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EQUILIBRIUM VERIFICATION AND REPORTING POLICIES
IN A MODEL OF TAX COMPLIANCE*

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1. INTRODUCTION

One of the major applications of the economic approach to crime and punishment, pioneered by Becker [1968], has been tax compliance. This literature, which focuses primarily on taxpayer behavior, treats the choice of a compliance level (or, more generally, participation in illegitimate activities) as a portfolio problem, in which an agent must allocate his budget (of income, or effort) between a risky asset (unreported income, or criminal activity) and a risk-free asset (reported income, or legal activity) (see e.g., Stigler [1970]; Allingham and Sandmo [1972]; and Polinsky and Shavell [1979]). These authors all assume a fixed and uniform probability that proscribed behavior will be detected, although some attention has been given to the determination of the optimal probability of detection (except see Yarbrough and Yarbrough [1984]). Srinivasan [1973] allows the probability of detection to depend on true income (not reported income) but takes that function as given. He also analyzes the optimal probability of detection, but again under the assumption that it is uniform. More recently, there have been repeated-games analyses of tax evasion (Landsberger and Meilijson [1982]; and Greenberg [1984]), but these models are equally applicable to tax evasion and other proscribed behavior (e.g., Rubinstein [1979]).

We consider the problem of tax compliance to be fundamentally different from that of many other illegitimate activities because taxpayers are required to submit a preliminary accounting of their behavior. This preliminary round of information transmission will tend to differentiate individuals, and raises the possibility that it may not be optimal to apply the same verification policy to all taxpayers. In this paper, we incorporate the information content of the income reporting process into an equilibrium model of tax compliance and enforcement. It may be possible to apply a similar analysis to other legal contexts since many regulatory agencies require initial reporting by the entity that is being monitored. Examples include public utilities regulation (Baron and Besanko [1983]) and

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environmental regulation (Epple and Visscher [1984]). In addition, the administration of many public welfare and transfer programs begin with the submission of a claim for relief. We do not pursue any of these applications in this paper, but our results in the tax compliance context suggest that doing so may yield useful insights.

In order to take account of the special informational features of the tax compliance problem, we assume the taxpayer possesses private information about his own income level, while the IRS knows only the probability distribution according to which the taxpayer’s income is realized. By investing resources, the IRS can (perhaps only stochastically) verify the taxpayer’s income. We assume the cost of verification depends on the probability of verification chosen by the IRS. We then characterize the optimal reporting rule for an individual taxpayer, given his private information, and given the verification policy of the IRS. Similarly, we characterize the optimal verification policy of the IRS, given the reporting behavior of taxpayers, and given its incomplete information regarding the taxpayer’s true income.

Our approach to this problem represents an application of the sequential equilibrium solution concept (Kreps and Wilson [1982a]), and is very much in the spirit of Spence [1974]. This methodology has yielded interesting results when applied to problems in limit pricing (Milgrom and Roberts [1982a]; Matthews and Mirman [1983]; Saloner [1983]), reputation and predation (Kreps and Wilson [1982b]; Milgrom and Roberts [1982b]), bargaining (Fudenberg and Tirole [1983]; Cramton [1984]), signalling (Spence [1974]; Kreps [1984]; Milgrom and Roberts [1984]) and convertible debt call policy (Harris and Raviv [1984]).

Other recent analyses of related problems can be found in Townsend [1979], Baron and Besanko [1983], Reinganum and Wilde [1985a] and Border and Sobel [1985]. These papers do not use the equilibrium approach adopted here; instead they employ a principal-agent structure to examine the problem of optimal auditing. In our context, the principal-agent structure endows the IRS with the ability to pre-announce and commit to an audit policy irrespective of its optimality once taxpayers’ actual reports are received. Although one can imagine persuasive arguments for considering this framework, the alternative formulation in which the IRS cannot commit itself (but must choose a policy which is optimal given taxpayers’ actual reports) is equally interesting. The very power and magnitude of government mitigates against its being able to commit itself; there is no third-party higher authority to enforce the commitment. Moreover, the individual taxpayer cannot subsequently determine whether the IRS has actually used its pre-announced verification policy; under these circumstances there is little reason to believe that the IRS will follow its pre-announced policy.

In section 2 we set up our basic game-theoretic model of tax compliance. In section 3 we describe how to construct an equilibrium, given that opportunities for income are bounded. This equilibrium has the feature that, for any given “audit class,” taxpayers with greater true income under-report less than those
with lower true income, and efforts at verification are lower the greater is reported income. This is the fundamental result of this paper, and initially may seem counter-intuitive (or even counter-factual). It is important to keep in mind that it applies to comparisons within audit classes, not across audit classes. It is also important to recognize that it depends crucially on the assumption that the IRS cannot precommit to any audit strategy. Since, in general, the probability of audit must decline with reported income in order to keep high income taxpayers from under-reporting too much (whether or not there is precommitment), ex post optimality for the IRS implies the absolute amount of under-reporting must also decline with true income.

In section 4 we present two algebraic examples. In section 5 we use a comparative static analysis to examine the impact of separating taxpayers into different audit classes on the basis of some characteristic which is directly observable. We find that classes of taxpayers who enjoy greater opportunities for high income under-report to a greater extent (for any given level of true income); accordingly, more effort is devoted to detecting under-reporting by them. This result applies to comparisons across audit classes and is therefore not inconsistent with our fundamental result concerning comparisons within audit classes. We also examine the limiting case in which opportunities for income are unbounded; in this case, taxpayers under-report by a constant amount and the IRS’ verification policy is independent of reported income. Therefore, this set of circumstances rationalizes the assumption of a constant audit probability which is so prevalent in the extant literature. Section 6 summarizes the results presented in this paper, discusses briefly their robustness with respect to various assumptions, and suggests possibilities for future research.

2. THE MODEL

The timing of moves in the model is as follows: the taxpayer observes his true income; we will often refer to the taxpayer’s true income as the taxpayer’s “type”. Based upon true income, the taxpayer conveys a statement of reported income to the IRS. Since the IRS does not observe the taxpayer’s true income, it must make some conjecture about the type of taxpayer who would report a given level of income. Based on the level of income reported and these conjectures, the IRS chooses a level of effort to be devoted to investigating the taxpayer. This effort will be assumed to generate a particular probability that the taxpayer’s true income will be verified, with the property that greater effort leads to a greater probability of verification; investigation does not imply certain apprehension.

Suppose that true (taxable) income for the taxpayer is a random variable \( I \in [\bar{I}, \bar{I}] \), where \( 1 < \bar{I} < \infty \), with distribution function \( F(\cdot) \). Let \( x \) denote reported income for a taxpayer. A strategy for the taxpayer is a reporting policy \( x = r(I) \), where \( r: \bar{I} \to (\infty, \infty) \); that is, the taxpayer may report any level of income, not just those which might possibly occur. If the taxpayer is not investigated, then he is asked to pay a tax of \( tx \) dollars; that is, we assume proportional taxation.
This assumption is analytically and expositionally convenient but not crucial; the basic qualitative features of the model are preserved under the assumption of progressive taxation, a point we return to in section 6.

If the taxpayer is investigated and his true income is ascertained, then a tax plus a fine proportional to evaded tax is assessed: if \( I \) is the taxpayer's true income, this amount is \( tI + t\pi(I - x) \). As Yitzhaki [1974] pointed out, the assumption that penalties are proportional to evaded tax, as opposed to unreported income, is consistent with U. S. tax laws. However, no feasibility restrictions are placed on these transfer payments in our model; individuals who report negative income and are not investigated receive a transfer (or negative income tax), and taxpayers whose payment exceeds their income due to over-reporting or a fine are bound by their reports, and hence suffer negative income. We assume that the IRS takes as given this tax and penalty structure; its only latitude is in the amount of resources it devotes to income verification activities. This rules out the strategy of (costlessly) imposing immense penalties to compensate for very little (costly) investigative effort. The tax schedule is in fact not subject to IRS control. Neither are penalties, in principle, even though in practice they are often the outcome of negotiation between the auditor and the taxpayer.

