1. Introduction

Allingham and Sandmo's (1972) seminal contribution to the theory of tax evasion evoked the following response from Kolm (1973): 'But this is hardly public economics; in fact it is very private' (p. 265). Kolm's attempt to rectify this situation has since spawned a literature which does consider what government policy should be in the face of evasion.¹ The problem typically addressed is how the government should set the parameters of the tax and penalty system if it has to collect a fixed amount of revenue from tax-payers who are prone to evade. In general, however, no attempt is made to consider what the government might be doing with that revenue.² This seems a curious oversight, since while the government taketh away, it also giveth back, and the latter activity surely exerts some influence on evasion. This paper attempts to remedy this neglect of the expenditure side of government activity and so provide the basis for a more public theory of tax evasion.

Considering both sides of the government budget not only makes the theory more comprehensive, but it also allows the tax evasion decision to be embedded within a general model of the allocation of public and private goods. This treats the problem of efficient good provision as a case of the Prisoner's Dilemma or Isolation Paradox game form: as is well known, such a set-up contains an in-built motive for individuals to cheat, and in the

¹We thank A.B. Atkinson, J.B. Davies, J.R. Kesselman, A. Sandmo, seminar participants at Bonn, Geneva, Kiel, Southampton and U.B.C. and an anonymous referee for helpful and constructive comments. Errors remain our own.
²See, for example, Sandmo (1981), Cowell (1985), and Slemrod and Yitzhaki (1987). Exceptions are the original Kolm paper, Gottlieb (1985) and Hansson (1985). The effect of how the government spends taxes on the labour supply decision has however been considered. See Lindbeck (1982).
present context such cheating takes the form of tax evasion. Such an approach introduces naturally a 'public dimension of interdependence amongst tax-payers rather than imposing interdependence by invoking 'stigma' or 'social custom' as some recent work has done. It also permits investigation of a determinant of tax evasion recently emphasised by Becker, Büchner and Sleeking (1988): the use to which tax revenue is put. One effect that will be considered is the impact of providing different types of public goods on the private evasion decision.

The approach is also able to throw some light on the empirical evidence concerning the effect on tax evasion of a change in the tax rate. The standard theoretical prediction, following Yitzhaki's (1974) modification of the Allingham–Sandmo model, is that this will be negative, which is both counter-intuitive and at odds with the econometric evidence. Moreover, this is a theoretical result which appears to be relatively robust; for example, Gordon (1988) shows that even with stigma and reputation costs present, there is no guarantee that the tax rate effect will be positive. The issue is an important one, however, since it has a bearing on the long-run development of the size of the irregular economy: will this increase or decrease as the public sector grows relative to the private sector? Will this increase or decrease with the current trend towards tax cuts? In this paper the conventional assumptions about decision-making under uncertainty are made, but account is also taken of the impact of the presence of public goods on the making of those decisions. In so doing it is found, in a sense to be made precise, that it is the relative abundance of private and public goods that is crucial to the relationship between tax rates and tax evasion.

2. Model structure

We retain the standard framework of decision-making under uncertainty

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3See, for example, Benjamini and Maital (1985) and Gordon (1988).

4To see this relationship, recall that in the Allingham–Sandmo model a tax rate change has both an income and a substitution effect. If absolute risk aversion is decreasing, these work in opposite directions, so the net effect on the amount of income concealed is ambiguous. This problem is resolved by Yitzhaki (1974), who shows that if the penalty is levied on tax evaded rather than income concealed (the system in the United States), then the substitution effect disappears. Under decreasing absolute risk aversion an increased tax rate then unambiguously implies less income concealed. Christiansen (1980) subsequently shows that not only will more income be reported, but also that less tax will be evaded (the Yitzhaki result appears necessary for this but not sufficient). The existing theory thus predicts that tax evasion decreases with the tax rate.

5The time series evidence of Crane and Nourzad (1986) and Poterba (1987) seems more relevant than Clotfelter's (1983) cross-sectional study. As in the present model, there is a (single) uniform tax rate in the time-series studies (where tax evasion is regressed on an average of the various marginal tax rates found in the cross-section). In any case the model presented here needs time-series data to be tested, since at least at the national level, the cross-section will contain no variation in the level of public good provision.
for the individual tax-payer. This individual has income $Y$ liable to tax, not all of which may be reported to the tax authority. There is a fixed probability $\pi$ that the individual is investigated, caught (if evading) and punished with a surcharge on tax liability proportional to the evaded tax. Let $t$ be the (uniform) tax rate, $E$ the amount of income concealed ($0 \leq E \leq Y$), and $s$ the surcharge rate. Then the individual has to decide the amount $E$ (and hence $tE$) of a 'risky asset' (evaded tax) to 'purchase'; $tE$ has the random rate of return $r$ given by:

