Tax Evasion, Concealment and the Optimal Linear Income Tax*

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Abstract
This paper incorporates tax evasion into the analysis of optimal linear income taxation. Tax-evaders can influence the probability of being caught, if audited, through expenditures on concealment. It is proved that tax evasion can make a given tax system more as well as less progressive depending on the “concealment technology”. The paper derives and interprets simple formulas for characterizing the optimal tax rate and audit probability. It also gives sufficient conditions under which tax evasion lowers the optimal tax rate, while showing that an increase in the optimal tax rate is also possible.

I. Introduction
The subject matter of tax evasion is not a simple academic curiousus. In a survey of this literature, Marelli (1987) comes to the conclusion that “The hidden economy is a profound phenomenon of our times; however measured, however defined, one conclusion is common to all the authors who attempted to deal with it: the problem of the hidden economy cannot be dismissed as quantitatively trivial” [p. 221]. More recently, using individual data from the Internal Revenue Service Taxpayer Compliance Measurement Programs, Feinstein (1991) estimates the income tax evaded in the U.S. for 1987 to have been $83.7 billion and for 1982, $63.4 billion.

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This paper attempts to incorporate tax evasion into the optimal linear income tax literature. It distinguishes itself from earlier studies on this subject, e.g. Sandmo (1981) and Cowell (1990), by considering a model in which taxpayers can influence the probability of being caught cheating on their taxes. Specifically, we assume that the probability that an audit reveals a taxpayer’s true income depends on how much money he spends to conceal his cheating.

In the optimal income tax literature, starting with Mirrlees’ (1971) classic paper, the government chooses income tax rate(s) to maximize social welfare. The underpinning of this theory is the assumption that while the government cannot separate taxpayers out on the basis of their differing abilities (so as to impose optimum lump-sum taxes), it can distinguish between them on the basis of their earned income. Consequently, it raises its required revenues through the taxation of income. This, in turn, distorts taxpayers’ work and consumption decisions. The optimum income tax strikes the “right” balance between efficiency and government’s redistributive considerations. This literature devotes particular attention to the question of the progressivity of optimal income taxes.

The literature on tax evasion makes the point that division of the variables in an economic system into publicly known (i.e., earned income) and publicly unknown (i.e., earning ability) is rather arbitrary and in fact unnecessary. (The original papers are Allingham and Sandmo (1972), Kolm (1973) and Srivivasan (1973); see Cowell (1990) for a survey.) This literature is built around the fact that the government can obtain the information it needs (on incomes) at a cost, i.e., by conducting audits. This cost is assumed to be sufficiently high so that auditing every taxpayer is not optimal. As a result, an individual’s tax liability is based on his reported income, with a certain number of audits being performed in order to assure some degree of compliance.

The notion that taxpayers’ true incomes may be observed publicly only through costly audits has important implications for the question of optimal tax design. In particular, it implies that the set of government policy tools include an audit strategy as well as tax rates. This opens up interesting questions regarding the optimal audit strategy, interaction of tax rates and audit strategy, reformulation of the optimal income tax schedule, particularly the degree of progressivity of the income tax, and the tradeoff between efficiency and equity considerations. We derive and interpret simple formulas for characterizing the optimal tax rate and audit probability. As for the progressivity question, we first prove that depending on the “concealment technology”, the presence of tax evasion and concealment cost can make a given tax system more as well as less progressive. We then address the issue in the context of the government’s optimal tax
design. We derive sufficient conditions under which the presence of tax evasion leads to a lower optimal (marginal) tax rate, but show that an increase in the optimal tax rate is also possible.

The closest precursor to our study is Sandmo (1981). He considers a model consisting of two types of workers, with only one type being able to underreport his income. In this paper, we assume a continuum of individuals with different earning abilities. Moreover, we do not introduce any asymmetry among taxpayers and allow everyone to evade taxes if he so chooses. We also relax two of Sandmo’s other assumptions: (i) that there are no costs associated with concealing tax evasion, and (ii) that taxpayers cannot influence the probability of being caught cheating on their taxes.

Our treatment of concealment technology sheds some new light on the elusive distinction between “tax evasion” and “tax avoidance”, at least for the purpose of optimal tax design. According to Cowell (1990): “we see that ‘evasion’ activities typically involve the individual taxpayer either in making decisions under uncertainty ... or in trying to eliminate that uncertainty by more through concealment” [p. 12]. Given that uncertainty is the crucial element, the logic of this definition must also carry to activities that Cowell, Sandford (1980) and Slemrod and Sorum (1984) have termed “discretionary” compliance costs, i.e., “costs of obtaining advice on tax avoidance” [Sandford, p. 153]. The point is that there generally exists some degree of uncertainty about one’s tax liability. How much a person must pay in taxes depends on how well he “shelters” his income. Moreover, the effectiveness of tax shelters are often uncertain and not all tax shelters are equally effective. The implied uncertainty in a taxpayer’s net income may thus be reduced through tax advice just as through concealment.1

While we allow for tax-evaders to affect the probability that they are caught cheating if audited, we consider only the class of simple audit strategies with purely random audits. The government chooses the “best” policy within this class by setting the audit probability for each market.2 This allows us to keep the discussion simple, but still capture some of the essential features of tax evasion. We make two other important assumptions. First, the penalty rate is given. This assumption is quite common in the literature on tax evasion; see Kolm (1983) for a discussion.3 Second,

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1 Indeed Cowell goes on to point out that “if a particular ‘avoidance’ scheme is actually the subject of legal doubts ... the perceived consequences to the taxpayer of engaging in that scheme may be equivalent to those of participating in manifestly illicit tax evasion” [p. 13].

2 Such random audits would generally be suboptimal if a broader range of audit policies (such as cut-off rules) were available to the government; see e.g., Reinganum and Wilde (1985), Border and Sobel (1987), Scotchmer (1988) and Cremer et al. (1990).

3 The rationale for this assumption is that for motives of social cohesion and political ethics, the government is unable to impose “very high” penalties on tax evaders. As punishing is
for reasons of tractability, we assume that preferences are quasi-linear. This assumption is also not uncommon in the literature; see Cremer et al. (1990).

