In particular, we assumed risk-neutrality for all agents and that the principal desired to maximize net expected revenue. In spite of its simple nature, the model yields interesting results not found in the previous literature, and several extensions seem worth pursuing.
An important extension would be to consider risk-averse agents. It is easy to show that the problem under the audit cutoff rule is unchanged; agents face no real uncertainty, and (as long as utility is increasing in income) they will behave precisely as described in section 4. However, the risk-averse agent will respond differently to the random audit policy. This analysis leaves open the possibility that a policy of random audits may be more effective than an audit cutoff policy when taxpayers are risk averse. Indeed, some of Townsend’s (1979) analysis suggests this possibility rather strongly. He finds that for risk-averse agents, random verification can reduce verification costs yet still support self-selection when there are only two states. It seems quite likely that these results will hold with a continuum of states as well.

Obviously, it is important to consider alternative objective functions; for example, the tax authority may desire to minimize the cost of raising a specified amount of revenue. One might also consider the more traditional objective of maximizing a utilitarian criterion.

There is a presumption in the tax policy literature that some taxpayers are always honest, even when they could increase their expected net income by under-reporting. To capture this formally, we could assume some fraction of taxpayers, say \( z \), always report honestly. The issue is how this affects the principal’s optimal strategy; in particular, what happens as \( z \) decreases? One might speculate that in the lump-sum case the presence of an ‘honest’ group will drive a wedge between \( i^* \) and \( T^* \) when audit cutoffs are used. Beyond this, however, it is not clear what might happen. A simple model which addresses this issue in a slightly different context can be found in Graetz, Reinganum and Wilde (1983).

Instead of knowing the function \( p(\cdot) \), taxpayers might only know \( p = E[p(\cdot)] \), particularly if they do not know the distribution of income, \( G(\cdot) \). In this case, the issue is whether it is in the interest of the principal to reveal \( p(\cdot) \) or whether net revenue can be increased by exploiting taxpayer ignorance of the audit rule.

Finally, we should point out that the general problem described in section 2 remains unsolved; that is, the optimal audit/tax/fine policy (in the absence of constraints) remains to be characterized. The solution of this problem would be an extremely illuminating and important contribution to the literature on tax evasion.

References
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