Given the taxpayer's report, the IRS must choose a level of effort to devote to income verification and compliance enforcement. This effort is costly, and must be expended even if the taxpayer is known to be under-reporting (the IRS must prove that the individual has not complied, and must expend resources to collect the tax due plus any applicable fines). Thus reports below \( I \) and above \( \tilde{I} \), although they obviously imply noncompliance, are not met with immediate penalties because the imposition and collection of such penalties is costly. The investigation procedure yields a probability of verification which is monotonically increasing in effort, so that we can treat the IRS as choosing a probability of verification, with an associated effort cost. Let \( \rho \) denote the probability of verification.

A strategy for the IRS is a verification policy \( \rho = \rho(x) \), where \( \rho: (-\infty, \infty) \to [0, 1] \). Since any report is permissible, the verification policy must be defined for any possible report. This formulation assumes that the probability of verification is independent of the extent of under-reporting, and that investigation results either in accurate income verification, or no new information (that is, there are no partial discoveries). Both of these assumptions seem strong (or at least unrealistic), and relaxing them would be desirable. However, the latter is unlikely to have much impact on the qualitative nature of our results. Furthermore, since we focus on separating equilibria, in which the IRS can infer a taxpayer's true income from his report, the former is, in effect, already a feature of our model. It is possible, of course, that for a given absolute amount of under-reporting, higher income taxpayers face a lower probability of detection than lower income taxpayers since they may have greater opportunities to evade, but we ignore this possibility for now.

Let \( c(\rho) \) denote the cost to the IRS of sustaining probability \( \rho \) of verification.
We assume that \( c(0) = 0 \), and that \( c(\cdot) \) is twice continuously differentiable for \( \rho \in [0, 1) \) and satisfies the following restrictions: for \( \rho \in [0, 1), \)

1. \( 0 < c'(\rho) < \infty \) and \( 0 < c''(\rho) < \infty \).
2. \( \lim_\rho \frac{c'(\rho)c''(\rho) + \rho}{\rho^2} = \infty \).

Restriction (A3) is a curvature condition which ensures that the marginal cost of verification does not rise too quickly.

Finally, since the IRS does not observe \( I \) directly, it must form beliefs or conjectures which relate reports to types of taxpayers. Let \( \mu(s|x) \) denote the IRS’s prior probability assessment that the true type of a taxpayer who reports \( x \) belongs to the set \( s \subseteq [I, \bar{I}] \). We require that \( \mu([I, \bar{I}]|x) = 1 \); that is, the IRS’s beliefs cannot assign to any report a taxpayer type which is known to be nonexistent.

We assume that both the IRS and the taxpayer are risk-neutral, and maximize expected net revenue and expected net income, respectively. It is possible to relax somewhat the assumption that taxpayers are risk-neutral; another point we discuss more fully in section 6. The assumption that the IRS maximizes expected net revenue (rather than some measure of social welfare reflects our decision to take a positive (rather than normative) approach to government behavior. A widely accepted tenet of modern political theory is that bureaucrats act so as to maximize their own discretionary budgets (Niskanen [1971]). Since net tax revenues represent the discretionary budget for the federal government as a whole, it seems plausible that the fund-raising arm of the federal government, the IRS, should act so as to maximize net tax revenues. Wertz [1979] presents some evidence that this assumption provides a good starting point, although other objective functions are possible. We have analyzed some of these and the results of that effort are summarized in section 6.

Expected net revenue to the IRS when it observes a report of \( x \) and chooses a probability \( \rho \) of verification, conditional upon its beliefs \( \mu(s|x) \), is

\[
R(x, \rho; \mu) = \rho [E_\mu(I|x) + \pi_t(E_\mu(I|x) - x)] + (1 - \rho)tx - c(\rho),
\]

where \( E_\mu(I|x) \) represents the expected value of the taxpayer’s income, given that he reported income \( x \), based upon the IRS’s prior probability assessment \( \mu \); that is, \( E_\mu(I|x) = \int_{I, \bar{I}} I d\mu(I|x) \).

Expected net income to a taxpayer who has true income \( I \) and reports income \( x \), conditional upon the verification policy \( p(\cdot) \), is

\[
N(I, x; p) = p(x) [I - tI - \pi_t(I - x)] + (1 - p(x))(I - tx).
\]

Note that the taxpayer bears no costs when being investigated, suffering a penalty only if noncompliance is ascertained. The functions \( R(x, \rho; \mu) \) and \( N(I, x; p) \) represent the payoffs to the IRS and the taxpayer, respectively.

**Definition 1.** A triple \((\bar{\mu}(s|\cdot), \bar{\rho}(\cdot), \bar{\nu}(\cdot))\) is an equilibrium if
(a) Given the beliefs \( \tilde{\mu}(s; \cdot) \), \( \tilde{p}(x) \) maximizes \( R(x, \rho; \tilde{\mu}) \);
(b) Given the verification policy \( \tilde{p}(\cdot) \), \( \tilde{r}(I) \) maximizes \( N(I, x; \tilde{p}) \); and
(c) \( \tilde{\mu}(s|x) \) and is the conditional probability (under the prior distribution \( F(\cdot) \)) that \( I \in s \cap \tilde{r}^{-1}(x) \) given that \( I \in \tilde{r}^{-1}(x) \), whenever \( \tilde{r}^{-1}(x) \neq \emptyset \).

This definition admits the possibility of pooling equilibria, in which \( \tilde{r}^{-1}(x) \) is set-valued. Although an analysis of pooling equilibria in this context would be interesting, we will not deal with this issue here (for a class of problems in which only pooling equilibria exist, see Crawford and Sobel [1982]). Rather, we will be specifically interested in a separating equilibrium in which \( \tilde{r}(I) \) is monotonically increasing; in this case, \( \tilde{r}^{-1}(x) \) is single-valued. Consequently, we define point beliefs, which assign a unique taxpayer type to each report \( x \). Let \( \tau : (-\infty, \infty) \rightarrow [I, \tilde{I}] \) denote these beliefs. Given the beliefs \( \tau(\cdot) \), we can rewrite the expected net revenue to the IRS as follows.

\[
R(x, \rho; \tau) = \rho[\tau(x) + \pi(\tau(x) - x)] + (1 - \rho)tx - c(\rho).
\]

**Definition 2.** A triple \( (\tilde{\tau}(\cdot), \tilde{p}(\cdot), \tilde{r}(\cdot)) \) is a separating equilibrium if \( \tilde{r}(\cdot) \) is monotone increasing, and

(a) Given the beliefs \( \tilde{\tau}(x), \tilde{p}(x) \) maximizes \( R(x, \rho; \tilde{\tau}) \);
(b) Given the verification policy \( \tilde{p}(x), \tilde{r}(I) \) maximizes \( N(I, x; \tilde{p}) \); and
(c) \( \tilde{\tau}(\tilde{r}(I)) = I \) for all \( I \in [I, \tilde{I}] \).

Alternatively, a consistency condition equivalent to (c) is

\[
\tilde{\tau}(x) = \tilde{r}^{-1}(x) \quad \text{for all} \quad x \in [\tilde{r}(I), \tilde{r} \tilde{I}].
\]

**Necessary conditions for a separating equilibrium.** The IRS maximizes \( R(x, \rho; \tau) \) by a choice of \( \rho = p(x) \). Pointwise optimization requires that

(1) \( R_p(x, p(x); \tau) = t(1 + \pi)(\tau(x) - x) - c'(p(x)) \geq 0 \) \((\leq 0)\) and \( p(x) \leq 1 \) \((\geq 0)\).

When \( p(x) \) is interior, the necessary condition is

(1') \( R_p(x, p(x); \tau) = t(1 + \pi)(\tau(x) - x) - c'(p(x)) = 0. \)

Since \( c''(\rho) > 0 \) for all \( \rho \), equation (1) is necessary and sufficient to determine the optimal verification policy \( p(x) \) (given \( \tau(x) \)).