II. The Model

The economy consists of individuals who are identical in all respects except earning ability, \( w \). Each individual is endowed with one unit of time and derives utility from net income, \( c \), and leisure, \( 1 - L \), where \( L \) is labor supply. Preferences are assumed to be quasi-linear and represented by

\[
u = c + f(1 - L)\]

where \( f(\cdot) \) is strictly increasing, twice differentiable and strictly concave.

The tax schedule is linear with a constant marginal tax rate of \( \theta \) and a guaranteed income \( a \). A taxpayer's true income is unknown to the government; it may be observed only through a costly audit. The taxpayer may then attempt to evade taxes by reporting only a proportion, \( 1 - \alpha \), of his income. The tax administration audits a randomly selected fraction, \( \beta \), of individuals. Each individual thus faces a probability \( \beta \) of being audited independently of his actions. However, a tax-evader can affect the probability of being caught, if audited, by spending money to conceal his evasion. Specifically we assume that this probability, which represents the "concealment technology", is given by

\[\gamma = \gamma(m, \alpha, z),\]

where \( m \) is the expenditure on concealment. \(^4\) \( \alpha \) is the proportion of income not reported and \( z \) is the amount concealed. Note that we have

\[z = \alpha wL.\]

We assume that \( \gamma \) is increasing in both \( \alpha \) and \( z \) and strictly increasing in at least one of them: \( \gamma_{\alpha} \geq 0 \) and \( \gamma_z \geq 0 \) (with at least one of the inequalities being strict). \(^5\) This assumption captures the intuitively appealing idea that ceteris paribus both the absolute amount and the proportion of income concealed may matter. Note also that either of the derivatives can be zero such that the special cases in which only \( z \) or only \( \alpha \) matters are not ruled

\(^4\)Expenditures to conceal tax evasion may entail "bribes" as well as real resource costs. Bribes are income transfers with no efficiency loss, unless their existence results in rent-seeking activities. To emphasize the efficiency loss aspect of concealment costs, we assume that \( m \) denotes only the real resource costs of concealment.

\(^5\)Subscripts denote partial derivatives.

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out. A second intuitive assumption is that $\gamma$ (strictly) decreases with $m$ (i.e., $\gamma_m < 0$), but that the returns to concealment expenditures occur at a decreasing rate ($\gamma_m > 0$).\textsuperscript{6}

A person who is caught cheating is taxed on the true amount of his income. In addition, he is fined by an amount proportional to the tax evaded. The penalty rate is denoted by $r - 1$. The consumption of a tax-evader is thus a random variable, with two possible values, $c_g$ in the “good” state of nature where he is not caught and $c_b$ in the “bad” state where his cheating is discovered. These values are given by

\begin{align}
  c_g &= wL - \theta(wL - z) - m + a, \quad (2a) \\
  c_b &= wL(1 - \theta) - (r - 1)\theta z - m + a. \quad (2b)
\end{align}

It is easily established that $c_g$ occurs with probability $(1 - \beta \gamma)$, while $c_b$ occurs with probability $\beta \gamma$. It follows that an individual's expected utility, $v$, is given by

\begin{align}
  v &= (1 - \beta \gamma)u(c_g, L) + \beta \gamma u(c_b, L) \\
  &= (1 - \theta) wL + (1 - \beta \gamma) \theta z + f(1 - L) - m + a. \quad (3)
\end{align}

The individual chooses $m$, $z$ and $L$ to maximize $v$. It is convenient to introduce the notation $\theta^e$ for the individual's "expected" marginal tax rate defined as

\begin{equation}
  \theta^e = [1 - (1 - \beta \gamma) \alpha] \theta. \quad (4)
\end{equation}

Substituting (4) into (3) leads to the following formal statement of the individual's problem

\begin{equation}
  \max_{m, z, L} v = (1 - \theta^e) wL + f(1 - L) - m + a.
\end{equation}

This reformulation has a nice interpretation: it suggests that the taxpayer acts as if he had the opportunity to change his marginal tax rate from $\theta$ to $\theta^e$ by spending $m$ dollars.

Assuming an interior solution, and taking into account the relationship between $\alpha$ and $z$ given by (1), we obtain the following first-order conditions:

\begin{align}
  \frac{\partial v}{\partial m} &= -\beta r \theta z \gamma_m - 1 = 0, \quad (5a)
\end{align}

\textsuperscript{6} As to the remaining second-order derivatives, we assume $\gamma_m \geq 0$ and $\gamma_z \geq 0$. These conditions are imposed for technical reasons but have obvious intuitive interpretations.
\[ \frac{\partial v}{\partial z} = \theta [1 - (\gamma + \alpha \gamma + z \gamma) \beta_\tau] = 0. \]  
(5b)

\[ \frac{\partial v}{\partial t} = (1 - \theta) w + \beta \tau \theta \gamma w \gamma_a - f' = 0. \]  
(5c)

Since \( \gamma_a \geq 0 \) and \( \gamma \geq 0 \), it follows from (5b) that a necessary condition for an interior solution is that

\[ 1 - \beta \tau \gamma > 0. \]  
(6)

We assume that this inequality is satisfied and impose the second-order sufficient conditions for a maximum.

It is important to note that in equations (5a) and (5b) \( w \) and \( L \) only appear as \( wL \approx y \), where \( y \) denotes pre-tax earnings. It is then possible to use these two equations to express \( m \) and \( z \) as functions of \( y \). This is convenient, as it makes it meaningful to study how \( m \) and \( z \) vary as \( y \) varies.

### III. The Progressivity of the Income Tax Schedule

In the absence of tax evasion, given quasi-linear preferences, labor supply and pre-tax income both increase with earning ability. The linear tax schedule, which is based on earnings, will then work as a redistributive scheme taking money away from people with higher earning ability and redistributing it to people with lower earning ability. Moreover, this taxation scheme is progressive in that the average tax rate increases as pre-tax income increases.

We begin by investigating whether these properties continue to hold in the presence of tax evasion and its concealment. Our first result concerns the relationship between earnings and earning ability:

**Proposition 1.** The higher the earning ability, \( w \), the higher the pre-tax income, \( y \). Specifically we have:

\[ \frac{dy}{dw} = - L \left( \frac{f'}{L} - f'' \right) \frac{|V_2|}{|V|} > 0, \]

where \( f' \) and \( f'' \) are the first and the second derivatives of \( f \), \( |V| \) is the determinant of \( V \), the Hessian matrix of \( v \), and \( |V_2| \) is its principal minor of order 2.

