Piketty in the long run

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Abstract

I examine the idea of ‘the long run’ in Piketty (2014) and related works. In contrast to simplistic interpretations of long-run models of income- and wealth-distribution Piketty (2014) draws on a rich economic analysis that models the intra- and inter-generational processes that underly the development of the wealth distribution. These processes inevitably involve both market and non-market mechanisms. To understand this approach, and to isolate the impact of different social and economic factors on inequality in the long run, we use the concept of an equilibrium distribution. However the long-run analysis of policy should not presume that there is an inherent tendency for the wealth distribution to approach equilibrium.

Keywords: Long run; income distribution; wealth distribution; inequality; inheritance; equilibrium

1. Introduction

One of the great things achieved by Thomas Piketty and his collaborators is to make people reflect once again on what happens, or what may happen, in the long run. The scale of the project is interesting. Robert Solow, reviewing J. K. Galbraith’s New Industrial State, remarked that ‘Economists are determined little thinkers’ (Solow 1967). Galbraith was, of course, not a ‘little thinker’; neither is Piketty.

The Piketty research project focuses on the analysis of long-run developments of the distribution of income and of wealth (Piketty 2005; Atkinson and Piketty 2007, 2010; Alvaredo and Piketty 2010; Piketty and Saez 2014). This is among the biggest of big questions and is closely related to other broad distributional questions that have featured prominently in the literature over the last decade or so, such as the relationship between inequality and growth.
(Banerjee and Duflo 2003) or whether there is overall convergence or divergence in the world income distribution (Sala-i-Martin 2006).

To better appreciate the contribution of Piketty (2014) to our understanding of the forces underlying inequality we should perhaps begin by asking what is meant by ‘the long run’.

2. The long run

Within the economics literature on inequality there has long been an interest in the long run. It is reflected in the recent interest, cited in the introduction, in documenting income shares, income distribution and wealth distribution. However, there are difficulties in trying to provide a serious analysis of the long run. Setting aside the familiar empirical problems of forecasting and of backwards extrapolation into previous centuries, there is a problem in determining what the long run could tell us in principle. How does the time scale of the long run relate to the human time scale? In what way does long-run analysis connect to practical policy making?

There is a danger in trying to force a big-picture account of how distributions develop through time into an almost uninformative mantra rather than conducting a compelling reasoned analysis. Here are two examples of the kind of mantra that I have in mind, taken from popular interpretations of the work of two important predecessors of the Piketty project.

A development mantra

A prominent example of a mantra concerning the dynamics of distributions is the popular discussion and use of the Kuznets curve. This is taken to characterise a specific pattern of income inequality through the process of economic development (Kuznets 1955). It is based in part on an empirical claim about the observed relationship between the level of economic development and income inequality: plot the level of development (according to some metric) on the horizontal axis and a measure of inequality on the vertical axis and you should see an inverted-U shape. This empirical claim may be true of a collection of economies at a particular moment in time; a bolder step is to translate this to a statement about the development of a particular economy – perhaps a single country, perhaps the world as a whole. Then the low-high-low of the cross-sectional relationship between measured inequality and level of development becomes a claim about the development process – rising inequality followed by falling inequality (Piketty 2014: 13–15). The problem with the way in which the Kuznets vision has sometimes been interpreted is that the inverted-U story becomes a kind of inevitable pattern for the development of income and wealth distributions. In my view the Kuznets process is better
interpreted as a way of posing some motivating broad questions about fundamental relationships at a time when longitudinal data were scarce. It is usually not seen as so helpful nowadays in the light of the broad range of evidence that is now available (Atkinson, Piketty, and Saez 2011).

I want to emphasise that my reference to ‘mantra’ is not an attack on the pioneering work of Kuznets. Kuznets did not try to over-claim in his key papers. Furthermore, he was careful about what one could say with the then available data: indeed Kuznets’ earlier careful work (Kuznets 1953) was a model for the way in which Piketty and his collaborators have assembled evidence from tax data.

**A statistical mantra**

It is often convenient to characterise the shape of the interpersonal distribution of wealth or income using a statistical functional form. If there seems to be a regularity of shape across a broad collection of data sets then it is tempting to refer to this shape as a ‘law’ of distribution. Perhaps the most widely known of these is *Pareto’s Law*. But what is it and what is the attraction of the Pareto analysis? The supposed law can be interpreted in several ways, some of which could be misleading.

