Tax compliance and firms' strategic interdependence☆

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A B S T R A C T

We focus on a relatively neglected area of the tax-compliance literature in economics, the behaviour of firms. We examine the impact of alternative audit rules on receipts from a tax on profits, allowing for both compliance responses and market responses by the firms. Why does this alternative focus make such a difference to the analysis? Most models in the literature focus on a simple proportionate audit rule in an adapted version of the Allingham and Sandmo (1972) model, as though firms habitually play the dual roles of producers and gamblers. In many of the standard models of corporate compliance there is a fundamental separation result between the production and concealment activities. This conclusion appears to be robust to alternative assumptions about market structure and the specifications of firms' objectives. However, taxes are not neutral in a setting where the behaviour of the tax authority depends on all the declarations in a particular market. The tax authority can exploit this market-based information and so, in the light of this, we investigate the implications of using a more intelligent audit rule that is easily implementable. Specifically we focus on a relative audit rule — where the probability of audit of a particular firm depends on that firm's observable behaviour relative to others operating in the same market.2

Our aim in this paper is to provide an economic rationale for relative audit rules that are used by some tax authorities and to explain why it is

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1 See Nur-tegin (2008) and Slemrod (2004) for recent overviews of the literature.

2 This type of audit rule and the associated compliance behaviour has been examined in the laboratory (Aim et al., 1993; Clark et al., 2004; Collins and P牧Shee, 1991). In order to be effective it is necessary that the tax-payer's environment be one that permits observation by each agent of a signal related to the tax liability of others. In the laboratory this can be artificially arranged: within an industry this may occur naturally from specialised knowledge of market conditions.
to a tax authority's advantage to use a relative rule. We also demonstrate the advantages of such a policy beyond the authority's narrow tax-compliance objectives: the idea is that introducing a relative audit rule would introduce a regime where tax enforcement can influence output decisions. By conditioning an individual audit on the declaration of all firms the authority creates an externality. The externality can be seen as generating two dividends: (a) less tax evasion and (b) an efficiency improvement. The reduction in tax evasion is a direct result of the tax authority's making better use of available information from all firms taken together. The move toward static efficiency arises because of an induced increase in output generated by the switch in enforcement regime.

The paper is organised as follows. Section 2 reviews the story of corporate tax compliance as conventionally presented in the literature and outlines the model presented here; Section 3 examines the equilibrium behaviour of firms in the two main dimensions of decision-making and Sections 4 to 6 present the main results, for both non-collusive and collusive behaviour. Finally, Section 7 concludes. Proofs of propositions and lemmas are provided in Appendix A.

2. The setting and model outline

2.1. Background

The literature on models of corporate tax compliance usually focuses on one of two relatively simple market structures — competitive price-taking or monopoly. The elements of such a model are as follows: a risk-neutral price-taking firm with constant marginal costs and a determinate demand curve faces a proportionate profits tax. The sole source of uncertainty is created by a combination of the firm’s actions (the firm can conceal profit, but at a cost) and the government’s tax audit (a given audit probability with a known penalty proportionate to the amount concealed). The firm conceals up to the point where the marginal cost of concealment equals the marginal reduction of expected tax rate, a rule that is independent of the firm’s output level (Cowell, 2004).

The advantage of this approach is its simplified behavioural analysis of the tax-evading firm: the “production department” can get on with determining the level of output in the light of market conditions; the “tax-management department” separately decides on matters of profit declaration. But there are three causes for concern:

- The separability result is artificial and it is not clear that it would survive in a more interesting model of the industry.3
- The type of audit rule used is naive in that it does not make use of low-cost or costless information that would be available to the tax authority from the firms’ reports.
- The argument that taxation policy has no effect on output seems inappropriate in the light of the perception that corporate taxation does influence firms’ activities. Of course this perception may be misplaced, but it would be useful to know whether there is a good theoretical case for considering a real effect of taxation and tax-enforcement policy.

To address these questions we develop a simple model that will permit a somewhat richer version of market structure and behaviour by the tax authority. The model consists of a conventional story of individual firms, an industry with a given number of firms, a simple tax function and an audit rule. We will briefly examine each of these in turn.

2.2. The industry

We focus on an oligopolistic market with a fixed number of firms each producing a single output; the outputs of the firms in the market are substitutes. The firms compete in a standard quantity-competition (Cournot) model of market interaction. The details of each firm’s simple production technology are subsumed within a conventional profit function. It faces a requirement to pay tax and knows that it has opportunities for evasion. This enables us to focus on perhaps the most appealing and relatively uncomplicated case of strategic interdependence amongst firms in order to examine the potential role of taxation policy in a market form that is not purely mechanistic. One consequence of this is that we would expect the standard Cournot–Nash equilibrium to emerge in which output is above the level corresponding to joint-profit maximisation but below that characterising economic efficiency. In what follows we describe the environment for firms competing in quantities — for the case where firms compete in terms of prices see Bayer and Cowell (2006).