$$r = 1, \quad \text{with probability } 1 - \pi,$$

$$= -s, \quad \text{with probability } \pi,$$

so disposable income (consumption) is a random variable $c$ given by:

$$c = (1 - t)Y + rtE. \quad (1)$$

Suppose there are $n$ identical individuals, all of whom face the same decision problem. The revenue raised by the government is:

$$R = \sum [Y - c] - n\phi(\pi) = ntY - \sum rtE - n\phi(\pi), \quad (2)$$

where $n\phi(\pi)$ is the dollar cost of enforcing a probability of detection $\pi$ in a community of size $n$. It is assumed that $\phi(\pi) \geq 0$. If $n$ is not too small, given random enforcement, $\sum r$ in (2) may be replaced by $n$ times $\mathbb{E}[r]$ (with $\mathbb{E}$ the expectations operator).

The revenue raised could be thrown away, or it could be used to provide public goods which yield utility to tax-payers. Assume that the quantity of public good services, $z$, enjoyed by each (identical) tax-payer is proportional to $R$. To allow for the possible effect of the size of the community, let $z = R/\psi(n)$, where $\psi$ is a function such that $1 \leq \psi(n) \leq n$. Assume further that:

$$\lim_{n \to \infty} \frac{1}{\psi(n)} = 0,$$

$$\lim_{n \to \infty} \frac{n}{\psi(n)} = \frac{1}{\psi} > 0. \quad (3)$$

$^6$The probability of detection is assumed fixed to keep the analysis simple. In reality individuals will have some control over $p$. For example, Allingham and Sandmo (1972) suggest that $p$ might decrease with $Y - E$ (income declared) because the authorities have some idea of what $Y$ should be for any particular profession and the greater the discrepancy between that $Y$ and declared income the more likely they are to investigate.
so, for example, the function $\psi = n^\sigma$, $0 < \sigma \leq 1$, would do. In a small economy, where the individual feels large enough to matter, he or she obviously has to take account of the impact of his or her decision to evade on the supply of public goods. The individual may also take into account the effect of his or her decision on other tax-payers' decisions on how much to evade. Assume that the individual behaves in the classic Cournot fashion. This implies taking as given the evasion activity $E$ of any other tax-payer, along with the tax and enforcement parameters $(\pi, s, t)$ and the functions $\phi$ and $\psi$. Hence, the individual takes as given the level of $z$ that would be provided if he or she were dead honest:

$$\tilde{z} = \frac{n[tY - \phi(\pi)] - [n - 1]\bar{t}E}{\psi(n)},$$

where $\bar{r} = \delta[r] = 1 - \pi - n\pi$, the expected private rate of return per dollar of evaded tax; a positive number by assumption.

In view of the above, the amount of public goods enjoyed by each taxpayer can be written:

$$z = \tilde{z}(n, t) - \frac{rtE}{\psi(n)}.$$

The individual has a concave cardinal utility function $U(c, z)$, and preferences in the face of uncertainty conform to the von Neumann–Morgenstern axioms. Accordingly, the individual seeks to maximise

$$\sigma[U(c, z)]$$

by choice of $E$, where $c$ and $z$ are determined by (1) and (5), above.

The first-order condition for the maximum of (6) yields:

$$\delta \left[ \left\{ U_c - \frac{U_z}{\psi(n)} \right\} r \right] - 0,$$

if $0 < E < Y$ (with subscripts denoting partial derivatives). A corner solution with perfect compliance is ruled out if $(U_c - U_z/\psi)\bar{r}$ evaluated at $E = 0$ is positive. Hence, in a small economy, positive evasion requires that both (i)

7 Note that $\psi(n)/n$ is the marginal rate of transformation between $c$ and $z$, and the precise form of $\psi(n)$ depends on the extent to which public goods are rival [see Borcherding (1985)]. For example, in the isoelastic case the degree of rivalness increases with $\sigma$, with $\sigma = 1$ implying that public goods are absolutely rival. Note that the other extreme – pure non-rivalness ($\sigma = 0$) – does not have the required limiting properties and is thus not permitted.