Pareto was responsible for some pioneering work that in some way can be seen as similar to Piketty’s research project. Using the scarce data available at the end of the nineteenth and the beginning of the twentieth centuries he postulated that the distribution of incomes regularly conformed to a characteristic pattern (Pareto 1896, 1965, 2001). Use $F$ to denote the distribution function of wealth, so that $F(W)$ means the proportion of the population with wealth less than or equal to $W$. Then the characteristic shape that Pareto uncovered in his data analysis can be summarised by

$$\log(1 - F(W)) = k - \alpha \log W,$$

where $k$ and $\alpha$ are constants. In other words, if one plots the distribution of wealth in a diagram with the log of wealth $W$ on the horizontal axis and the log of the proportion of the population possessing $W$ or more on the vertical axis, then we should get a negatively sloped straight line. The constant $k$ (the intercept in the equation) is not of great importance in the interpretation of the supposed relationship; by contrast the slope – that has come to be known as ‘Pareto’s $\alpha$’ – has sometimes been seen as of enormous importance. Clearly this simple formulation raises a number of questions.

First, is the shape empirically reasonable? A straight-line relationship is not reasonable for all definitions of income and wealth over the broad mass of the population. If one draws the associated density function, one finds a single-tailed distribution that appears to be quite unlike the empirical income distributions that emerge from contemporary official statistical sources in most
countries. But Pareto was only able to observe part of the income distribution and it worked for his data; it is still useful for some parts of the data available today. However, although it is handy as a shortcut indicator of inequality (the lower is coefficient $\alpha$, the higher is measured inequality) and although it is convenient to use as a tool for filling in the gaps where data in the upper tail are sparse, it is not clear that even there the Pareto functional form always works well in practice (Cowell 2013).

Second, is there a ‘tendency’ for distributions to take this particular form? Here one is going beyond a simple description of shape, where suitability could be determined by statistical criteria. A Pareto distribution could be the equilibrium outcome of a particular economic process (see section 4 below for a discussion of the meaning of this concept) but then its status as a ‘law’ is contingent upon whether the economic model associated with that process is appropriate.

Third, is there some kind of social dynamic that suggests that a particular Pareto distribution should be or ought to be the characteristic form of the distribution of income or wealth? Should one expect a particular value of $\alpha$ to emerge as part of some grand social and economic process over time or ought social planners aim at at a particular $\alpha$ in order to achieve some specific policy objective? These types of approach do little to illuminate the forces underlying wealth inequality and in some interpretations amount to little more than statistical determinism.

Again, I want to stress that this is not an attack on Pareto’s empirical methods but on the (mis-)interpretations of Pareto. Changing a useful empirical insight about the upper tail of distributions into a law does not advance understanding.

To avoid being derailed by a mantra approach and to obtain a more productive interpretation of the long run we need to examine carefully the nature of the economic modelling underlying the income distribution and the structure of wealth.

3. Modelling wealth

It is not hard to find examples of economic analysis and economic models that are unnecessarily complicated. A cynic might suggest that making it hard for the non-technical reader to grasp the elements of an economic model is an essential part of academic respectability. However, in the present case cynicism is misplaced: the analysis of the development of wealth distributions is necessarily complicated.

The two main reasons for the complicated nature of the problem are the diversity of forces that bear upon the economic mechanisms involved and the nature of time within any reasonably fleshed-out economic model. Let us briefly consider these in sections 3.1 and 3.2.
3.1 Distribution mechanisms

If we want an economic theory of distribution where should we look? The economist’s instinctive response is to compartmentalise the answer into (a) the role of the market mechanism and (b) the role of non-market mechanisms, with a strong presumption that the emphasis be on (a).

(a) Obviously the market is central to a lot of the long-run story in that it ultimately determines the rate of return to labour and capital, the factors of production. Furthermore, it is possible to construct fairly useful long-run models using an extreme simplification of the fine structure of the economy. Typically such economic models would consist of infinite-lived agents who trade with each other. Within such a framework it is easy to slip into the mindset that the market takes care of everything. But does it? Obviously not in situations where ‘market failure’ is important – imperfect and asymmetric information, externalities, public goods and the like. But in the context of analysing the long-run distribution of wealth we do not need to appeal to failure of the market in order to question the universality of the market as a distribution mechanism.

(b) If economists admit the importance of non-market mechanisms then perhaps their thoughts next turn to government. The role of government is obviously important in taxation and in providing and enforcing the legal and institutional framework that underpins wealth ownership and wealth transfers. But, arguably the government’s role is secondary to another class of non-market mechanism – voluntary transfers and inheritance.