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The proposition is significant because it indicates that the tax system displays the same properties with respect to the individuals' earning abilities as to their actual earnings.

Turning now to the progressivity of the tax system, we consider a given (linear) tax schedule and investigate: (i) if the tax scheme continues to be progressive in the presence of tax evasion, and (ii) if it is more or less progressive than in the absence of tax evasion. To ascertain progressivity, both marginal and average tax rates are considered. With tax evasion, the marginal tax rate is no longer constant as \( \theta \) is replaced by \( \theta^e \) (the expected tax rate). This is personalized and, for progressivity, we have to examine the sign of \( \partial \theta^e / \partial y \). Regarding the average tax rate, we first define \( T = \theta^e y - a \) to be the expected net tax payment (taxes paid over and above the lump-sum payment received). An individual's average tax rate is then given by \( T/y \). Note, however, that in the presence of tax evasion, the burden of taxation on individuals is not just the taxes and fines paid to the government, but also the concealment cost associated with evading some of the taxes. To capture this we define \( T_m = T + m \) as the net tax payment and concealment cost, and consider the expression for \( T_m/y \) which gives the average tax rate including concealment cost.

It follows directly from the above discussion that without tax evasion

\[
\frac{\partial \theta^e}{\partial y} = 0 \quad \text{and} \quad \frac{\partial (T/y)}{\partial y} = \frac{\partial (T_m/y)}{\partial y} = \frac{a}{y^2}.
\]  

(7)

To obtain the counterpart to these expressions in the presence of tax evasion and concealment, we use the property that equations (5a) and (5b) of the first-order conditions give \( m \) and \( z \) as functions of \( y \). Consequently, the marginal and average tax rates (achieved at the consumer's optimum) can be expressed as functions of pre-tax income \( y \). The derivatives of these functions are given in:

**Lemma 1.** Marginal and average tax rates vary with pre-tax income according to

\[
\frac{\partial \theta^e}{\partial y} = \theta \beta x \gamma z - \frac{1}{y} \frac{\partial m}{\partial y},
\]  

(8a)

\[
\frac{\partial (T/y)}{\partial y} = \theta \beta x \gamma z + \frac{a}{y^2} \frac{1}{y} \frac{\partial m}{\partial y},
\]  

(8b)

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The proofs of propositions 1-3 and lemmas 1 and 2 are contained in the Appendix.

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\[
\frac{\partial(T_m/y)}{\partial y} = \theta \beta \tau \alpha^2 y^2 + \frac{a - m}{y^2}.
\]

(8c)

All of the expressions (8a)-(8c) have ambiguous signs. Comparing these expressions to their counterparts for the case without tax evasion also leads to ambiguous signs. This suggests that tax evasion (combined with concealment) can in some cases make the tax system more progressive, while in other cases it may lead to a less progressive or even a regressive tax system. To show that this is indeed the case, we consider two special cases (\(\gamma_a = 0\) and \(\gamma_z = 0\)) for which opposite and intuitive results obtain.

Consider first the case where \(\gamma_a = 0\) and \(\gamma_z > 0\); that is, when the probability of being caught if audited is influenced by the dollar amount of income concealed but not (independently) by what fraction of income that amount is. This case entails two interesting properties. First, all individuals choose the same \(m\) and \(z\) regardless of their income. This follows because equations (5a) and (5b) of the first-order conditions will now be independent of \(L\) and \(w\) and hence of \(y\). Second, it can be seen from (5c) that when \(\gamma_a = 0\), \(L\) depends only on \(w(1 - \theta)\), so that \(L\) and hence \(y\) will be the same with and without tax evasion. The formal results are:

**Proposition 2.** If \(\gamma_a = 0\) for all \(m, \alpha\) and \(z\), then:

\[
\frac{\partial \theta^c}{\partial y} = \theta \beta \tau \alpha^2 y^2 \frac{(1 - \beta \tau \gamma) \theta z}{y^2} > 0.
\]

(9a)

\[
\frac{\partial (T/y)}{\partial y} = \theta \beta \tau \alpha^2 y^2 + \frac{a}{y^2} \frac{(1 - \beta \tau \gamma) \theta z + a}{y^2} > 0,
\]

(9b)

\[
\frac{\partial (T_m/y)}{\partial y} = \theta \beta \tau \alpha^2 y^2 + \frac{a - m}{y^2} \frac{(1 - \beta \tau \gamma) \theta z + a - m}{y^2} > 0.
\]

(9c)

Proposition 2 shows that expected marginal and average tax rates increase with pre-tax income. The tax system is progressive. Furthermore, comparing the expressions in proposition 2 to those given in (7), it can easily be seen that all three derivatives are larger than their counterparts without tax evasion (for the same values of \(\theta\) and \(a\)). We can therefore conclude that in this case, tax evasion and its concealment make the tax system more progressive. Intuitively, because \(m\) and \(z\) are independent of pre-tax earnings, everyone evades the same amount and gets caught with the same probability. Hence the expected benefit from tax evasion is the same for everyone. This is, in essence, equivalent to a uniform lump-sum transfer; and increasing the lump-sum transfer increases progressivity of the tax system.
Consider next the second special case where $\gamma_z = 0$ together with $\gamma_a > 0$, that is, when $\gamma$ is influenced by the proportion of income concealed but not (independently) by its dollar amount. We have:

**Proposition 3.** If $\gamma_z = 0$ for all $m$, $a$ and $z$, then

\[
\frac{\partial \theta^a}{\partial y} = \frac{-1}{y} \frac{\partial m}{\partial y} < 0, \quad (10a)
\]

\[
\frac{\partial (T/y)}{\partial y} = \frac{a}{y} - \frac{1}{y} \frac{\partial m}{\partial y}, \quad (10b)
\]

\[
\frac{\partial (T_m/y)}{\partial y} = \frac{a - m}{y}, \quad (10c)
\]

Equations (10b) and (10c) indicate that the average tax rates may increase as well as decrease with pre-tax income depending on the size of guaranteed income $a$. Thus the tax system is no longer necessarily progressive. However, using (7), it can easily be checked that all three derivatives are smaller than their counterparts in the absence of evasion. Consequently, tax evasion now makes the system necessarily less progressive.