The taxpayer maximizes \( N(I, x; p) \) by a choice of \( x = r(I) \). If \( p(\cdot) \) is differentiable, then the optimal report (given \( p(x) \)) solves

(2) \[
N_p(I, r(I); p) = p'(r(I))[-t(1 + \pi)(I - r(I))] + p(r(I))t\pi - t(1 - p(r(I))) = 0.
\]

If \( p(\cdot) \) is twice differentiable, then a second-order necessary condition is

(3) \[
N_{xx}(I, r(I); p) = p''(r(I))[-t(1 + \pi)(I - r(I))] + 2p'(r(I))(1 + \pi) \leq 0.
\]

Equations (1), (2) and (3) hold simultaneously at equilibrium. Incorporating the consistency condition that \( I = \tau(x) = \tilde{r}^{-1}(x) \), we can rewrite equations (1'), (2) and (3) as follows:
\begin{align*}
(4) \quad & t(1+\pi)(r^{-1}(x) - x) - c'(p(x)) = 0, \\
(5) \quad & p'(x)\left[-t(1+\pi)(r^{-1}(x) - x)\right] + p(x)t\pi - t(1-p(x)) = 0, \\
(6) \quad & p''(x)\left[-t(1+\pi)(r^{-1}(x) - x)\right] + 2p'(x)t(1+\pi) \leq 0.
\end{align*}

Equations (4) and (5) can be combined to give equation (7) below, an ordinary differential equation:

\begin{equation}
- p'(x)c'(p(x)) + t\pi p(x) - t(1-p(x)) = 0.
\end{equation}

We can rewrite (7) as follows:

\[
p' = \frac{[t\pi p - t(1-p)]/c'(p)}{c'(p)}.
\]

The expression \([t\pi p - t(1-p)]/c'(p)\) is continuously differentiable in \(p\) on \([0, 1]\) under assumption (A1). It then follows that for any given boundary condition \(p_o(a) = b\), where \(a \in (-\infty, \infty)\) and \(b \in [0, 1]\), a unique solution \(p_o(x)\) to (7) exists (at least in a neighborhood of \(x = a\)). Moreover, \(p_o(x)\) will be twice differentiable (Hestenes [1980, p. 49, Theorem 14.1]). The solution \(p_o(\cdot)\) implies an (inverse) reporting policy \(r_o^{-1}(\cdot)\), which can be obtained by solving equation (4). Finally, we need to add appropriate beliefs. Consistency requires that \(\tau_o(x) = r_o^{-1}(x)\) for \(x \in [r_o(I), r_o(\bar{I})]\). What are appropriate beliefs for income levels which would not be reported in equilibrium by any existing type of taxpayer? We think that the most “natural” beliefs would assign the type with the nearest equilibrium report; that is, \(\tau_o(x) = \bar{I}\) for \(x > r_o(\bar{I})\), and \(\tau_o(x) = I\) for \(x < r_o(I)\). However, it is shown in the appendix that any other set of out-of-equilibrium beliefs will generate another equilibrium which is equivalent in terms of observable behavior. The interested reader is also referred to Kreps [1984], who makes rigorous the notion of “appropriate beliefs” in the Spence [1974] signalling model, and thereby generates a unique equilibrium which is a Pareto optimal separating equilibrium.

We will refer to a solution of equation (7), along with its associated reporting policy and beliefs, as a candidate for equilibrium if it also satisfies equation (6); that is, if the second-order necessary condition for the taxpayer’s optimum is satisfied. Note that equation (7) is merely suggestive of candidates for a sequential equilibrium (because at best it embodies necessary, but not sufficient conditions); these candidates must be verified, modified or eliminated by checking whether the implied verification and reporting policies actually are best against each other.

Consider the triple \((\tau_o, p_o, r_o)\). Because \(p_o(x)\) is twice differentiable, \(r_o^{-1}(x)\) will also be differentiable. By construction, the pair \((p_o, r_o^{-1})\) satisfy equations (4) and (5) for all \(x\). Equations (4) and (5) can be differentiated to obtain:

\begin{align*}
(8) \quad & t(1+\pi)(r_o^{-1}(x) - 1) - c''(p_o)p_o'(x) = 0 \\
\text{and} \\
(9) \quad & p_o''(x)\left[-t(1+\pi)(r_o^{-1}(x) - x)\right] + 2p_o'(x)t(1+\pi) \\
& \quad - p_o'(x)t(1+\pi)r_o^{-1}(x) = 0.
\end{align*}

**Lemma 1.** If \((p_o(x), r_o^{-1}(x))\) satisfies (6) with a strict inequality, then \(p_o'(x) < 0\)
and $r_o^{-1}(x) \in (0, 1)$.

**Proof.** From equation (8), either (i) $p_o'(x) > 0$ and $r_o^{-1}(x) - 1 > 0$, or (ii) $p_o'(x) < 0$ and $r_o^{-1}(x) - 1 < 0$. From equation (9) and assuming that inequality (6) is strict, either (iii) $p_o'(x) > 0$ and $r_o^{-1}(x) < 0$ or (iv) $p_o'(x) < 0$ and $r_o^{-1}(x) > 0$. Since (i) and (iii) are mutually inconsistent, it follows that (ii) and (iv) hold. That is, $p_o'(x) < 0$ and $r_o^{-1}(x) \in (0, 1)$.

Q.E.D.

Lemma 1 implies that the candidate $(\tau_o, p_o, r_o)$ for an equilibrium has the property that a taxpayer who reports greater income faces a lower probability of verification. Since $r_o^{-1}(x) = 1/r_o'(I)$, it follows that $r_o'(I) > 1$; that is, under-reported income $I - r_o(I)$ declines with true income. This is perhaps our most fundamental result, but it may strike some readers as counter-intuitive. To see why it makes sense, note that $p_o'(x) < 0$ is natural since it gives taxpayers an incentive to report high income when their true income is high. But ex post optimality with respect to IRS audit activity then requires that $r_o'(I) > 1$, since otherwise the IRS would want to set $p_o'(x) > 0$. Also, it is important to remember that Lemma 1 applies to comparisons within audit classes, not across them. Finally, Lemma 1 is robust to the assumption of progressive taxation but may fail if taxpayers are sufficiently risk-averse. These points are discussed in more detail in section 4.

3. A CONSTRUCTIVE APPROACH TO EQUILIBRIUM

In this section, we use a constructive approach to characterize equilibrium for cost functions which satisfy assumptions (A1), (A2) and (A3). Define $\bar{x} = I - c'(0)/t(1 + \pi)$. Next solve the ordinary differential equation (7) using $p(\bar{x}) = 0$ as a boundary condition. Denote this solution by $p_o(x)$. Next define $r_o^{-1}(x) = x + c'(p_o(x))/t(1 + \pi)$. Note that $r_o^{-1}$ is differentiable. If $p_o(x)$ satisfies the following condition (B), then the pair $(p_o, r_o^{-1})$ satisfies equations (4), (5) and (6) (with a strict inequality).

(B) $-p_o'(x)c'(p_o(x)) + 2p_o(x)t(1 + \pi) < 0$ for all $x \leq \bar{x}$.

Lemma 1 then implies that $p_o'(x) < 0$ and $r_o^{-1}(x) \in (0, 1)$. Thus $r_o^{-1}(x)$ is invertible to obtain $x = r_o(I)$. Define $x \in (-\infty, \bar{x}]$ such that $x = r_o(I)$. Since $r_o^{-1}(\cdot)$ is monotone increasing, if $x$ exists, then it will be unique and $x < \bar{x}$; if no such value exists, define $x = -\infty$.

**Theorem 1.** If $p_o(x)$ exists throughout $[x, \bar{x}]$ and satisfies condition (B), then the following triple is an equilibrium.