2.3. Taxation and firms’ objectives

Let us set out the role of the tax system in the objective function for the firms. Assume a given population \( N = \{1, \ldots, n\} \) of firms. Firm \( i \) makes gross profit \( \Pi_i(q) \) where

\[
q := (q_1, q_2, \ldots, q_n)
\]

is the vector of quantities produced. Each firm \( i \) makes a declaration of profits \( d_i \) on which taxes and any penalties are based. There is no loss-offset or compensation: subsidies are not given for losses, nor are bonuses paid for revealed over-compliance. For simplicity we impose the condition that gross profits and declaration are non-negative; we also assume a linear profits tax \( t \) so that the legal tax liability is \( t \Pi_i(q) \) and the tax actually paid in the absence of an audit is \( td_i \). Profit net of taxes if no audit takes place is therefore:

\[
\Pi_i(d, q) := \Pi_i(q) - td_i. \tag{1}
\]

After an audit, if a firm is found to have underpaid tax, it is required to make up the shortfall and also to pay a fine. We assume that this fine is proportional to the concealed profit although this is not essential for our results. Net profit if there is an audit is therefore

\[
\Pi_i(d, q) := \Pi_i(q) - tf_i(q) - f[\Pi_i(q) - d_i]. \tag{2}
\]

where \( f \) is the proportionality factor of the fine.

Creating the opportunity for evasion requires that the firm incur a real resource cost covering fees for specialist advice, reorganising transaction patterns or purchasing avoidance schemes. The cost can also be interpreted as the cost of effective concealment. A firm spends real resources in order to hide evasion. Incurred these costs becomes mandatory for an evading firm, since evasion without concealment effort would lead to a discrepancy between book profit and declared profit, which may be easy for the authority to pick up; the marginal cost of concealment will be higher for more “visible” firms and visibility may increase with the scale of the firm’s operations. In the light of this it is reasonable to assume that the concealment-cost function may differ from one firm to another and is a non-negative function of

\[\text{3} \text{See Goerke and Runkel (2006) for a recent treatment of this issue: they examine a simple Cournot model with entry.}\]
the profit concealed \( C_i(\Pi_i(q) - d_i) \geq 0 \), with the properties \( C_i(0) = 0, C_i'(\cdot) \geq 0, C_i''(\cdot) \leq 0 \), where primes denote derivatives.\(^5\)

We take the managers of the firm to be risk neutral, although the analysis does not crucially depend on the risk preferences assumed.\(^6\) So the objective function of the firm is its expected payoff

\[
E r_i(d,q) := \beta_i(d) \Pi_i(d,q) + [1 - \beta_i(d)] \Pi_i(\Pi_i(q) - d_i).
\]

where \( E \) is the expectations operator, \( \beta_i(d) \in [0,1] \) is the probability of audit for firm \( i \) and

\[
d := (d_1, d_2, ..., d_i, ..., d_n)
\]

is the vector of declarations by all firms in the industry. The specification (3) explicitly allows for the possibility that the audit probability for firm \( i \) may depend on other firms’ declaration decisions.

### 2.4. Audit rule

The tax authority can choose an audit policy, but takes \( t \) and \( f_i \) as given. The probability of audit for firm \( i \) is determined by the tax authority using an audit rule,\(^7\) in other words a function \( \beta : N \times \mathbb{R}^n \rightarrow (0,1) \). We distinguish two types of audit rule that may be employed: fixed and relative rules.

**Definition 1.** A fixed audit rule is one where \( \beta_i(d) = f_i^0 \) for any \( i \) and for all \( d \).

**Definition 2.** A uniform fixed audit rule is a fixed audit rule such that \( f_i^0 = f_j^0 \) for all \( i \).

**Definition 3.** A relative audit rule is a function \( \beta \) that satisfies the following conditions wherever \( 0 < \beta_i < 1, \forall i \in N \):

\[ \frac{\partial \beta_i(d)}{\partial d_i} < 0 \quad \forall i \in N \tag{D1} \]

\[ \frac{\partial \beta_i(d)}{\partial d_j} > 0 \quad \forall j \neq i \in N \tag{D2} \]

\[ \sum_{i=1}^{n} \frac{\partial \beta_i(d)}{\partial d_j} = 0 \quad \forall j \in N. \tag{D3} \]

Under the fixed audit rule \( \beta \) does not depend on either the declaration of the firm in question or the declarations of the other firms in the industry and the firm’s objective function (3) can be simplified by replacing expression \( \beta_i(d) \) by the given number \( f_i^0 \).