3.2 Inheritance

Time linkages are central to the modelling of the inheritance process. It can be thought of as a story of two chapters, each with a different conception of time (Champernowne and Cowell 1998):

**Chapter 1: Intragenerational.** Think of a lifetime as a time interval that runs from initial age \( t_0 \) to final age \( t_1 \), where \( t_0 \) could be physical birth, or an economic starting point such as entry into the labour market and \( t_1 \) could be physical death or some other key economic exit point, such as the date of retirement; time could be thought of as flowing continuously from \( t_0 \) to \( t_1 \), or as passing in discrete units (years). Within the lifetime individuals or households plan economic activity that leads to either capital accumulation or the dissipation of assets. This sets the scene for the second chapter.

**Chapter 2: Intergenerational.** Think of society as the projection through time of a collection of dynasties, generation by generation. The sequence of generations could be imagined as a discrete-number conception of time. The link between the generations is the inheritance process: financial capital is
passed from generation \( n \) to generation \( n + 1 \). Of course, depending on the way in which institutions within society operate, other forms of wealth, such as human capital, may be passed too.

Taking a snapshot of wealth at any moment in history, one sees elements drawn from each of these two chapters. Representatives of different dynasties who happened to be born in different years will be alive at the same time. So to track the course of inequality one could look at the wealth distribution either in terms of what the overall picture looks like in a specific year or the in terms of the distribution among the dynasties for a given generation \( n \).

3.3 A meta-model

We need a simple sketch of how an economic model of the dynamics of wealth distribution can be constructed. The potential components of a model can conveniently be introduced using the two-chapter breakdown mentioned under the discussion of inheritance (section 3.2).

Mainstream economics is stronger on Chapter 1 of the wealth-dynamics story. Life-cycle models are conventionally used to explain savings behaviour; augmenting these models to cover a bequest motive is a simple, logical way of closing out the ‘Chapter 1’ account. Standard microeconomic modelling provides apparently reasonable answers to questions about the nature of risk taking, entrepreneurial activity; microeconomics gives us clear answers about the forces affecting work decisions and thus about labour market outcomes that affect the wealth accumulation process during the lifetime.\(^7\)

The Chapter 2 elements – linking one generation with the next – consist of a diverse collection of processes, social norms and institutions. A brief checklist would surely include the following three elements.

**Family formation**

If the rich form marriage alliances with the rich this clearly has a tremendous impact on the process of wealth concentration in the long run in contrast to social settings where partnership arrangements are much more diffuse (Blinder 1973). But other features of the family are also crucial in terms of the potential division or concentration of wealth during the inheritance process: the number of children in the family and the relationship between family size and parental wealth for example.

**Bequest motive**

Bequests may be consciously designed as an extension of the Chapter 1 planning process. We can imagine altruistic individuals taking into account the
interests of their direct and distant descendants. For those who have good
planning skills – or can afford to hire such skills – this may be a good account
of the bequest/inheritance process (Kopczuk 2007, 2013); often, however,
tergenerational transfers just come about by accident. Whether the planning
or the accident story is a better representation of reality is clearly going to
affect the way in which this distributional mechanism operates as the structure
of the wealth distribution develops over time.

Bequest division

The long-run dynamics of the wealth distribution must be affected by the way
the bequest of generation \( n \) is divided amongst the inheritors of generation
\( n + 1 \). In some cases this distribution is determined by formal legal restrictions
(usually with the objective of equal division) or sometimes by social custom
(often with highly unequal division):

Unlike the male codfish which, suddenly finding itself the parent of three
million five hundred thousand little codfish, cheerfully resolves to love them
all, the British aristocracy is apt to look with a somewhat jaundiced eye on
its younger sons. (Wodehouse 1924)

Clearly economics has things to say on each of these three elements: marriage
partners may be selected for pecuniary advantage as well as for love; strategic
issues may influence how much is left to posterity and to which members of
posterity. But many of these processes and institutions are commonly seen as
being as being essentially non-economic in nature.

The parts of the wealth-modelling problem described so far apply to isolated
agents – to individuals and to families. These disparate parts need to be
brought together.

The modelling of the behaviour of individual dynasties within society could
be combined with a macroeconomic model that jointly determines the rate of
growth of output and the returns to the productive factors, labour and capital
(Stiglitz 1969, Conlisk 1977, Bourguignon 1981). This then closes the model
since the market return to labour determines the earnings that feature in
Chapter 1 of the inheritance story and the ownership of capital appears in the
portfolios of the individual members of the dynasties mentioned in the
description of Chapter 2.