As (10b) and (10c) indicate, if $a$ is zero, the system is necessarily regressive. This is because a high-income individual can always achieve a lower average tax rate (including concealment cost) than that of a low-income individual simply by choosing the same $a$. He then achieves the same expected tax rate at the same absolute cost $m$; but in relative terms (per dollar of income) he faces a lower cost. (Of course, if he chooses $a$ optimally he can even do better.) If $a$ is positive, this regressive effect is mitigated by the progressive lump-sum transfer and the overall effect becomes ambiguous.

To sum up, it is apparent from the analysis of the two special cases that the "concealment technology" captured by the function $\gamma$ plays a crucial role in determining the redistributional impact of tax evasion.

**IV. Optimal Taxation**

The possibility of tax evasion and its concealment affects the optimal tax design. In the optimal tax literature, starting with Mirrlees' (1971) classic
paper, the government chooses income tax rate(s) to maximize social welfare. In the presence of tax evasion, the audit strategy becomes a crucial second ingredient in this problem. Furthermore, the resources allocated to concealing evasion constitute an additional deadweight loss created by the tax. This section focuses on the issue of optimal tax design. Our inquiry is based on the assumption that the government chooses the "best" audit policy within the class of purely random audits by setting the audit probability \( \beta \) for all individuals. Another crucial assumption is that the penalty rate \( (r - 1) \) cannot exceed a certain ceiling. Because punishing is assumed costless, while auditing involves a cost, the government will always set the penalty rate at its maximum level.

**Problem and First-Order Conditions**

Assume that the earning ability \( w \) is distributed over \([w^-, w^+]\) with the cumulative distribution given by \( F(w) \). Normalize the population size by assuming that \( F(w^+) = 1 \). Let \( R(w; \theta, \beta) = \theta c(w) \) be the expected tax collected from an individual with earning ability \( w \). The government's net revenue requirement is \( R_0 \). Audit costs are given by a strictly increasing function \( c(\beta) \). The government sets \( \theta, a \) and \( \beta \) to maximize a social welfare function given by

\[
\int_{w^-}^{w^+} \psi(v) \, dF,
\]

where \( \psi \) is an increasing and strictly concave function, subject to its revenue constraint and the behavior of taxpayers as described in the previous sections.

The Lagrangian expression associated with the government's problem is

\[
\Lambda = \int_{w^-}^{w^+} \psi(v(w; \theta, a, \beta)) \, dF + \lambda \left[ \int_{w^-}^{w^+} R(w; \theta, \beta) \, dF - a [c(\beta) - R_0] \right].
\]

Assuming an interior solution, the first-order conditions for this problem are

\[
\frac{\partial \Lambda}{\partial \theta} = \int_{w^-}^{w^+} \psi \frac{\partial v}{\partial \theta} \, dF + \lambda \int_{w^-}^{w^+} \frac{\partial R}{\partial \theta} \, dF = 0, \tag{11a}
\]

\[
\frac{\partial \Lambda}{\partial a} = \int_{w^-}^{w^+} \psi \frac{\partial v}{\partial a} \, dF + \lambda \int_{w^-}^{w^+} (-1) \, dF = 0, \tag{11b}
\]

\[10\] \( R \) is independent of \( a \) because preferences are quasi-linear.

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\[ \frac{\partial \Lambda}{\partial \beta} = \int_{u^-}^{u^+} \psi' \frac{\partial v}{\partial \beta} \, dF + \lambda \left\{ \int_{u^-}^{u^+} \frac{\partial R}{\partial \beta} \, dF - c'(\beta') \right\} = 0. \] (11c)

**The Optimal Tax Rate**

Introduce the notation \( E(\cdot) \) for the expected value and \( \text{cov}(\cdot, \cdot) \) for the covariance. Using the property that \( \partial v/\partial a = 1 \) (from differentiating (3) and applying the envelope theorem), we can rewrite expression (11b) as

\[ \lambda = E(\psi'). \] (12)

Substituting from (12) into (11a) and applying the envelope theorem to calculate \( \partial v/\partial \theta \) and \( \partial R/\partial \theta \), we can establish:

**Proposition 4.** The optimal tax schedule satisfies the following relationship

\[ - \text{cov} \left[ \psi', \left( \frac{\theta^*}{\theta} \right) y \right] = - E(\psi') E \left[ (w - f') \frac{\partial L}{\partial \theta} \right] + E(\psi') E \left( \frac{\partial m}{\partial \theta} \right). \] (13)

Equation (13) illustrates the tradeoff that determines the optimal tax rate. It shows that the impact of a change in \( \theta \), and of the resulting adjustment in \( a \), on welfare may be decomposed into three components: (i) the redistributive benefit (measured by the covariance term on the l.h.s.), (ii) the traditional excess burden of the tax (measured by the first term on the r.h.s.), and (iii), the resource cost of concealment (measured by the second term on the r.h.s.). The equation states that the tax rate should be set such that at the margin, the benefits and costs just balance one another.

Condition (13) differs from the traditional optimal income tax formula in two important ways. First, it includes an additional term (concealment cost), and second, the other terms have a somewhat different structure than the traditional ones. This is not surprising because, as we have seen in the preceding section, tax evasion affects the redistributive properties of the tax. Moreover, as can be seen from the individual's first-order condition (5c), unless \( \gamma_a = 0 \), tax evasion also affects labor supply.

To gain further insight into determination of the optimal tax rate, we have to examine the three terms in (13) more closely. First, rewrite the covariance term as

\[ - E \left[ \psi' \left( \frac{\theta^*}{\theta} \right) y \right] + E(\psi') E \left[ \left( \frac{\theta^*}{\theta} \right) y \right]. \] (14)

\(^{11}\)The Appendix contains the expressions for \( \partial v/\partial \theta \) and \( \partial R/\partial \theta \), as well as for \( \partial v/\partial \beta \) and \( \partial R/\partial \beta \).
The first term in (14) measures the loss in social welfare resulting from the decrease in after-tax income caused by an increase in $\theta$. On the other hand, the increase in $\theta$ (and the tax revenue) also induces an increase in the lump-sum transfer $a$. The benefits of this are measured by the second term in (14). The covariance term thus measures the net redistributive gains achieved by collecting tax revenue through a proportional income tax and redistributing the proceeds uniformly. An interior solution for $\theta$ requires this term to be positive at the optimum. This is the case as a positive marginal tax rate is only justified if it is beneficial in terms of redistribution. The strict concavity of $\psi$ (which implies that $\psi^\prime$ decreases with $y$) implies that the covariance term is indeed positive as long as $(\theta^c/\theta)y$ increases with $y$.