8 See, for example, Cornes and Sandler (1985). This assumption is relaxed in section 4.
\[ U_c > U_z/\psi \] (evaluated at \( E = 0 \)), and (ii) \( r > 0 \). The value of taking a pound away from the government (gain in \( c \) less loss of \( z \)) must be positive \textit{and} the evasion gamble must be better than fair. With a very large population, \( n \to \infty \), so (7) becomes:

\[ \delta[U_c] = 0, \quad (7') \]

the Allingham–Sandmo first-order condition. Since the individual is too small to personally affect \( z \), a corner solution at \( E = 0 \) is now ruled out simply by the evasion gamble being better than fair. It is clear that some of the comparative static results for this model will be very similar to the elementary model without public goods. For example, it is not hard to show that, under plausible conditions, an increase in the probability of investigation or in the surcharge will reduce evasion. However, the effect of other parameter changes is more subtle.

3. Tax rates and evasion in a large economy

The important application of this model is to examine the effect of an across-the-board increase in the uniform tax rate \( t \). Differentiating the first-order condition (7) with respect to \( t \):

\[ \frac{\partial (tE)}{\partial t} = -\delta[U_c]Y + \frac{\delta[U_c]E + \delta[(U_{cz}Y - U_{zz}E)]E}{\psi(n)D}, \quad (8) \]

where \( D = -\delta[\{U_c r - 2U_{cr} \psi(n) + U_{cr} \psi(n)^2 \} r^2] \), is non-negative in view of the concavity assumption, and strictly positive if the individual is strictly risk averse. In this section a large economy \( (n \to \infty) \) is assumed, so (8) becomes:

\[ \frac{\partial (tE)}{\partial t} = \frac{\delta[U_c]Y - \delta[U_c]E \hat{E}}{\psi(n)D}, \quad (8') \]

which shows the initial change in tax evasion conditional on \( \hat{E} \), the expectation of the change in the level of public good provision. Without perfect foresight this predicted change will almost certainly be falsified by the actual change, thus triggering off a process of adjustment and readjustment. The final effect is found by using (4) to solve (8') for the change in the (symmetric) Nash equilibrium considered in section 2:

\[ \frac{\partial (tE)^N}{\partial t} = \frac{\delta[U_c]Y - \delta[U_c]Y \psi}{\delta[U_c]r^2 - \delta[U_c]r \psi}. \quad (9) \]
Note that the denominator of (9) is negative if the Cournot–Nash equilibrium is stable.\footnote{Stability requires that the reaction function be a contraction, so the absolute value of $\frac{\partial \delta}{\partial (tE)}$ must be less than one. This is satisfied if $|\delta[U_{c}r]| > |\delta[U_{c}r]|/\bar{\theta}$, which makes the denominator of (9) negative.}

The numerator of (9) is less easy to sign. Some progress is made by differentiating (7') with respect to true after-tax income (while maintaining the Cournot assumption):

$$\frac{\partial (tE)}{\partial Y(1-t)} = -\frac{\delta[U_{c}r]}{\delta[U_{c}r^2]}$$

which is familiar as the income effect on the demand for a risky asset. It is known from Arrow (1970) that $\delta[U_{c}r] > 0$ given decreasing absolute risk aversion. This assumption would thus make (10) positive. Note that if $U_{cz} = 0$, then (9) is $-Y$ times (10), so given additively separable preferences the change in the Cournot–Nash equilibrium level of tax evasion is negative under decreasing absolute risk aversion. This is the ‘Yitzhaki–Christiansen’ effect discussed earlier (see footnote 4) which leads to an inverse relationship between tax evasion and the tax rate. By contrast, if $U_{cz} \neq 0$, the sign of (9) becomes ambiguous and the possibility cannot be excluded that an increased tax rate increases evasion even under standard assumptions on risk aversion.

An insight into the likely effect on the sign of (9) of a non-zero $U_{cz}$ is gained by differentiating (7') with respect to $\bar{z}$ (again assuming zero conjunctural variation):

$$\frac{\partial (tE)}{\partial \bar{z}} = -\frac{\delta[U_{c}r]}{\delta[U_{c}r^2]}$$

The sign of $\delta[U_{c}r]$ thus determines how an individual making the Cournot assumption would respond to a change in public good provision. A tax change in a large economy can thus be seen to have two different income effects. The first is the conventional one stressed by Yitzhaki (1974). The second arises from the post tax-change (balanced-budget) adjustment to government expenditure. If this second effect is positive, it can be thought of as in some sense compensating for the first one; if it is negative it only serves to reinforce the first. Given that the denominator of the expression in (9) is negative (see footnote 9), it can now be seen that $\delta[U_{c}r] > 0$ is necessary if tax evasion is to increase with a rise in the tax rate. Sufficiency obviously requires that this second income effect also be larger in absolute terms.