3.4 Assembling the pieces

Without specifying the exact form of any one of the components listed above
it is clear that the combined model could indeed be very complicated. So, what
do we do? Here are three ideas:
**Numerical simulation.** When formulas are too tough for a closed-form solution it is tempting to use numbers to cut the knot. Too often this approach leads to mind-numbing reams of tables and not much insight.\textsuperscript{10}

**Focus on one feature.** A good economist might follow that characteristic economist’s instinct and go back to being a determined little thinker. There is some merit in this technique, in that it enables one to examine precisely the impact of a particular model component under the *ceteris paribus* assumption. However, this is fine for a neat point in a technical journal but it is not much good for the big picture.\textsuperscript{11}

**Big picture.** Trying to represent the big-picture view of this complicated process might seem at best mere arm waving and at worst not just arm waving but drowning in a sea of speculation. But it does not have to be. As Piketty shows, there is a sensible way to carry the academic discourse forward: to do so one needs to understand the nature of equilibrium in this context.

### 4. Equilibrium distribution

The meta-modelling approach enables us to take a closer look at the nature of the long run. Given a specific model within the scheme set out in section 3.3 we can characterize a long-term equilibrium. To do this requires some care.

Again use $F$ to denote the distribution function of wealth, but let us make clear the specific generation to which the distribution applies: the wealth distribution that is present in generation $n$ is given by the function $F_n$ (a snapshot of the actual wealth distribution at a given moment in time will depend on the age distribution as well as $F_n$). The meta model can be developed further to take into account the generational sequence of wealth distributions. Suppose that the economic mechanisms and institutions present in the two chapters of the inheritance story set out in section 3 have been precisely specified: summarize these as a combined process $P$ that takes a historically pre-existing wealth distribution in generation $n$ as given and transforms it into another distribution in generation $n+1$. We can depict this in sketch form as follows:

$$F_n \mapsto P \mapsto F_{n+1}.$$  

The first arrow depicts the ‘input’ to the combined process $P$ and the second arrow depicts the ‘output’ from $P$. Applied recursively this gives a development story of the way in which the wealth distribution evolves through time, here interpreted as meaning over the generations $n = 1, 2, 3, \ldots$. This development story is valid for as long as the process $P$ can reasonably be assumed
to remain unchanged. For a given process $P$ it may be that we can find some particular distribution $F^*$ that has the following property:

$$F^* \implies P \implies F^*.$$ 

If so, then we can call the self-reproducing distribution $F^*$ an *equilibrium distribution* (see also Piketty 2014: 361–66). Clearly the nature of the equilibrium distribution depends on the process $P$.

So, in answer to the question ‘what happens in the long run?’ there broadly appear to be three possibilities:

1. *Convergence to equality*. In principle $F^*$ is a degenerate distribution with all the probability mass at the same point.
2. *Convergence to inequality*. In principle $F^*$ is a non-degenerate distribution with positive inequality. It need not represent a situation of stasis: there could be mobility with individuals or families who are among the poor in one generation being among the rich in a later generation; but the cross-sectional distribution remains the same in perpetuity.
3. *Divergence*. For the specified $P$ no such $F^*$ exists.

This is a broad-brush picture and needs to be qualified. First, the ‘in principle’ clause in Cases 1 and 2 above is important. Convergence to equality (Case 1) need not literally mean that in the long run every agent has the same wealth level or the same income. There may be some noise arising from stochastic components of individual endowments, variability in tastes or random shocks. In the same way, in Case 2 it is an over-simplification – although maybe a useful over-simplification – to see the equilibrium wealth distribution consisting just of permanent structural inequality (Champernowne and Cowell 1998, Cowell 2012); again there may be noise which blurs the otherwise clear picture of rigid inequality in the long run. Second, Case 3 can take more than one form: there could be indefinitely increasing inequality as long as $P$ remains unchanged, or there could be long term fluctuations in the distribution of wealth and income.

What kind of shape would this equilibrium distribution have? There is a substantial sub-literature that shows how tightly specified models of wealth dynamics will in the long run yield an equilibrium that can be characterized as a familiar functional form, such as the Pareto distribution, mentioned above. But it is also quite common to find that an interesting story of wealth dynamics yields an outcome that cannot be easily represented in a standard formula. However, in the absence of a simple formula, the precise shape of the equilibrium distribution may not be so important: it is sometimes enough to know that an equilibrium exists and that there is convergence to the equilibrium. Within that context it may then be possible to discuss the effect of substantial shocks to or changes in the model environment – wars, depression,
technological change – in order to understand how the equilibrium will change and it may be possible to understand how policy instruments will affect the equilibrium distribution.