The first term on the r.h.s. of (13) measures the increase in excess burden of the tax. As $\theta$ increases, labor supply responds by $\partial L/\partial \theta$. Given quasi-linear preferences, this change is in total a substitution effect. Multiplying it by $(w-f)$, which is positive from (5b) and (5c) and measures the excess of marginal product of labor over marginal utility of leisure, gives the change in utility. To obtain the corresponding loss in social welfare, we have to sum over individuals and multiply by $E(\psi^\prime)$ (to convert dollars into social welfare). The third term in equation (13) measures the social value of the change in the resource cost of concealment as $\theta$ increases.

The Tax Progressivity Issue — A First Look

The next interesting question is whether tax evasion and concealment cost make the tax system more or less progressive. We have already dealt with this issue in Section III, but only in a partial equilibrium setting. There, we assumed that the tax schedule ($\theta, a$) was the same with and without evasion. In general equilibrium, however, the tax schedule cannot remain the same in the presence of evasion as it would violate the government's budget constraint. We should thus compare the optimal tax schedule in the presence of evasion to the optimal schedule in the absence of evasion. We

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12 This welfare loss is evaluated given $L$ and $m$; the welfare impacts of the changes in $L$ and $m$ are captured by the other terms in (13). If $L$ and $m$ are given, an individual's optimization problem simply consists in minimizing expected tax payment $R = \theta y$ with respect to $z$ (with $\alpha = z/y$). By the envelope theorem, a change in $\theta$ induces a change of $(\partial \theta^c/\partial \theta)y = (1 - (1 - \beta \gamma) \alpha) y = (\theta^c/\theta) y = R$, and thus in the individual's net income.

13 This makes an interior solution possible but, of course, does not guarantee it. Essentially, we assume away situations in which $\partial m/\partial \theta$ is negative and "very large" in absolute value. Even though the derivative has an ambiguous sign, this does not appear to be a very plausible situation.

14 We have from (5c) $w - f^* = \theta y (1 - \beta \alpha^2 \gamma y)$, and from (5b) $1 - \beta \alpha^2 \gamma y = \beta (y + z \gamma y) > 0$. The last inequality implies that $\beta \alpha^2 \gamma y < \beta \alpha^2 < 0$, so that $w - f^* > 0$. 

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focus our attention on the marginal tax rates, while keeping in mind that the guaranteed income is adjusted to meet the government's budget constraint.

Write the equation characterizing the optimal tax rate in an alternative way. Substituting for $\theta^c$ from \(4\) into \(13\) and eliminating $w-f'$ by using \(5b\) and \(5c\) yields

$$
\theta = \frac{-\text{cov}(\psi', [1 - (1 - \beta \gamma) \alpha] y] - E(\psi') E(\partial m/\partial \theta)}{E(\psi') E[w[1 - (1 - \beta \gamma) \alpha + \beta \tau z \alpha][- \partial L/\partial \theta]]}.
$$

The counterpart to this expression in the absence of tax evasion, see Dixit and Sandmo (1977), is

$$
\theta = \frac{-\text{cov}(\psi', y)}{E(\psi') E[w(- \partial L/\partial \theta)]}.
$$

The numerator in \(15\), in comparison with \(16\), has an additional term, reflecting the resource cost of evasion. It is clear that if $\partial m/\partial \theta > 0$ this additional term would tend to reduce the optimal tax rate. Intuitively, in the presence of tax evasion, taxation creates an additional distortion and therefore the optimal tax rate might be expected to be smaller. However, comparison of the denominators does not lead to a clear-cut conclusion. In any event, comparing optimal tax rates solely on the basis of first-order conditions is very dangerous.\(^{15}\)

In order to obtain more precise results, consider again the special case where $\gamma$ is independent of $\alpha$ (i.e., $\gamma_a = 0$). In this case, \(5c\) implies that for a given marginal tax rate, labor supply is the same with and without tax evasion. This fact makes comparisons much easier. Using \(5b\) and the property that $m, z$ and thus $\gamma$ are now the same for everyone, \(15\) simplifies to

$$
\theta = \frac{-\text{cov}(\psi', y) - (\partial m/\partial \theta) E(\psi')}{E(\psi') E[w(- \partial L/\partial \theta)]}.
$$

This expression differs from \(16\) only in its inclusion of the additional term: $-(\partial m/\partial \theta) E(\psi')$. Moreover with $\gamma_a = 0$, it can be shown that $\partial m/\partial \theta$ is necessarily positive.\(^{16}\) Still, as will become clear below, this is not sufficient to imply that tax evasion lowers $\theta$. To draw that conclusion, an entirely different kind of argument is required.

\(^{15}\) Neither \(16\) nor \(15\) provides an explicit solution for the optimal tax rate since their r.h.s.'s also depend on $\theta$.

\(^{16}\) This may be done by substituting $\gamma_a = 0$ in the first-order conditions \(5a)\)–\(5c\) and then differentiating them with respect to $\theta$. 