(i) The equilibrium verification policy is

$$
\bar{p}(x) = \begin{cases} 
0 & x \geq \bar{x} \\
p_o(x) & x \in [x, \bar{x}] \\
c^{-1}(t(1 + \pi)(I-x)) & x \leq x
\end{cases}
$$
(ii) The equilibrium reporting policy $\bar{r}(I)$ is the unique value of $x \in [x, \bar{x}]$ such that

$$I = \bar{r}(I) + c'(p_0(\bar{r}(I)))/t(1+\pi);$$

that is, $\bar{r}(I) = r_s(I)$ as defined above.

(iii) Finally, the equilibrium beliefs are

Figure 1
\[ \tilde{\tau}(x) = \begin{cases} 
\bar{I} & x \geq \bar{x} \\
\tilde{r}_o^{-1}(x) & x \in [\bar{x}, \bar{x}] \\
\tilde{I} & x \leq \bar{x} 
\end{cases}. \]

The proof of Theorem 1 is rather lengthy and can be found in the Appendix; equilibrium verification and reporting policies are illustrated in Figure 1.

The appendix also shows that alternative out-of-equilibrium beliefs generate separating equilibria which are equivalent to the one described in Theorem 1 in terms of observable behavior: \( \tilde{\tau}(I) \) is identical for all \( I \in [\bar{I}, \bar{I}] \) and \( \tilde{\rho}(x) \) is identical for all \( x \in [\bar{x}, \bar{x}] \).

Let \( T(I) \) denote the expected tax paid by a taxpayer with income \( I \). Since only reports \( x \in [\tilde{\tau}(I), \tilde{\tau}(I)] \) will be observed in equilibrium, it follows that \( \tilde{\rho}(x) = p_o(x) \) for all observed reports \( x \). Then the expected tax for a type \( I \) taxpayer can be written as follows.

\[ T(I) = p_o(r_o(I)) [I + t\tau(I - r_o(I))] + (1 - p_o(r_o(I))) \tau r_o(I). \]

**Corollary 1.** For the equilibrium verification and reporting policies of Theorem 1, \( dT(I)/dI > 0 \) and \( d(T(I)/I)/dI < 0 \) for all \( I \in [\bar{I}, \bar{I}] \).

That is, the expected tax \( T(I) \) increases and the expected average tax rate \( T(I)/I \) decreases with an increase in true income \( I \). Thus the effective tax schedule in the presence of incomplete information \( T(I) \) is regressive, although the statutory tax schedule is linear. The analogue to corollary 1 when taxes are progressive is that the presence of incomplete information makes the effective tax schedule less progressive than the statutory tax schedule.

4. **Examples**

In this section, we compute the equilibrium verification and reporting policies \( \tilde{\rho}(x) \) and \( \tilde{\tau}(I) \) for two specific cost functions. The first is \( c(\rho) = -c\ln(1 - \rho) \), and is important because it satisfies (A1), (A2) and (A3), thus showing that our restrictions on the cost of verification function can be met. The second is \( c(\rho) = c\rho \). The constant cost case is important because even though it fails to satisfy (A1) and (A2) it is a natural choice and yields a particularly simple outcome.

Consider first \( c(\rho) = -c\ln(1 - \rho) \). In this case, the upper limit \( \bar{x} = I - c/(1 + \pi) \). To complete the solution we need only solve equation (7) using as a boundary condition that \( p_o(\bar{x}) = 0 \) and verify that \( p_o \) satisfies condition (B).

Solving equation (7) with this boundary condition yields

\[ p_o(x) = 1 - \pi/[1 + \pi - \exp\{-((\pi/c)(\bar{x} - x))\}]. \]

The associated (inverse) reporting policy is

\[ r_o^{-1}(x) = x + \frac{1 + \pi - \exp\{-((\pi/c)(\bar{x} - x))\}}{(\pi/c)(1 + \pi)}. \]
The lower limit \( \bar{x} \) is defined implicitly by \( I = r_o^{-1}(\bar{x}) \). To see that \( \bar{x} \) exists and is unique, note that \( r_o^{-1}(\bar{x}) = \bar{I} \), \( \lim_{x \to -\infty} r_o^{-1}(x) = -\infty \), and \( r_o^{-1}(x) > 0 \) for all \( x \leq \bar{x} \). Therefore there exists a unique value \( \bar{x} \in (-\infty, \bar{x}) \) such that \( I = r_o^{-1}(x) \).

It is tedious but straightforward to check that

\[
- \frac{\partial(x)c'(p_o(x)) + 2p_o(x)\pi(1+\pi)}{c[1+\pi - \exp\{-t(\pi/c)(\bar{x} - x)\}] < 0 \]

for all \( x \leq \bar{x} \). That is, condition (B) holds for all \( x \leq \bar{x} \).

**Corollary 2.** Let \( c(\rho) = -c\ln(1-\rho) \). Then the following triple is a separating equilibrium.

(i) The verification policy is given by

\[
\bar{\rho}(x) = \begin{cases} 
0 & x \geq \bar{x} \\
1 - \frac{\pi}{1+\pi - \exp\{-t(\pi/c)(\bar{x} - x)\}} & x \in [\bar{x}, \bar{x}] \\
1 - \frac{c}{\pi}(1+\pi)(1-x) & x \leq \bar{x} 
\end{cases}
\]

(ii) The reporting policy \( \bar{r}(I) \) is the unique value of \( x \in [\bar{x}, \bar{x}] \) such that

\[
I = \bar{r}(I) + \frac{1+\pi - \exp\{-t(\pi/c)(\bar{x} - \bar{r}(I))\}}{(t\pi/c)(1+\pi)}, I \in [\bar{I}, \bar{I}] ;
\]

that is, \( \bar{r}(I) = r_o(I) \).

(iii) The beliefs which assign a taxpayer type \( I \) to each observed report \( x \) are given by

\[
\bar{\pi}(x) = \begin{cases} 
I & x \geq \bar{x} \\
r_o^{-1}(x) & x \in [\bar{x}, \bar{x}] \\
I & x \leq \bar{x} 
\end{cases}
\]

The following example demonstrates that we can relax somewhat our assumptions on the costs of verification. Suppose that \( c(\rho) = c\rho \). This cost function is consistent with the interpretation of \( \rho \) as the probability of audit, when auditing exhibits constant returns to scale, but the assumption that \( c''(\cdot) > 0 \) and (A2) are not satisfied for \( c(\rho) = c\rho \). Assumption (A2) was simply for analytical convenience, so that the constraint \( p(x) \leq 1 \) could be ignored. The assumption that \( c''(\cdot) > 0 \) ensured a unique solution to the IRS' optimization problem. In the case considered below, the IRS is indifferent about its verification policy, given the taxpayer's equilibrium reporting policy. Nevertheless, there is a unique equilibrium verification policy.

**Corollary 3.** For \( c(\rho) = c\rho \), let \( \bar{x} = \bar{I} - c/t(1+\pi) \) and \( \bar{x} = I - c/t(1+\pi) \). Then the following triple is a separating equilibrium.

(i) The equilibrium verification policy is
\[
\bar{p}(x) = \begin{cases}
0 & x \geq \bar{x} \\
\frac{1}{1+\pi} \left(1 - \exp \left( \frac{-\tau(1+\pi)}{c} (\bar{x} - x) \right) \right) & x \in [\bar{x}, \bar{x}] \\
1 & x < \bar{x}
\end{cases}
\]

(ii) The equilibrium reporting policy is \( \bar{r}(I) = I - c/I (1+\pi) \), for \( I \in [\bar{I}, \bar{I}] \).