5. The long run and policy

There are only two jokes in economics; neither of them is very funny and one of them has the punchline ‘assume that there’s a can-opener’. But in this case, the statement ‘assume that there’s an equilibrium’ is not a flippant aside: the ‘can-opener’ provided by the equilibrium distribution device really helps us to distinguish different types of long-run behaviour. The key question about the long run is not something like ‘how long is the long run?’ but ‘is long-run equilibrium the appropriate focus for understanding the nature of economic inequality and the effect of policy toward inequality?’

Piketty’s own primary focus is on the long-run relationship between the rate of return to capital ($r$) and the growth rate of output ($g$). The reason for this is that if the difference $r - g$ is positive and the elasticity of substitution between capital and labour is large then, in long-run equilibrium the share of capital in total income must be rising. Given that capital income (income from wealth) is more unequally distributed than labour income, this $r - g$ criterion can be seen as the motor in the process $P$ driving inequality in the long run (Piketty 2014: 350–353). He argues that the force generated by the $r - g$ gap is likely to be stronger than, say, a policy reform affecting the inheritance transmission mechanism, such as the introduction of the Napoleonic Civil Code.

But, to my mind, a question of similar magnitude concerns whether or not $P$ is associated with a long-run equilibrium at all. If it is, then marginal modifications of a tax on wealth or on the transfers of wealth (in the form of an inheritance tax) can be expected to nudge the equilibrium in the direction desired by the designer of economic policy. If it is not, then the presence or absence of a governmental redistributive mechanism may be crucial in influencing whether there is an explosion of inequality.

So, is the $r - g$ criterion going to become a new mantra of inequality? I hope not. But the insights provided by Piketty (2014) will certainly become the basis for serious economic analysis of inequality and associated policy questions.

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Notes

1. In fact popular discussion of Kuznets focuses on just one of two forces mentioned in his 1955 paper. He also discusses the role of growth and the concentration of savings in a manner that is very similar to Piketty: ‘one can then say, in general, that the basic factor militating against the rise in upper income shares that would be produced by
the cumulative effects of concentration of savings, is the dynamism of a growing and free economic society’ (Kuznets 1955: 11; see also Atkinson, Piketty, and Saez 2011: 57).

2. Disarmingly he admits: ‘In concluding this paper, I am acutely conscious of the meagerness of reliable information presented. The paper is perhaps 5 per cent empirical information and 95 per cent speculation, some of it possibly tainted by wishful thinking.’ (Kuznets 1955: 26).

3. However, he only used a proportion of the statistical evidence available from income-tax data available at the time.

4. This might more properly be termed ‘Pareto’s first Law’. The formula is for what is formally described as a Pareto Type-1 distribution – Pareto also suggested more sophisticated formulas for characterizing income and wealth distributions (Kleiber and Kotz 2003; Cowell 2011: Appendix A).

5. As examples of the first see the versions of a rather odd ‘mathematical theory of history’ to be found in Davis (1941) and Bernadelli (1944); as an example of the second see Adams (1976).

6. Piketty (2014) discusses both of these mantras but does not get sidetracked.

7. Of course, this version of Chapter 1 is not the only apparently reasonable approach: the standard account based on well-informed maximizing agents has been called into question by the behavioural-economics literature.

8. See the discussion of the impact of the French Civil Code contrasted with the earlier system of primogeniture – Piketty 2014, 361–65. Much of the US literature focuses on bequests as offsetting differences between children and thus acting as a force promoting equality – see, for example Becker and Tomes (1979), Menchik (1979), Sheshinski and Weiss (1982). However Chu (1991) provides an interesting alternative argument for primogeniture as a rational outcome within families.


10. In contrast Piketty (2014) does this very successfully – see for example ‘Will China Own the World’ on pages 460–63.

11. Piketty himself does this to illuminate specific issues in his technical contributions to the literature – see the on-line appendix to Piketty (2014).

12. Becker and Tomes (1979) provide a general model of the long run which in essence has this kind of outcome.


14. Of course the first question is important for understanding the empirical relevance of analytical devices such as equilibrium growth models (Atkinson 1969).

15. Technically the requirement is that the elasticity should be greater than 1. See Atkinson (2014) in this issue for a detailed discussion.

Bibliography


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