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The Tax Progressivity Issue — A Closer Look

Let us fix the audit probability at its optimal level $\beta^*$. Then examine the behaviour of social welfare, $W$, as $\theta$ changes, while $a$ is adjusted to satisfy the government’s budget constraint. Substituting (12) into (11a) and making the same kind of simplifications as those which led to (17) yields

$$
\frac{dW}{d\theta} = -\text{cov}(\psi', y) - E\left[\theta w\left(-\frac{\partial L}{\partial \theta}\right)\right] E(\psi') + \frac{\partial m}{\partial \theta} E(\psi').
$$

(18)

Denote the optimal values of the variables in the absence of tax evasion with a ‘tilde’. We can then find out how the presence of tax evasion affects the optimal $\theta$ by studying the sign of (18) at $\tilde{\theta}$; a negative sign implies that tax evasion reduces $\theta$, while a positive sign yields the opposite result.17 From Section III, setting $\theta = \tilde{\theta}$ results in

$$
L = \tilde{L}, \quad y = \tilde{y}, \quad a = \tilde{a} - (1 - \beta^* \gamma) \tilde{\theta} z - c(\beta^*),
$$

and

$$
v = \tilde{v} - m - c(\beta^*),
$$

(19)

where $m$, $z$ and $\gamma$ are constants (because $\gamma_a = 0$). We can now establish:

Lemma 2. Given $\gamma_a = 0$, we have

$$
-\text{cov}\left[\psi'(\tilde{v} - m - c(\beta^*)), \tilde{y}\right] < -\text{cov}\left[\psi'(\tilde{v}), \tilde{y}\right] = \frac{dW}{d\theta} \tilde{\theta} < 0.
$$

Lemma 2 provides a sufficient condition for tax evasion to lower the optimal tax rate (i.e., for (18) to be negative). The condition in the lemma depicts a situation where society becomes less concerned with redistribution (the covariance term diminishes) as a result of a uniform lump-sum tax of $m + c(\beta^*)$ levied on each person. This is, in essence, what happens to taxpayers because of tax evasion. As mentioned in Section III, when $\gamma_a = 0$, everyone conceals the same amount and incurs the same concealment cost. It follows that the concealment cost $m$ together with the reduction in $a$, brought about by the audit cost,18 reduces everyone’s income by the same amount and equal to $m + c(\beta^*)$. It is intuitively expected that when, for whatever reason, society is less concerned with redistribution, it will want a less progressive tax. Lemma 2 suggests that if the lessening of redistributive concerns arises when society is poorer, tax

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17 We assume that $W$ is concave in the relevant range.

18 This may appear paradoxical at first. Given $a$, each individual is better off by evading taxes; but since everyone evades the same amount, $a$ has to decrease.

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evasion by making society poorer calls for a lowering of the optimal tax rate.

Lemma 2 indicates that the social welfare function plays a crucial role in determining the impact of tax evasion on optimal taxation. However, it falls short of providing a precise characterization of the class of social welfare functions that would actually imply a lower optimal tax rate under tax evasion. The next step in that direction is:

**Proposition 5.** Assume \( \gamma_\alpha = 0 \). Then \( \psi'' < 0 \) implies that the optimal marginal tax rate is lower with tax evasion and concealment than in the absence of evasion.

**Proof:** First note that \( \psi'' < 0 \) implies \( \text{cov} \left[ \psi'(\bar{t} - m - c(\beta^*)), \hat{y} \right] < 0 \). This condition can then be written as

\[
\frac{\partial \left[ - \text{cov} \left[ \psi'(\bar{t} - m - c(\beta^*)), \hat{y} \right] \right]}{\partial \bar{v}} > 0, \tag{20}
\]

which implies the condition in lemma 2.

Proposition 5 gives a condition on the social welfare function that is sufficient for ensuring that the optimal value of the tax rate will be lower with tax evasion (when \( \gamma_\alpha = 0 \)). The result is quite intuitive as \( \psi'' < 0 \) ensures that inequality (20) holds, so that society becomes less concerned with redistribution as its members become poorer. And as lemma 2 has shown under this circumstance, tax evasion, by making society uniformly worse off, will call for less redistribution and a lower value for \( \theta \).

There are many social welfare functions which violate this condition, however. They include the commonly used class given by \( \psi = \psi(\bar{v}) \) with \( \rho < 1 \). Of course, even if the sufficient condition is violated, the result may still go through. Examples to illustrate this can easily be constructed. On the other hand, it is also possible to find examples where the opposite result holds, such that tax evasion would lead to an increase in the optimal tax rate.¹⁹

While these results have been obtained for the special case of \( \gamma_\alpha = 0 \), they have important implications for the more general situation. Specifically, the following three lessons emerge: (i) we cannot hope for an

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¹⁹ A simple example illustrating this is obtained from the following specifications:
\[ f(1 - L) = (1 - L)^2, \; \gamma = \gamma^1 \{1000(\text{m} + 1)\}, \; R_\alpha = 0, \; \psi(r) = r^{1/2}, \; \tau = 2, \; \omega \text{ takes only two values} \; \omega_1 = 100 \; \text{and} \; \omega_2 = 1000 \; \text{with equal probability}, \; \text{and the audit cost function is such that} \; \beta^* = 0.2. \; \text{Solving the optimal tax problem without tax evasion yields} \; \theta = 0.2444. \; \text{With tax evasion the solution depends on the value of} \; c(\beta^*). \; (\text{Note the assumption} \; \beta^* = 0.2 \; \text{only specifies the audit cost function up to a constant.}) \; \text{Then if for instance} \; c(\beta^*) = 1, \; \text{we find} \; \theta^* = 0.2443 < \theta \; \text{while if} \; c(\beta^*) = 1000, \; \text{we have} \; \theta^* = 0.24562 > \theta. \]

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unambiguous result in the general case, (ii) simple inspection of the first-order conditions can indeed be misleading, and (iii), to assess the impact of tax evasion on optimal tax rates, it is not sufficient to consider the additional distortions created by evasion and concealment; it is also necessary to determine how it affects the redistributive benefits of taxation.

The Optimal Audit Probability

We now turn to the characterization of the optimal audit probability. Substituting (12) into (11c) and applying the envelope theorem to calculate $\partial v/\partial \beta$ and $\partial R/\partial \beta$ yields:

Proposition 6. The optimal audit probability satisfies

$$- \text{cov}(\psi', \theta \tau y z) + E(\psi')E\left[(w - f') \frac{\partial L}{\partial \beta}\right]$$

$$- E(\psi')E\left(\frac{\partial m}{\partial \beta}\right) - E(\psi') c'(\beta) = 0. \tag{21}$$

Equation (21) characterizes the optimal value of $\beta$ under the assumption that the solution is interior.\textsuperscript{20} It is somewhat similar to expression (13) except that there is one additional term. Moreover, it is now even more difficult to assess which terms constitute benefits and which terms constitute costs of an increase in the audit probability.