(iii) The equilibrium point beliefs are

\[
\bar{\tau}(x) = \begin{cases}
\bar{I} & x \geq \bar{x} \\
x + c/I (1+\pi) & x \in [\bar{x}, \bar{x}] \\
I & x \leq \bar{x}
\end{cases}
\]

Proof. For \( x \geq \bar{x} \), \( \bar{\tau}(x) = \bar{I} \), so \( R_\rho(0, x; \bar{\tau}) \leq 0 \), which implies that \( \bar{p}(x) = 0 \) for \( x \geq \bar{x} \). For \( x \in [\bar{x}, \bar{x}] \), \( R_\rho(\rho, x; \bar{\tau}) = 0 \) for all \( \rho \). Since the IRS is indifferent about its strategy, the policy given above is a best reply (and is found by solving equation (7) with the boundary condition \( p(\bar{x}) = 0 \)). For \( x < \bar{x} \), \( \bar{\tau}(x) = I \) and \( R_\rho(\rho, x; \bar{\tau}) > 0 \) for all \( \rho \). Thus \( \bar{p}(x) = 1 \) for \( x < \bar{x} \). To see that \( \bar{r}(\cdot) \) given above is a best reply to \( \bar{p}(\cdot) \), note that reports \( x < \bar{x} \) are dominated by \( \bar{x} \), while reports \( x > \bar{x} \) are dominated by \( \bar{x} \). Thus the best report is the solution to \( N_\rho(x, I; \bar{p}) = 0 \) (provided \( N_{xx} < 0 \) at that point). Solving \( N_\rho = 0 \) implies \( \bar{r}(I) = I - c/I (1+\pi) \), and it is easy to check that \( N_{xx}(\bar{r}(I), I; \bar{p}) < 0 \).

Q. E. D.

5. DIFFERENT CLASSES OF TAXPAYERS

We have seen in sections 3 and 4 that when equilibrium verification and reporting policies are (at least partially) characterized by equation (7), then the extent of under-reporting, \( I - \bar{r}(I) \), falls as true income \( I \) rises, and the verification policy \( \bar{p}(x) \) declines as reported income \( x \) rises.

However, our analysis assumed that all individuals (or types of taxpayers) were \textit{ex ante} indistinguishable. If some other characteristic of individuals is observable, then the IRS can, in some cases, condition its verification policy upon this observable characteristic. Such characteristics may be relatively immutable (e.g., sex or race), or subject to choice but largely determined by other considerations (e.g., occupation or place of residence). For instance, suppose that two individuals, one residing in Beverly Hills and the other in Death Valley, report the same taxable income \( x \). Since the \textit{ex ante} distribution of income opportunities is likely to differ in a predictable way between these two locales, the equilibrium verification policy (and consequently the equilibrium reporting policy) should differentiate between these two identical reports.

Since we focus on separating equilibria, the only relevant aspect of the distribution of types is its support (i.e., the form of the distribution appears nowhere in Theorem 1). Thus, we can consider changing \( \bar{I}, I \) or both. For simplicity, suppose that \( I \) is unchanged. We will refer to taxpayers with different values of
as belonging to different classes of taxpayers; types of taxpayers will then vary within each class.

The following Lemma, which characterizes the relative positions of the functions

\[ p(x) = p_0(x) \text{ and } p(x) = c^{-1}(t(1 + \pi)(I - x)) \]

will prove useful later.

**Lemma 2.** Suppose \( p_0(x) \) is a solution to equation (7) which also satisfies condition (B). Then

(a) \( p_0(x) > c^{-1}(t(1 + \pi)(I - x)) \) if \( x > x' \);
(b) \( p_0(x) = c^{-1}(t(1 + \pi)(I - x)) \) if \( x = x' \); and
(c) \( p_0(x) < c^{-1}(t(1 + \pi)(I - x)) \) if \( x < x' \).

**Proof.** \( p(x) = p_0(x) \) solves equation (1') with \( \tau(x) = r_o^{-1}(x) \) and \( p(x) = c^{-1}(t(1 + \pi)(I - x)) \) solves (1') with \( \tau(x) = I \). The solution to (1') is increasing with inferred income \( \tau(x) \). Since \( r_o^{-1}(x) \) is monotone increasing with \( r_o^{-1}(x) = I \), it follows that \( r_o^{-1}(x) > I \) if \( x > x' \), \( r_o^{-1}(x) = I \) if \( x = x' \), and \( r_o^{-1}(x) < I \) if \( x < x' \). The claim follows.

Q. E. D.

We can now consider the effect of increasing \( I \) upon the equilibrium verification and reporting policies. To do this, explicitly denote this dependence as follows: \( \bar{p}(x; \bar{I}) \) and \( \bar{\tau}(I; \bar{I}) \). A solution to equation (7) through 0 at \( \bar{x} = I - c'(0)/t(1 + \pi) \) is denoted \( p_o(x; \bar{I}) \). Since the solution to (7) is unique through any given value of \( p \in [0, 1] \), \( p_o(x; \bar{I}) \) and \( p_o(x; I') \) cannot cross, for any \( I \neq I' \). Since \( p_o'(x; I) < 0 \), it follows that \( p_o(x; \bar{I}) \) is increasing in \( I \). That is, if \( I' > \bar{I} \), then \( p_o(x; I') > p_o(x; \bar{I}) \) for all \( x < \bar{x}' = I' - c'(0)/t(1 + \pi) \).

Recall that the inverse reporting policy

\[ r_o^{-1}(x; \bar{I}) = x + c'(p_o(x; \bar{I}))/t(1 + \pi) \]

is increasing; that is, \( r_o^{-1}(x; \bar{I}) > 0 \). The reporting policy \( \bar{\tau}(I; \bar{I}) \) is defined by \( r_o^{-1}(\bar{\tau}(I; \bar{I}); \bar{I}) = 0 \). Since \( r_o^{-1}(x; \bar{I}) > 0 \) and \( r_o^{-1}(x; \bar{I}) \) is increasing in \( \bar{I} \), it follows that \( \bar{\tau}(I; \bar{I}) \) is decreasing in \( \bar{I} \). Thus for any given level of true income \( I \), reported income falls as \( \bar{I} \) rises; alternatively put, unreported income \( I - \bar{\tau}(I; \bar{I}) \) rises with \( \bar{I} \). Since \( \bar{x} = \bar{\tau}(I; \bar{I}) \), we see that \( \bar{x} \) also falls as \( \bar{I} \) rises. Thus if \( I' > \bar{I} \), then \( \bar{x}' < \bar{x} \).

**Theorem 2.** If the equilibrium verification and reporting policies are as described in Theorem 1, then for \( I' > \bar{I} \),

(a) \( I - \bar{\tau}(I; I') > I - \bar{\tau}(I; \bar{I}) \) for \( I \in [I, \bar{I}] \); and
(b) \( \bar{p}(x; I') > \bar{p}(x; \bar{I}) \) for \( x \in (x', \bar{x}) \); the equilibrium verification policies coincide outside this interval.

Figure 2 illustrates these results.

**Proof.** We can partition the interval \( (-\infty, \infty) \) into five sets: \( (-\infty, x'] \), \( (x', \bar{x}] \), \( (x, \bar{x}] \), \( [\bar{x}, \bar{x}'] \) and \( [\bar{x}', \infty) \). For \( x \in (-\infty, x'] \), the optimal verification policies agree: \( \bar{p}(x; I') = \bar{p}(x; \bar{I}) = c^{-1}(t(1 + \pi)(I - x)) \). For \( x \in (x', \bar{x}] \), \( \bar{p}(x; I') = p_o(x; I') > c^{-1}(t(1 + \pi)(I - x)) = \bar{p}(x; \bar{I}) \). The inequality follows from Lemma 2.
For $x \in (x, \bar{x})$, $\bar{p}(x; \bar{I}) = p_o(x; \bar{I}) > p_o(x; \bar{I}) = \bar{p}(x; \bar{I})$. For $x \in [\bar{x}, \bar{x}']$, $\bar{p}(x; \bar{I}) = p_o(x; \bar{I}) > 0 = \bar{p}(x; \bar{I})$. Finally, for $x \in [\bar{x}, \infty)$, the policies again agree, with $\bar{p}(x; \bar{I}) = \bar{p}(x; \bar{I}) = 0$.

That is, if two individuals have the same true income $I$, then that individual with the larger value of $\bar{I}$ will fail to report a greater amount of income than the
one with the smaller value of \( \bar{I} \), and will accordingly find a greater amount of effort devoted to his investigation. Alternatively, the individual who resides in Beverly Hills, but reports the same income as a resident of Death Valley, will (be inferred to) have concealed more income, and is more likely to be investigated, than his counterpart who resides in Death Valley.