A cursory glance at this necessary condition suggests that the tax-payer’s behaviour can be fully and conveniently characterized by a condition on the
cross-partial derivative $U_{cz}$. However, since it is the expectation of the product of this partial derivative with $r$ that is important, and since $r$ takes both positive and negative values, merely imposing a sign restriction on $U_{cz}$ is not helpful. And this is so even if one tries to simplify the problem by requiring that $U_{c}$, be independent of $r$, so that (11) has the sign of $\delta[U_{cz}]$. This is because such a restriction implies that $U(c,z)$ must have the form $	heta_1(c) + \theta_2(z) + c \theta_3(z)$, where the $\theta$'s are non-decreasing concave functions, which in turn implies that $U_{cz}$ must be non-negative. Thus, it is clear that this approach would exclude a whole class of interesting cases (i.e. those in which $U_{cz}<0$) without throwing much light on the relative magnitudes of the components of the numerator in (9). It will be seen that the important issue which determines the sign of (9), and hence the behaviour of tax evaders with respect to changes in $t$, is not the sign of $U_{cz}$, but a condition which can be interpreted in terms of the relative abundance of public and private goods. This is shown for two special cases; in one of which $c$ and $z$ are Edgeworth-Pareto substitutes, in the other $c$ and $z$ are complements.

3.1. Ziff public goods

Let the utility function be such that $z$ has zero income effects (Ziff). This implies that $m = U_z/U_c$, the marginal valuation of public goods using private goods as numeraire, is independent of $c$. This situation is illustrated in fig. 1. With $U_z/U_c$ constant, the Ziff assumption implies $U_{cz}U_c - U_{cz}U_z = 0$, so that $\delta[U_{cz}r] = \delta[U_{cz}r]m$. Thus, (9) becomes:

$$\frac{\partial(\psi Y)}{\partial t} = \frac{\delta[U_{cz}r](1-m/\bar{r})Y}{\delta[U_{cz}r^2] - \delta[U_{cz}r]m}$$

since in a large economy $m$ is independent of the realisation of $r$.

Under decreasing absolute risk aversion the denominator in (12) is negative, so the sign of (12) turns on the sign of $1-m/\bar{r}$. Observe that the effect on utility of an increase in the size of the public sector is given by:

$$\frac{\partial \delta[U]}{\partial t} = Y[-\delta[U_c] + \delta[U_z]/\bar{r}]$$

This expression defines a sense in which public goods can be thought of as being under- or over-provided. If it is positive, then public goods are under-

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10The case $U_{cz}=0$ is trivial since this obviously reduces the model to the Allingham-Sandmo case: the public sector becomes, by definition, irrelevant to private choices. Gottlieb (1985) makes this assumption and replicates their comparative static results.

11This assumption is quite common in the analysis of public good provision - see, for example, Green and Laffont (1979).
provided since the individual would welcome an expansion of the public sector with its consequent implications for enforced taxation. Corresponding interpretations apply in the cases where the expression is zero or negative. In the case of Ziff public goods, the term in square brackets in (13) can be written as \( \partial U / \partial t \) has the sign of \( m / \psi - 1 \). From (12), \( \partial (tE)^N / \partial t \) has the sign of \( \partial [U_{cr}] / [1 - m / \psi] \); so if individuals display decreasing absolute risk aversion, the effect on tax evasion of an increase in the tax rate is positive or negative as public goods are under- or over-provided.

3.2. Homogeneous utility functions

Suppose that \( U(c, z) \) is a cardinal utility function, homogeneous of degree \( \beta \). Then \( U_c \) will be homogeneous of degree \( \beta - 1 \). It must therefore be the case that \( U_{cc} + U_{cz} = [\beta - 1] U_c \) (using Euler's theorem). Hence, \( \partial [U_{cr}] = \partial [U_{cr}] / z + (\beta - 1) \partial [U_{cr}] / z \). In view of (7') the last term is zero. Thus, (9) becomes:

\[
\frac{\partial (tE)^N}{\partial t} = \frac{\partial [U_{cc}] Y + \partial [U_{cc}] Y / \psi z}{\partial [U_{cr}] Y + \partial [U_{cr}] / \psi z}.
\]

(14)

It has been noted that the first term in the numerator is positive under decreasing absolute risk aversion. Arrow (1970) also reveals that the second
term is negative under increasing relative risk aversion. Since Arrow's preferred hypothesis is that risk aversion decreases absolutely, but increases relatively, this makes for an interesting problem. Given that the denominator is negative, (14) is more likely to be positive under these assumptions the smaller is $\psi_2$. Thus, tax evasion is more likely to increase with the tax rate the lower the existing level of tax collected. Again under-provision of public goods may lead to tax evasion increasing with the tax rate.