The covariance term continues to measure the redistributive impact of the change in the government's instrument. It has an ambiguous sign, which depends on the relationship between tax evasion and income. If $\gamma_\alpha = 0$, everyone has the same $z$ and the same $y$ so that increasing $\beta$ has no effect on redistribution (the covariance term is zero). On the other hand, if high-income individuals evade more taxes, the term is likely to be positive (unless these persons also spend much more money on concealment).

The second term in (21) reflects the traditional deadweight loss of taxation.\textsuperscript{21} The third term reflects the resource cost of concealment. Not

\textsuperscript{20} It can easily be shown that if the government has a positive revenue requirement (to rule out $\beta = 0$), if $c''(\beta)$ is large enough (the audit cost increases fast enough to ensure that auditing a sufficiently high fraction of the population to induce truth-telling would be too costly), and if some additional regularity conditions are satisfied (to guarantee that the function has in fact a maximum), then there will be an interior solution for $\beta$. None of these conditions appears to be particularly restrictive.

\textsuperscript{21} Note, however, that $\partial L/\partial \beta$ has an ambiguous sign. Consequently, it is possible that increasing the penalty rate reduces the distortion in labor supply created by the tax itself.
surprisingly it has an ambiguous sign. A higher audit probability could
indeed induce individuals to spend more resources on concealment, but it
could also reduce evasion altogether, thereby reducing concealment
expenditures. The last term reflects the increase in audit cost which affects
individuals' welfare through a reduction in the lump-sum transfers \(a\).

V. Concluding Remarks

We have studied how the presence of tax evasion and concealment costs
affects the design of an optimal linear income tax scheme, when taxpayers
can influence the probability that they are caught cheating. Our main
results fall into two categories. First, we have provided simple and intuitive
formulas to characterize the optimal tax rate and audit probability
(propositions 4 and 6). Second, we have been able to draw two important
lessons regarding the progressivity of the tax system (propositions 2, 3, and
5): (i) the nature of the concealment technology is quite important in
determining the degree of progressivity of a given tax system, and (ii) if
society becomes less concerned with redistribution when all its members
become poorer, the optimal tax rate must necessarily be lower with tax
evasion. This has been shown to be the case if the third derivative of the
social welfare function is negative (and if the probability of being caught
when audited is independent of the proportion of income concealed).

The paper may be extended in some important directions by consider-
ing more sophisticated behavior on the part of the government. First, one
could move away from the assumption of purely random audits and allow
the tax administration to base its decision on whether to audit a particular
taxpayer on his reported income. Second, the tax administration could be
allowed to choose not only the audit probability, but also the "quality" of
an audit. The quality of an audit may then be conditioned on reported
income. This should affect the concealment technology, as it changes the
nature of the relationship between the taxpayer's reported income and the
probability of being caught if audited. This may prove important, and
policy relevant, given our finding that the concealment technology plays a
crucial role in determining the redistributive impact of the tax system.
Third, one may make the concealment technology depend on the
taxpayer's supply of effort. Since the opportunity cost of the time an
individual spends to conceal his cheating is equal to his earning ability,
such a possibility is likely to affect both the progressivity of a given tax and
the design of an optimal tax scheme. Finally, it would be interesting and
informative to examine under what conditions our optimal tax rules with
tax evasion can be obtained as the equilibrium outcome of a median voter
model. One could even attempt to endogenize the government's behavior.
We leave these different avenues open for further research.
Appendix

Proof of Proposition 1

Totally differentiating the first-order conditions (5a)-(5c) results in

\[
\begin{align*}
\frac{\partial m}{\partial w} & = -\beta \tau \theta \left( \begin{array}{c}
\alpha^2 L \gamma_{\alpha ax} \\
(2 \gamma_a + \alpha \gamma_{\alpha ax} + z \gamma_a)(a/w) \\
(1 - \theta)/\beta \tau \theta \end{array} \right), \\
\frac{\partial z}{\partial w} & = -\alpha^{2} z \gamma_{\alpha ax}, \\
\frac{\partial L}{\partial w} & = -\alpha^{2} \gamma_{\alpha ax} + z \gamma_a \gamma_{\alpha ax},
\end{align*}
\]

(A1)

where \( V \) is the Hessian matrix of \( u \) and \( V/\beta \tau \theta \) is given by

\[
\begin{pmatrix}
z \gamma_{\alpha ax} & \gamma_{\alpha ax} + \alpha \gamma_{\alpha ax} + z \gamma_a \\
\gamma_{\alpha ax} + \alpha \gamma_{\alpha ax} + z \gamma_a & (2 \gamma_a + \alpha \gamma_{\alpha ax} + z \gamma_a)/wL + 2 \gamma_a + 2 \alpha \gamma_{\alpha ax} + z \gamma_{\alpha ax}
\end{pmatrix}

- \alpha^{2} \gamma_{\alpha ax} + z \gamma_a \gamma_{\alpha ax}, (2 \gamma_a + \alpha \gamma_{\alpha ax} + z \gamma_a)(-a/L)

- \alpha^{2} \gamma_{\alpha ax} + z \gamma_a \gamma_{\alpha ax}, (2 \gamma_a + \alpha \gamma_{\alpha ax} + z \gamma_a)(-a/L)

- \alpha^{2} \gamma_{\alpha ax} + z \gamma_a \gamma_{\alpha ax}, (2 \gamma_a + \alpha \gamma_{\alpha ax} + z \gamma_a)(w/L)

In order to solve (A1) we first rewrite its r.h.s. as

\[
\begin{pmatrix}
-\frac{L}{w} & v_{13} \\
v_{23} & v_{ss} + ((f'/L))^t - f''
\end{pmatrix}
\]

where \( v_{ij} \) denotes the \( i, j \)th element of \( V \). Solving (A1) and introducing the notation \( v_{ij}^* \) for the elements of \( V^* \) the adjoint of \( V \), yields

\[
\begin{align*}
\frac{\partial m}{\partial w} & = -\frac{L}{w} \left( \begin{array}{c}
f'/L - f'' \\
v_{13}
\end{array} \right), \\
\frac{\partial z}{\partial w} & = -\frac{L}{w} \left( \begin{array}{c}
f'/L - f'' \\
v_{23}
\end{array} \right), \\
\frac{\partial L}{\partial w} & = -\frac{L}{w} \left[ 1 + \left( \frac{f'/L - f''}{V} \right) v_{13}^* \right].
\end{align*}
\]