Another obvious solution to equation (7) exists (which corresponds, at least in our examples, to the boundary condition \( p_s(\bar{x}) = 1/(1 + \pi) \)). This is the constant solution \( \bar{\rho} = 1/(1 + \pi) \); to see that this policy and its implied reporting policy \( \bar{r}(I) \) do not yield a separating equilibrium for cost functions which satisfy our domain restrictions, recall that we have ruled out cost functions with the property that \( c'(0) = 0 \). This was done initially to ensure that equation (7) had a unique solution for the given boundary condition; \( c'(0) = 0 \) is also ruled out by (A3).

Solving equation (4) for the (inverse) reporting policy associated with \( \bar{\rho}(x) = 1/(1 + \pi) \) yields

\[
\bar{r}^{-1}(x) = x = c'(1/(1 + \pi))/(1 + \pi).
\]

Let \( \alpha = c'(1/(1 + \pi))/(1 + \pi) \). Then the putative equilibrium has \( \bar{r}(I) = I - \alpha \) for \( I \in [\bar{I}, \bar{I}] \), and \( \bar{r}(x) = x + \alpha \) for \( x \in [\bar{I} - \alpha, \bar{I} - \alpha] \), while \( \bar{r}(x) = \bar{I} \) for \( x \leq \bar{I} - \alpha \) and \( \bar{r}(x) = \bar{I} \) for \( x \geq \bar{I} - \alpha \). To calculate the remainder of the putative equilibrium, we compute the optimal verification policy \( p(x) \) given \( \bar{r}(x) \).

Again let \( \bar{x} = \bar{I} - c'(0)/(1 + \pi) \). Then for \( x > \bar{I} - \alpha \), \( \bar{r}(x) = \bar{I} \). Since

\[
R_{\rho}(x, 0; \bar{r}) = t(1 + \pi)(\bar{I} - x) - c'(0) < 0
\]

for all \( x > \bar{x} \), it follows that \( \bar{\rho}(x) = 0 \) for \( x \geq \bar{x} \), while \( \bar{\rho}(x) \) solves

\[
R_{\rho}(x, \bar{\rho}(x); \bar{r}) = t(1 + \pi)(\bar{I} - x) - c'(\bar{\rho}(x)) = 0
\]

for \( x \in [\bar{I} - \alpha, \bar{x}] \). This set is non-empty because \( c''(\cdot) > 0 \) implies that \( \bar{x} = \bar{I} - c'(0)/(1 + \pi) > \bar{I} - c'(1/(1 + \pi))/(1 + \pi) = \bar{I} - \alpha \). Thus \( \bar{\rho}(x) = c'^{-1}(t(1 + \pi)(\bar{I} - x)) \) for \( x \in [\bar{I} - \alpha, \bar{x}] \). For \( x \in [\bar{I} - \alpha, \bar{I} - \alpha] \), \( \bar{\rho}(x) = \bar{\rho}(x) = 1/(1 + \pi) \). Finally, for \( x \leq \bar{I} - \alpha \), \( \bar{r}(x) = \bar{I} \), so \( \bar{\rho}(x) \) solves

\[
R_{\rho}(x, \bar{\rho}(x); \bar{r}) = t(1 + \pi)(\bar{I} - x) - c'(\bar{\rho}(x)) = 0
\]

or \( \bar{\rho}(x) = c'^{-1}(t(1 + \pi)(\bar{I} - x)) \) for \( x \leq \bar{I} - \alpha \).

To see that this combination does not constitute a separating equilibrium for cost functions satisfying our domain restrictions, note that \( N(I, \bar{r}(I); \bar{p}) = I(1 - t) \), while \( N(I, \bar{x}; \bar{p}) = I - t\bar{x} \). Thus \( N(I, \bar{x}; \bar{p}) > N(I, \bar{r}(I); \bar{p}) \) for all \( I \in (\bar{I} - c'(0)/t(1 + \pi), \bar{I}] \). Since we require \( c'(0) > 0 \), this set is non-empty. Thus a non-empty subset of taxpayer types would prefer to deviate from the "equilibrium" reporting policy \( \bar{r}(I) \) and to convey a report which falls outside the interval of "equilibrium" reports \( [\bar{r}(I), \bar{r}(\bar{I})] = [\bar{I} - \alpha, \bar{I} - \alpha] \).

Note that this argument succeeds because we have assumed \( \bar{I} < \infty \); that is, opportunities for income are bounded above. However, as \( \bar{I} \to \infty \), the interval \( (\bar{I} - c'(0)/t(1 + \pi), \bar{I}] \to \phi \); moreover, \( p_s(x; \bar{I}) \) converges to \( \bar{\rho}(x) \).
Lemma 3. \( \lim_{I \to \infty} p_s(x; I) \equiv p_s^\infty(x) = 1/(1 + \pi) \) for all \( x \in (-\infty, \infty) \).

**Proof.** Since \( p_s(\bar{x}; I) = 0 \) and \( p_s^\infty(\bar{x}; I) > 0 \) for all \( x < \bar{x} \), since \( p_s^\infty(x; I) \) increases with \( I \), taking the limit as \( I \to \infty \) gives \( p_s^\infty(x) = 0 \) for all \( x < \infty \). Moreover, from equation (7), we know that

\[
p_s(x; I) = 1/(1 + \pi) + p_s^\infty(x; I) + c(p_s(x; I))/t(1 + \pi) < 1/(1 + \pi).
\]

Now suppose that \( p_s^\infty(y) < 1/(1 + \pi) \) for some \( y < \infty \). Then we will show that \( p_s^\infty(z) = 0 \) for some \( z < \infty \), which is a contradiction.

Since \( p_s^\infty(y) < 1/(1 + \pi) \) and \( p_s^\infty(\cdot) \) satisfies equation (7),

\[
p_s^\infty(y) = [t(1 + \pi)p_s^\infty(y) - c(p_s^\infty(y)) < 0.
\]

Thus \( p_s^\infty(x) < 1/(1 + \pi) \) and \( p_s^\infty(x) < 0 \) for all \( x \geq y \). These imply that \( p_s^\infty(x) < 0 \) for all \( x \geq y \) as well. Thus there exists \( x < \infty \) at which \( p_s^\infty(z) = 0 \). This contradiction implies that \( p_s^\infty(x) = 1/(1 + \pi) \) for all \( x \). Q. E. D.

The convergence of \( p_s(x; I) \) to \( p_s(\cdot) = 1/(1 + \pi) \) suggests the following result; the proof is straightforward and will be omitted.

**Theorem 3.** The following triple constitutes a separating equilibrium when \( I = -\infty \) and \( I = \infty \). For \( x = c'(1/(1 + \pi))/(t(1 + \pi)) \),

(i) The equilibrium verification policy is \( \bar{p}(x) = 1/(1 + \pi) \) for \( x \in (-\infty, \infty) \);

(ii) The equilibrium reporting policy is \( \bar{r}(I) = I - \alpha \) for \( I \in [I, \overline{I}] \); and

(iii) the equilibrium point beliefs are \( \bar{\xi}(x) = x + \alpha \) for \( x \in (-\infty, \infty) \).

This equilibrium has the feature that individuals under-report by a constant amount, which renders optimal a constant verification policy. Given this verification policy, taxpayers are actually indifferent among all reports, but only the reporting policy specified above will support the constant verification policy. Previous work on tax compliance has generally assumed a probability of audit which is independent of reported income. This result describes some circumstances under which this assumption is valid. However, these circumstances are rather extreme, requiring that income opportunities are literally unbounded in both directions.