Hence, in both these special cases, an increase in the tax rate leads to an increase or a decrease in evasion according to whether public goods are in some sense under- or over-provided. The intuition behind this result is straightforward. If public goods are under-provided (as defined), then when the government increases their provision (via a higher tax rate), it runs the risk of making individuals on balance feel better off. Under decreasing absolute risk aversion this will make them increase the size of their gamble, i.e. evade more tax. In this section a second tax change induced wealth effect has thus been isolated and some insight has been gained into the conditions under which it is likely to neutralise the traditional one. A model of tax evasion which incorporates public good provision is thus able to explain a positive relationship between evasion and the tax rate.12,13

4. Rivalness, conformity and evasion

It seems reasonable to suppose that the type of public good provided will affect the evasion activity of tax-payers. This is indeed the case here, and may be examined by considering the two fundamental characteristics of public goods: non-rivalness and non-excludability. To do so, drop the assumption of a large economy. The degree of rivalness can be represented by $\psi(n)$: if there is a parametric shift so that $\psi(n)$ increases, this may be taken to indicate an increase in the rivalness of the publicly-provided good, since with $z = R/\psi(n)$ each person gets a smaller enjoyment of services out of a given total of public expenditure. The characteristic of non-excludability can be captured by re-examining the conjectured response of fellow tax-payers to an individual's own tax evasion.

Until now it has been assumed that each tax-payer makes the conjecture

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12 Agnar Sandmo has pointed out that the model presented in this paper readily generates $\partial(tE)/\partial t > 0$ within the original Allingham–Sandmo model – for example, if the two income effects were to offset each other only the substitution effect would remain.

13 A straightforward extension to the model would be to introduce a publicly supplied private good, consumption of which is linked to the individual's tax payments. Then instead of (1), $c = (1-t)Y + ntE + \theta(\bar{E} - E)$, where $\theta$ is the proportion of tax payments returned by the government as contribution-linked private goods. The condition for positive evasion in a large economy would be $U_\partial(\bar{E} - \theta) > 0$ which means that evasion must be more than better than fair. If $\bar{E}$ varies across individuals, an increase in $\theta$ would be expected to decrease evasion via an increase in the numbers at the $E = 0$ corner solution.
of a zero response by all the other tax-payers – the standard Cournot assumption. This assumption is now modified so that the conjectured response of any other individual to one's own evasion is given by:

$$\bar{E} = E_0 + qE,$$

(15)

where $E_0$ is some constant and $q$ is an 'index of conformity', $0 \leq q \leq 1$. The case $q=1$ is where each person assumes that everyone else will instantly conform their behaviour to his or her own; $q=0$ is of course the Cournot case again.

Substituting (15) into (6) the objective function becomes:

$$E v(1-t)Y + rtE,$$

$$-rtE - q(n-1)\bar{E},$$

(16)

where $K = n[tY - \phi(n)] - [n-1]\bar{E}$. And instead of (7) the first-order condition is now:

$$\frac{\partial V}{\partial E} - \frac{1}{\psi(n)}[r + (n-1)\bar{E}] = 0.$$

(17)

Note also that $q$ can be taken as an indicator of ‘excludability’ of the public good. To see this, consider a large economy where taxes are not enforced, then if there is a level $q$ of conformity this produces the same equilibrium as if a proportion $q$ of the publicly-provided goods could be charged for at price $\bar{\psi}/q$ (the proceeds being used to finance all public goods). The implication of this equivalence is that the higher the degree of conformity, the lower the ‘price’ the individual effectively pays for the public goods obtained in exchange for reducing his or her evasion.

We now investigate the effect of changes in $q$ and $\psi$ on the nature of the tax-evasion equilibrium. First, note that if $\bar{r} > 0$, $n > 1$ and $q < 1$, then the outcome will be Pareto inefficient and involve positive evasion.\(^{15}\) To see this note that the condition for a Pareto-efficient level of public good provision is that $U_e/U_z = \psi(n)/n$. Next we consider the effect of an individual’s expected utility of an increase in evasion in the neighbourhood of the zero-evasion Pareto-efficient allocation, taking as given the conjecture (15). Since $U_e$ and $U_z$ are independent of the state of nature if $E=0$.

\(^{14}\)Strictly speaking (16) only holds for $n\gg 1$ so that the (random) effect of everyone else’s evasion on public revenue can be represented by the expected value $[n-1]\bar{E}$.