(A2)

(A3)

(A4)

Differentiating \( y = wL \) with respect to \( w \) and substituting from (A4) in the resulting equation yields the expression given for \( dy/dw \) in proposition 1. The sign of \( dy/dw \) is established as follows: first note that \( v_{13} = |V_2| \), the second principal minor of \( V \); and from the second-order conditions \( |V_2| > 0 \) and \( |V| < 0 \). Moreover, because \( f \) is strictly increasing and strictly concave, we have \( f' > 0 \) and \( f'' < 0 \).
Proof of Lemma 1

Differentiating the expression for $\theta^*$ given by (4) with respect to $y$, noting that (1) implies $\partial z/\partial y = \alpha + y(\partial \alpha/\partial y)$ and simplifying, results in

$$\frac{\partial \theta^*}{\partial y} = -\theta \left[ -\beta \tau \alpha^2 \gamma_c + \left( 1 - \beta \tau (\gamma + \alpha \gamma_a + \alpha \gamma_c) \right) \frac{\partial \alpha}{\partial y} - \beta \tau \alpha \gamma_c \frac{\partial m}{\partial y} \right].$$

Substituting from (5a) and (5b) into this expression and simplifying yields (8a). To derive (8b) and (8c) we then only have to substitute for $\frac{\partial \theta^*}{\partial y}$ in

$$\frac{\partial^2 (l/y)'}{\partial y} = \frac{\partial [\theta^* - (a/y)]}{\partial y} \quad \text{and} \quad \frac{\partial (T_m'/y)}{\partial y} = \frac{\partial [\theta^* + (m-a)/y]}{\partial y}.$$

Proof of Proposition 2

In this case equations (5a) and (5b) are independent of $L$ and $w$ and hence of $y$. It follows that $\partial m/\partial y = \partial z/\partial y = 0$. Moreover, from the first-order condition (5b), we have that when $\gamma_a = 0$, $\beta \tau \gamma_c = (1 - \beta \tau \gamma)/z$. Substituting in (8a)-(8c), while noting $\alpha = z/y$, results in the expressions given by (9a)-(9c). The signs of (9a) and (9b) follow immediately from (6). To establish the sign of (9c), recall that $L$, and hence $y$, are the same with and without tax evasion. Now, given that the individual always has the option of reporting honestly, it follows from (3) that if he cheats, $(1 - \beta \tau \gamma) \theta z - m > 0$.

Proof of Proposition 3

To obtain these expressions, it is sufficient to substitute $\gamma_c = 0$ into the results of lemma 1. The signs follow from the fact that in this case $\partial m/\partial y > 0$. This is proved as follows. From the expression for $V'$ given in the proof of proposition 1 it follows that

$$V_{1,3}^+ = (\beta \tau \theta)^2 \left( (\gamma_r + \alpha \gamma_a) / (2 \gamma_a + \alpha \gamma_a) \right) \left( -\frac{\alpha}{L} \right) + \alpha^2 w \gamma_m \frac{2 \gamma_c + \alpha \gamma_a}{wL} +$$

$$= \frac{(\beta \tau \theta)^2 \alpha (2 \gamma_a + \alpha \gamma_a) (-\gamma_a)}{L} > 0.$$

Substituting in (A2) then results in $\partial m/\partial w > 0$. But we also have (because (5a) and (5b) depend on $w$ and $L$ through $y$ only)

$$\frac{\partial m}{\partial w} = \frac{\partial m}{\partial y} \frac{dy}{dw},$$

which together with proposition 1 implies $\partial m/\partial y > 0$. 

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Expressions Needed for the Proof of Propositions 4 and 6

Differentiating \( \nu \) as given by (3) and using the envelope theorem, we obtain

\[
\frac{\partial \nu}{\partial \theta} = - wL + (1 - \beta \tau) z. \quad (\text{A5a})
\]

\[
\frac{\partial \nu}{\partial \beta} = - \tau \gamma \theta z. \quad (\text{A5b})
\]

Next, using equations (3) and (1), and the definition of \( R \), we may write the following identity

\[
R(w; \theta, \beta) = wL - m + a + f(1 - L) - v(w; \theta, a, \beta),
\]

and differentiate it with respect to \( \theta \) and \( \beta \) to obtain

\[
\frac{\partial R}{\partial \theta} = (w - f') \frac{\partial L}{\partial \theta} - \frac{\partial m}{\partial \theta} + wL - (1 - \beta \tau) z, \quad (\text{A6a})
\]

\[
\frac{\partial R}{\partial \beta} = (w - f') \frac{\partial L}{\partial \beta} - \frac{\partial m}{\partial \beta} + \tau \gamma \theta z. \quad (\text{A6b})
\]

Proof of Lemma 2

Substituting (19) into (18) yields

\[
\left( \frac{\partial \tilde{w}}{\partial \theta} \right) = - \text{cov} [\psi'(\tilde{v} - m - c(\beta^*), \tilde{y})] \cdot \mathbb{E} \left[ \tilde{\theta} w \left( - \frac{\partial L}{\partial \theta} \right) \right] \mathbb{E} \left[ \psi'(\tilde{v} - m - c(\beta^*)) \right] \]

\[
- \frac{\partial m}{\partial \theta} \mathbb{E} \left[ \psi'(\tilde{v} - m - c(\beta^*)) \right]. \quad (\text{A7})
\]

By definition \( \tilde{\theta} \) satisfies (16) so that:

\[
0 = - \text{cov} [\psi'(\tilde{v}), \tilde{y}] - \mathbb{E} \left[ \tilde{\theta} w \left( - \frac{\partial L}{\partial \theta} \right) \right] \mathbb{E} \left[ \psi'(\tilde{v}) \right]. \quad (\text{A8})
\]

To complete the proof it is then sufficient to subtract (A8) from (A7), and use the properties that \( \mathbb{E} \left[ \psi'(\tilde{v} - m - c(\beta^*)) \right] > \mathbb{E} \left[ \psi'(\tilde{v}) \right] \) (which follows from the strict concavity of \( \psi \)), and \( \partial m/\partial \theta > 0 \).

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