6. **Conclusion**

This paper presents a model of tax compliance as a game of incomplete information. The sequence of moves has the taxpayer first reporting his income, and the IRS subsequently acting optimally on the basis of this report. Thus the IRS is not permitted to make non-credible threats about its verification policy. We find that an equilibrium verification policy involves devoting greater resources to verification for those taxpayers reporting lower levels of income. The equilibrium reporting rule for taxpayers implies that taxpayers with greater income under-
report less than those with lower income. We also find that although the statutory
form of the tax schedule is linear, the effective tax schedule under incomplete
information is regressive in the sense that the expected average tax rate declines
as income rises. For the special case of unbounded opportunities for income, we
find that the equilibrium verification policy will be constant, and taxpayers will
under-report by a constant amount.

It would be useful to compare these results to the optimal IRS audit policy and
associated taxpayer reporting rule when the IRS is allowed to precommit.
Unfortunately, the latter problem remains unsolved so that a full comparison
is impossible [see, however, Border and Sobel, 1985, and Reinganum and Wilde,
1985a]. However, it is clear that even with precommitment, the optimal audit
rule should decline with reported income. The difference between the approaches
is that the extent of under-reporting need not decline with true income if sequential
rationality is not imposed.

While our fundamental result (Lemma 1) may perhaps strike some readers as
counter-intuitive, at least initially, it is important to remember that it is derived
by considering only a single class of taxpayers; that is, under the assumption that
one cannot distinguish among taxpayers ex ante on the basis of some other
observable characteristic. When an observable characteristic which is related
to ex ante opportunities for high income is available, we find that those classes
of taxpayers with greater income opportunities (in terms of maximum possible
income) fail to report a greater amount of their income and face harsher verifica-
tion policies. That is, if two taxpayers have the same true income, then the
one who ex ante enjoyed a better range of income opportunities will report less
income; for any two taxpayers who report the same level of income, more effort
will be devoted to income verification for the taxpayer who ex ante enjoyed a
better range of income opportunities.

We have also derived a number of results not presented formally in this paper.
For example, a uniform decrease in the marginal cost of verification leads to an
increase in efforts devoted to verification, but the net effect of this change on
equilibrium compliance is ambiguous. We have also examined how robust our
fundamental result (Lemma 1) is to various underlying assumptions. It turns
out that progressive taxation has no effect on it; the optimal audit rule is still
decreasing in reported income and the associated taxpayer reporting rule still
requires under-reporting to decrease with actual income. Risk averse taxpayers,
however, can be problematic. A sufficient condition for Lemma 1 to hold in
this case is $t(1 + \pi) \geq 1$. Since this is only a sufficient condition, Lemma 1 might
still hold if it fails, but it may not. A problem could arise, for example, if absolute
risk aversion declines rapidly with income, as the combination $p'_a(x) < 0$ and $r'_a(I) >$
1 might not be possible in this case. However, since Lemma 1 deals with
comparisons within audit classes, absolute risk aversion may not vary enough
on the relevant range of incomes to create such a problem.

Finally, we have also considered other objective functions for the IRS. For
example, suppose the IRS only wants to maximize the expected amount of
unreported income it discovers in the audit process, net of audit costs. Then if 
$c(\rho) = c\rho$, we have

$$R(x, \rho; \tau) = \rho[\tau(x) - x] - c\rho.$$ 

In this case it can be shown that

$$\bar{p}(x) = \begin{cases} 
0 & x \geq \bar{x} \\
\frac{1}{1 + \pi} \left(1 - \exp\left(-\bar{x} - x)/c\right)\right) & x \in [x, \bar{x}] \\
1 & x < \bar{x}
\end{cases}$$

where \(\bar{x} = \bar{I} - c\), \(x = I - c\), and

$$\bar{r}(I) = 1 - c.$$  

Thus neither auditing nor reporting depend on the tax rate, and the penalty rate
only effects the audit rule; all taxpayers under-report by exactly the cost of an audit. Whether auditing increases or decreases compared to our original specification of IRS objectives depends on whether \(r(1 + \pi)\) is less than or greater than 1.

On a more technical level, our results have been derived under fairly stringent
restrictions on the cost function \(c(\cdot)\); in particular, we assumed that (A1), (A2)
and (A3) held. Corollary 3 showed that the restriction (A2) could be relaxed.
Assumptions (A1) and (A3) are objectionable in that they jointly rule out the class of cost functions in which \(c'(0) = 0\), and possibly other natural classes of cost
functions. The former assumption was used to assert the uniqueness of \(p_a(\cdot)\)
through the given boundary condition, and again in the proof that the constant
solution \(\bar{p}(x)\) does not generate an equilibrium. The latter was used in the proof
that \(p_a(x)\) does generate an equilibrium. A desirable extension of this work would
include the elimination of these restrictions. Although this might result in
multiple solutions to equation (7), for particular cases one can always directly
verify or reject these solutions as equilibria.

There are also technical issues regarding the uniqueness of the equilibrium we
analyze. We have shown elsewhere in a related model (Reinganum and Wilde
[1985b]) that the equilibrium given by \(p_a(\bar{x}) = 0\) is the unique separating
equilibrium when audit costs are constant. We have no analogous results for
cost functions satisfying (A1), (A2) and (A3) of this paper. Neither have we been
able to establish any results regarding pooling equilibria. Both of these problems
remain as topics for further research.

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APPENDIX

PROOF OF THEOREM 1. Since \(\bar{r}(I) = r_a(I) = \bar{x}\) and \(\bar{r}(I) = r_a(I) = x\) and since \(r_a\) is
invertible, $\bar{\tau}(x)$ satisfies the consistency requirement.

Given $\bar{\tau}(\cdot)$, we next show that $\bar{p}(x)$ maximizes $\mathcal{R}(x, \rho; \bar{\tau})$. For $x \geq \bar{x}$, $\bar{\tau}(x) = \bar{I}$. Then

$$R_\rho(x, 0; \bar{\tau}) = t(1 + \pi)(\bar{I} - x) - c'(0) < 0$$

for all $x > \bar{x}$,

since $R_\rho(\bar{x}, 0; \bar{\tau}) = 0$. Thus $\bar{p}(x) = 0$ for $x = \bar{x}$. For $x \in [x, \bar{x}]$, $\bar{\tau}(x) = r_\rho^{-1}(x)$.

Then

$$R_\rho(x, 0; \bar{\tau}) = t(1 + \pi)(r_\rho^{-1}(x) - x) - c'(0) = c'(p_\rho(x)) - c'(0).$$

Since $p_\rho(\bar{x}) = 0$ and $p_\rho'(x) < 0$, $p_\rho(x) > 0$ for all $x < \bar{x}$.

Thus $c''(\cdot) > 0$ implies that $c'(p_\rho(x)) - c'(0) > 0$ for all $x < \bar{x}$.

Thus $\bar{p}(x)$ is interior for $x < \bar{x}$. Solving $R_\rho(x, \bar{p}(x); \bar{\tau}) = 0$ yields $c'(p_\rho(x)) - c'(\bar{p}(x)) = 0$, or

$$\bar{p}(x) = p_\rho(x) \quad \text{for} \quad x \in [x, \bar{x}].$$

For $x \leq x$, $\bar{\tau}(x) = \bar{I}$. Then

$$R_\rho(x, 0; \bar{\tau}) = t(1 + \pi)(\bar{I} - x) - c'(0) > 0$$

and $R_\rho(x, 0; \bar{\tau})$ is decreasing in $x$, so $R_\rho(x, 0; \bar{\tau}) > 0$ for all $x \leq x$.

Solving

$$R_\rho(x, \bar{p}(x); \bar{\tau}) = t(1 + \pi)(\bar{I} - x) - c'(\bar{p}(x)) = 0$$

yields $\bar{p}(x) = c'^{-1}(t(1 + \pi)(\bar{I} - x))$ for $x \leq \bar{x}$.