\(^{15}\)See Gottlieb (1985). Note that $E$ has to be zero at a Pareto-efficient allocation, for otherwise there is a welfare loss to risk averse individuals because of the uncertain realisation of $e$. 
\[ \frac{\partial \mathcal{E}(U)}{\partial E} \bigg|_{E=0} = U_c \tilde{r} t [n-1][1-q]/n, \] (18)

which is clearly positive under the conditions specified above. Hence, unless \( q=1 \), and there is perfect conformity (excludability), all individuals choose a positive level of evasion which reduces public good provision below its optimal level.\(^{16}\) Note that this holds independently of the degree of rivalness of the public good.

Second, for any equilibrium with positive evasion, consider the effect of a pure increase in conformity. This may be modelled as a small increment in \( q \) and a corresponding adjustment in \( E \) such that \( E \) is kept constant. Eq. (17) yields:

\[ E_q = - \frac{\tilde{r} [n-1] \mathcal{E}(U_z)}{\tilde{D} \psi(n) t}, \] (19)

where

\[ \tilde{D} \equiv - \mathcal{E} \left[ U_{cz} r^2 - 2 U_{cz} (r + q(n-1)\tilde{r})/\psi + U_{zz} r^2 (r + q(n-1)\tilde{r})^2/\psi^2 \right]. \]

Since \( \tilde{D}, \tilde{r} \) and \( U_z \) are, by assumption, all positive, it is immediate from (19) that an increase in conjectured conformity leads to a decrease in everyone’s evasion. As noted above, an increase in conjectured conformity is equivalent to a decrease in the price of excludable publicly supplied goods. The latter would cause increased voluntary sacrifice of private goods and also reduce evasion.

A similar result is available for \( \tilde{\psi} \) from (3), the limit of \( \psi(n)/n \) in a large economy with some conformity. Let \( n = \infty \) and \( q>0 \). Then it can be shown that:

\[ E_{\tilde{\psi}} = \left[ \mathcal{E}(U_{cz} r) - q \frac{\mathcal{E}(U_{zz}) \tilde{r}}{\tilde{\psi}} \right] \frac{\tilde{\psi}}{t \tilde{D}} + \frac{q \tilde{r} \mathcal{E}(U_z)}{t \tilde{D} \tilde{\psi}^2}, \] (20)

where \( \tilde{\psi} < 0 \), is the effect of an increase in \( \tilde{\psi} \) on the supply of public goods. The first term on the right-hand side of (20) is the direct effect of an increase in \( \tilde{\psi} \) on evasion via the supply of public goods. It is easily checked that it is

\(^{16}\)Thus, the Prisoner’s Dilemma disappears with perfect conformity. However, \( q=1 \) is not sufficient to eliminate evasion unless \( U_z \) is unbounded below. For otherwise it is possible to have \( U_z/\tilde{r} < \psi(n)/n \) for all \( \tilde{\psi} > 0 \) and, for the case \( q=1 \) and \( E=0 \), instead of (18):

\[ \frac{\partial \mathcal{E}(U)}{\partial E} = U_c \tilde{r} t [1 - mn/\psi(n)] > 0; \]

hence the individual would want to evade for all values of \( \tilde{\psi} \) even with perfect conformity.
negative for Ziff public goods if absolute risk aversion is decreasing. The second term is the indirect effect via conformity: it works by raising the implicit price of excludable goods, and is clearly positive. Which of these two counteracting effects is stronger depends on the amount of conformity.

As to the effect of a tax change, differentiating (17) with respect to \( t \) yields:

\[
tE_t + E = H + t \frac{\partial \left[ U_{zz} Y - U_{zz} Z^2 \right]}{\hat{D} \hat{\psi}(n)} r[n - 1] q,
\]

(21)

where \( H \) is the right-hand side of (8) with \( D \) replaced by \( \hat{D} \). This illustrates the increased difficulty of discerning the sign of the tax rate effect once the Cournot assumption is relaxed. Even assuming a large economy where \( q \) remains positive is of little help, since the second term on the right-hand side of (2) would not then disappear. All that can be said is that in an economy (of whatever size) with some conformity, the relative abundance of public goods is not the sole determinant of how evasion responds to a tax rate change.

The device of using the number \( q \) to represent conformity also offers an insight into the likely relationship between the size of the economy and the prevalence of evasion. Within the confines of the present model it is reasonable to suppose that \( q \) is a non-increasing function of \( n \): for very small numbers the situation is akin to a classful of identical naughty children, in that each knows that if he or she starts throwing paper darts the others will do so too (\( q = 1 \)); for very large numbers sheer ignorance about some of one's fellows may lead to \( q \) being substantially less than unity. So, as the economy is replicated, the conformity effect can be expected to lead to an increase in evasion by each tax-payer.