We now show that, given $\bar{p}(\cdot)$, $\bar{\rho}(I)$ maximizes $N(I, x; \bar{p})$. Note that $N$ is continuous in $x$ because $\bar{p}(\cdot)$ is continuous. Clearly any report of $x > \bar{x}$ is dominated by a report of $x = \bar{x}$. This is because the taxpayer is not investigated at all, and thus pays a tax based only on reported income. For $x < \bar{x}$, $N$ is differentiable with

$$N_x(I, x; \bar{p}) = \bar{p}'(x)[ - t(1 + \pi)(\bar{I} - x)] + t\pi\bar{p}(x) - t(1 - \bar{p}(x))$$

$$= c'^{-1}(t(1 + \pi)(\bar{I} - x)) [t(1 + \pi)]^2(\bar{I} - x) + t\pi\bar{p}(x) - t(1 - \bar{p}(x)).$$

Since $t(1 + \pi)(\bar{I} - x) = c'(\bar{p}(x))$ and since $c'^{-1}(c'(\bar{p}(x))) = 1/c''(\bar{p}(x))$, we can evaluate $N_x$ at $\bar{I}$ to obtain

$$N_x(I, \bar{I}; \bar{p}) = t(1 + \pi)c'(\bar{p}(x))/c''(\bar{p}(x)) + t(1 + \pi)\bar{p}(x) - t > 0$$

by (A3). Since $N_x(I, x; \bar{p})$ is increasing in $I$, $N_x(I, x; \bar{p}) > 0$ for $x < \bar{x}$, for all $I \in [I, \bar{I}]$. Thus any report $x < \bar{x}$ is dominated by a report of $x = \bar{x}$. Finally, for reports $x \in [\bar{x}, \bar{I}]$, $N$ is differentiable with

$$N_x(I, x; \bar{p}) = p_\rho'(x)[ - t(1 + \pi)(\bar{I} - x)] + t\pi p_\rho(x) - t(1 - p_\rho(x)).$$

Note that $N_x(I, x; \bar{p}) = 0$ because $I - \bar{x} = c'(p_\rho(x))/t(1 + \pi)$ and because $p_\rho(x)$ satisfies equation (7). Since $N_x(I, x; \bar{p})$ is increasing in $I$, $N_x(I, x; \bar{p}) < 0$ for all $I > \bar{I}$. Similarly, $N_x(I, \bar{x}; \bar{p}) = 0$, and since $N_x$ is increasing in $I$, $N_x(I, \bar{x}; \bar{p}) < 0$ for all $I < \bar{I}$. Thus $\bar{\rho}(I) \in (x, \bar{x})$ for all $I \in (\bar{I}, \bar{I})$. Since $N(I, x; p)$ is differentiable in $x$ and its maximum is attained at an interior point of $[\bar{x}, \bar{x}]$ for $I \in [I, \bar{I}]$, $\bar{\rho}(I)$
must solve $N_x(I, \tilde{r}(I); \bar{p}) = 0$. To see that the value of \( \tilde{r}(I) \) so defined is unique and provides a global maximum of $N(I, x; \bar{p})$, note that $N_x(I, x; \bar{p})$ is differentiable in $x$ on $(x, \bar{x})$ and that $N_{xx}(I, x; \bar{p}) < 0$ whenever $N_x(I, x; \bar{p}) = 0$. That is,

$$N_{xx}(I, \tilde{r}; \bar{p}) = 2p_\alpha'(\tilde{r})(1 + \pi) - p_\alpha''(\tilde{r})(1 + \pi)(I - \tilde{r})$$

$$= 2p_\alpha'(\tilde{r})(1 + \pi) - p_\alpha''(\tilde{r})c'(p_\alpha(\tilde{r})) < 0,$$

where the second equality follows from the definition of $\tilde{r}(\cdot)$ and the inequality follows from condition (B).

Thus any stationary point of $N(I, x; \bar{p})$ provides a local maximum on $(x, \bar{x})$ (and dominates any report outside this interval by previous arguments). Suppose there were two such local maxima, provided by $\tilde{r}$ and $\hat{r}$. Then there would also have to be a local minimum provided by (say) $\hat{r}$, at which $N_x(I, \hat{r}; \bar{p}) = 0$ and $N_{xx}(I, \hat{r}; \bar{p}) \geq 0$. But this is a contradiction. Hence $\tilde{r}(I)$ as defined by $N_x(I, \tilde{r}; \bar{p}) = 0$ is unique and provides a global maximum of $N(I, x; \bar{p})$ for $x \in (-\infty, \infty)$.

Solving $N_x(I, x; \bar{p}) = 0$ for $x = \tilde{r}(I)$ yields

$$\tilde{r}(I) = I - [p_\alpha(\tilde{r}(I)) + t(1 - p_\alpha(\tilde{r}(I)))]/p_\alpha'(\tilde{r}(I))(1 + \pi).$$

Because $p_\alpha(\cdot)$ satisfies equation (7), this reduces to

$$\tilde{r}(I) = I - c'(p_\alpha(\tilde{r}(I)))/t(1 + \pi); \text{ that is, } \tilde{r}(I) = r_\alpha(I).$$

Q.E.D.

**THEOREM 1'**. Let $\mu(s|x)$, for $s \subseteq [I, \bar{I}]$, be arbitrary beliefs for $x < x$ or $x > \bar{x}$. Then under the same hypotheses as Theorem 1, the following triple is a separating equilibrium.

(i) The equilibrium verification policy is

$$\bar{p}_\mu(x) = \begin{cases} 
0 & x \geq \bar{x} \\
p_\mu(x) & x \in [x, \bar{x}] \\
c^{-1}(t(1 + \pi)(E_\mu(I|x) - x)) & x < x
\end{cases}$$

(ii) The equilibrium reporting policy is $\bar{r}_\mu(I) = r_\alpha(I)$, for $I \in [I, \bar{I}]$.

(iii) The equilibrium beliefs are

$$\bar{r}_\mu(x) = \begin{cases} 
E_\mu(I|x) & x > \bar{x} \\
r_\alpha^{-1}(x) & x \in [x, \bar{x}] \\
E_\mu(I|x) & x < x
\end{cases}$$

**PROOF OF THEOREM 1'**. For $x > \bar{x}$,

$$R_\mu(x, 0; \bar{r}_\mu) = t(1 + \pi)(E_\mu(I|x) - x) - c'(0) \leq t(1 + \pi)(I - x) - c'(0) < 0,$$
where the first inequality follows from the fact that $E_p(I|x) \leq I$, and the second follows from the definition of $\bar{x}$. Thus $\tilde{p}_\rho(x) = 0$ for all $x > \bar{x}$. For $x \in [x, \bar{x}]$, $R_p(x, 0; \tilde{\tau}_\mu) \geq 0$ so solving $R_p(x, \rho; \tilde{\tau}_\mu) = 0$ gives $\tilde{p}_\rho(x) = p_\rho(x)$. For $x < \bar{x},$

\[
R_p(x, 0; \tilde{\tau}_\mu) = t(1 + \pi)(E_p(I|x) - x) - c'(0) \\
\geq t(1 + \pi)(I - x) - c'(0) > 0,
\]

where the first inequality follows from $E_p(I|x) \geq I$ (for a justification of the second inequality, see the proof of Theorem 1 above). Thus $\tilde{p}_\rho(x)$ solves $R_p(\rho, x; \tilde{\tau}_\mu) = 0$; that is, $\tilde{p}_\rho(x) = c^{-1}(t(1 + \pi)(E_p(I|x) - x))$.

To see that $\tilde{p}_\rho(I) = r_\rho(I)$ maximizes $N(x, I; \tilde{p}_\mu)$ we need only check that reports $x < \bar{x}$ are still dominated by $x = \bar{x}$. Since $E_p(I|x) \geq I$, it is clear that $\tilde{p}_\rho(x) \geq \bar{p}(x)$. Moreover, since $N_p = -t(1 + \pi)(I - x) < 0$, it follows that $N(x, I; \tilde{p}_\rho) \leq N(x, I; \bar{p})$ for $x < \bar{x}$. That is, reports below $\bar{x}$, which were already dominated under $\bar{p}(\cdot)$, are even less attractive under $\tilde{p}_\rho(\cdot)$.

Q.E.D.

REFERENCES