5. Heterogeneous taxpayers

The robustness of the results obtained so far can be ascertained by allowing for the possibility that individuals differ. In order to avoid the problem of population size, assume that there is an infinite population of infinitesimal individuals. Suppose individuals are characterised by two distinguishing features: their income \( y \), and their personal attributes \( z \), both of which are exogenous. These personal attributes could include a wide variety of preference and attitude parameters. In particular, it is assumed that \( z \) captures the tax-payer's taste for public goods, risk aversion, honesty or conformity coefficient and investigation class. An individual's conformity coefficient \( q(y, z) \) is a measure of how he or she conjectures other people to adapt their behaviour to his or her own. This will be zero for most people, but there will be some whose influence exceeds their size. The investigation
class could be determined by the type of income, occupation or geographical region. The probability of investigation for any individual is now \( \pi(x) \).\(^{17}\) Some, but not all, of a person's \( x \) may be publicly observable. Let there be a joint distribution function \( F(y, x) \) in the population as a whole.

We denote the evasion by a \((y, x)\)-type person as \( e(y, x) \). Average income is defined as \( Y = \int y \, dF(y, x) \) and \( E = \int e(y, x) \, dF(y, x) \) is average evasion. It seems reasonable to suppose that an individual with positive \( q(y, x) \) will be concerned only with the overall reaction to his or her own evasion, but not the distribution of responses among the various types in the community. Hence, the analogues of (15) and (4) are:

\[
\bar{r}E = E_0(y, x) + q(y, x)e(y, x),
\]

\[
z = [tY - \bar{r}E]/\bar{\psi}.
\]

The optimisation problem is essentially the same as in the identical tax-payer case. Optimal evasion by any \((y, x)\)-type person must satisfy:

\[
\delta \left[ U_c r(x) - q(y, x) \frac{U_z}{\psi} \right] \leq 0, \quad \text{if } e(y, x) = 0, \tag{24i}
\]

\[
\delta \left[ U_c r(x) - q(y, x) \frac{U_z}{\psi} \right] = 0, \quad \text{if } 0 < e(y, x) < y, \tag{24ii}
\]

\[
\delta \left[ U_c r(x) - q(y, x) \frac{U_z}{\psi} \right] \geq 0, \quad \text{if } e(y, x) = y. \tag{24iii}
\]

So, in an equilibrium for this extended tax-evasion model it appears that three groups of tax-payers, corresponding to expressions (24i)–(24iii) above, have to be considered.

Case (ii) corresponds exactly to that discussed in sections 2, 3 and 4. If \( \bar{r}(x) > 0 \) for all \( x \), only 'people who matter' will be in the first group, i.e. those with \( q(y, x) > 0 \). Nonetheless, a positive conformity coefficient is not sufficient to ensure a corner solution at \( e - 0 \). It must also be the case that the expected gain in private consumption from evading a positive amount is inadequate compensation for the attendant loss of public goods through imitative behaviour by the rest of society. A simple way to illustrate this is to consider a \((y, x)\)-type indifferent between \( c \) and \( z \) at the margin. For (24i) to hold would then require \( \bar{r}(x) \leq q(y, x) \); so evasion would involve substituting

\(^{17}\)Note too that the domain of the function \( \phi \) must now be modified slightly, but this is unimportant to the analysis.
private consumption for public goods at too high an implicit price to make it worthwhile.

Changes in tax or enforcement parameters now have two types of effect: they can cause shifts of individuals between the groups just described (formerly honest, upright citizens might start concealing part of their income with the change in fiscal regime) or they can modify the evasion behaviour of those who remain within group (ii). We concentrate on the latter type of effect. Consider the implications of an infinitesimal increase in the tax rate \( t \) for those who evade some, but not all, of their income. For any individual the analogue of (8') and (21) is:

\[
te(y, \alpha) + e(y, \alpha) = -\frac{\partial}{\partial y} \left[ U_\alpha^{y} r \right] \frac{y}{D} + \frac{\partial}{\partial y} \left[ U_\alpha^{y} r \right] \frac{\partial^2}{\partial^2 y} \frac{y}{D} + \frac{\partial q}{\partial y} \frac{\partial}{\partial y} \left[ U_\alpha^{y} - U_\alpha^{y} \right] \frac{\partial^2}{\partial^2 y}.
\] (25)

where \( D > 0 \) is the analogue of \( \tilde{D} \), \( U_\alpha^{y} \), \( i, j = c, z \) are the appropriate second derivatives for a \((y, \alpha)\)-type person, and \( z_i^r = (Y - E_0(y, \alpha))/\Psi \). If the \((y, \alpha)\)-type's influence on his or her fellow tax-payers is of vanishingly small importance, then the third term in (25) disappears. Then in the case of Ziff public goods and a utility function exhibiting decreasing absolute risk aversion, such a tax-payer will evade more or less tax (\( tE \) rises or falls) according as

\[
m^2 \geq \frac{y}{z_r},
\] (26)

where \( m^2 \) is the marginal rate of substitution of private goods for public goods for a person with attributes \( \alpha \). Fig. 2 illustrates the evasion response to a tax change of high and low \( m^2 \) types.\(^{18}\)

What happens in the aggregate in this case obviously depends on the joint distribution of income \( y \) and the marginal rate of substitution \( m^2 \). But the outcome also depends on the way in which the responsiveness of tax evasion to income changes is distributed in the community. To see this, suppose that those in group (ii) with \( q = 0 \) constitute the whole community. Then using (10) and (25) it turns out that under Ziff:

\[
tE_i + E = \frac{t}{1-t} \int w(y, \alpha) [m^2 z_i - y] dF(y, \alpha),
\] (27)

where \( w(y, \alpha) = \partial e(y, \alpha)/\partial y \), a weight given by the income responsiveness of a \((y, \alpha)\)-type's evasion. Thus, if on average publicly supplied goods are under-provided in the community than, under decreasing absolute risk aversion, a tax increase leads to an increase in overall tax evaded: the criterion for under-/over-provision for each individual is as described in section 3 above, and the weighted average over the different types of individuals uses as

\(^{18}\)To keep matters simple equilibrium changes are not solved for.
weights the individual income effects. The response of aggregate evasion to a tax change therefore depends on a 'distributional characteristic' of the economy; the correlation between each individual's attitude towards over-/under-provision and income responsiveness of evasion.

As to the likely sign of (27), it can be seen from (26) that for any given $\alpha$-subclass under the assumption of decreasing absolute risk aversion, those individuals who increase their evasion will be poorer than those who evade less. There will therefore be a tendency for evasion to rise if those with lower incomes exhibit the greater income responsiveness. This effect will be reinforced if the joint distribution $F(y, \alpha)$ exhibits a strong negative correlation between $y$ and $m^*$, i.e. if on balance it is the poor who place the highest marginal valuation on public goods. Given that the rich are likely to have access to private alternatives to publicly-supplied goods and services, this is probably a reasonable assumption.

6. Conclusion

Introducing public goods 'closes' the standard model of tax evasion by taking into account the influence on evasion of the purpose for which taxes are raised. It highlights some fundamental interrelationships amongst individual tax-payers and the government and enables issues that underlie popular debate about 'big government' and the size of the irregular economy to be
formally addressed. In particular, it indicates one way of reconciling the theory with the evidence about the effects of tax rate changes. In a large economy with decreasing absolute risk aversion, the sign of the tax effect was seen to turn on the relative abundance of public versus private goods. In modified form this result survives the introduction of heterogeneous taxpayers. In a small economy, the analysis is no longer so straightforward. The individual now has to take account of the effect of his or her evasion decision on the amount of tax collected and on the evasion decisions of others. He or she thus influences public good supply both directly and indirectly. Concealing income becomes a means of personally redressing the balance in favour of private consumption, a method which increases in effectiveness the greater the degree of positive conjectured conformity between tax-payers. Thus, a reduction in such conformity, for example as the population grows larger, will tend to increase evasion.

The approach developed in this paper also has implications for the dynamics of tax evasion in the course of an economy's development. The large economy model suggests that if the public sector and the taxes to pay for it initially grows from a very low base then, because public goods are under-provided, the irregular economy also grows; but it also suggests that the situation may eventually be reversed, so that further tax hikes once the economy is over-endowed with public goods cause tax evasion to fall. However, resting an explanation for a positive tax rate effect on public goods being under-provided is unsatisfactory, since intuition seems to suggest the opposite, i.e. that individuals are more likely to increase evasion after a tax rise when they feel that government services do not represent good value for their tax contributions. This would be associated with over- not under-provision. Why is intuition such a poor guide here? One explanation is that even with public goods incorporated, the standard model still does not adequately capture the intricacies of the relationship between government and tax-payer. In any case, the reversal mentioned above may not happen in practice if, for example, as in Gordon (1988), there are reputation or stigma costs present. In such a situation, the initial phase of public sector growth may trigger off an 'epidemic' effect which sustains the growth of evasion even when public goods are abundant.

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