In the case of the relative audit rule, rather than limiting itself to treating each \( d_i \) in isolation, the tax authority conditions \( \beta \) on the information provided by all the declarations \( d \) as follows. The first property (D1) takes into account that ceteris paribus a higher profit declaration should lead to a lower audit probability. Lower declarations make the authority more suspicious and induce higher audit probabilities. Condition D2 captures the relative nature of the audit rule: if a competitor increases its declaration then this increases \( \beta \)’s probability of being audited. The third part of the definition (D3) captures the widely applied practice of tax authorities of assigning a certain amount of resources to an industry.\(^8\) Technically (D3) keeps the expected number of audits in an industry constant. This is consistent with authorities assigning a certain budget to an industry if the auditing technology exhibits constant returns.

The rationale behind property D2 is that the observation of a high declaration of one firm makes the authority believe that the profit situation in the industry was good for the fiscal year in question. Then it should shift its attention to the firms with comparatively low declarations, such as \( i \), because it becomes more likely that those firms have under-reported their profits. Put differently, an authority that does not know the profit situation in an industry on the one hand and that on the other hand believes that profits in an industry are correlated, should put a higher probability of tax evasion on a received declaration if the declarations of other firms increase.\(^9\) This strategy corresponds to an implicit model of industry profits that contains an industry-specific common shock component the authority cannot observe. However, by observing the level of declarations the authority can draw inferences on this common shock component. So differences in declarations indicate firm-specific unobserved shocks and/or tax evasion. A higher profit declaration of one firm renders tax evasion of the others more likely for their given declarations.

### 3. Equilibrium

In this section we outline the optimisation problem of the firms. We focus on quantity competition under a relative audit rule. For a fixed audit rule just set the partial derivatives of the detection probability with respect to the declaration of any firm to zero.

#### 3.1. Information and timing

We can imagine the following stylised sequence of decisions and actions.

0 Firms learn the tax and fine system and the audit rule that is in place for the coming tax year.
1 Firms choose quantities.
2 Firms observe the gross profits of all market participants and then choose their profit declarations.

It is the interaction of firms in stages one and two that makes the problem particularly interesting. Implicitly, we assume that the firms have an information advantage over the authority. Firms are better informed about the level of profits within an industry than the tax authority; this gives the relative audit rule “bite”. This situation could be applicable in an industry with stochastic demand shocks, for example: a firm can infer from its own profit what the equilibrium profits of the other firms must look like. In what follows we implicitly assume that a firm is able to estimate the tax-relevant gross profit of other firms more precisely than the authority, as the firm can use information from its own experience in the market, which the authority does not have. For this assumption to hold we have to assume that publicly

\(^5\) Similar assumptions are frequently used in the literature — see for example Virmani (1989) and Cremer and Gahvari (1993). For simplicity we ignore the point that firms typically have a choice of how much they invest in effective concealment. See Bayer (2006) for a model where concealment and detection efforts directly influence the detection probability.

\(^6\) In contrast to a large body of literature we do not analyze the impact of tax rates on evasion where risk preferences and fine functions are crucial for the results.

\(^7\) In what follows we use the term audit probability implying that the authority conducts random full-scale audits that perfectly reveal evasion. However, in reality authorities audit large firms every year. Due to the complexity these audits are only partial and do not reveal the true profit with certainty. Under such a scenario \( \beta \) can be seen as the detection probability, which can be increased by the authority by increasing the audit effort.

\(^8\) Typically, tax authorities are quite secretive about how they use their resources. In the United States, e.g., under the Market Segment Specialization Program (MSEP) and the Industry Specialization Program (ISP) introduce in 1993, for many industries a certain number of specialist auditors are employed and trained with respect to the specificities of industry (Department of the Treasury, 1997). This suggest the approach adopted here.

\(^9\) In reality, many authorities use computer-based scoring systems, which estimate the evasion potential of a declaration given some firm and industry characteristics relative to other declarations. In the US the system is based on a Discriminant Information Function (United States General Accounting Office, 1999). The Australian authority is most outspoken about using a relative rule for large business (Australian Taxation Office, 2008, p. 47).
available information (such as balance sheets) are not so closely tied to the accounts used for tax declarations that the information advantage of the firm over the authority disappears, a reasonable assumption in many tax jurisdictions. In standard models the extreme assumption is made that firms have no information advantage over the tax authority at all; for simplicity we take the opposite extreme where firms know the profits in the industry, while the authority does not.

Our model, where the tax authority’s behaviour is modelled by an audit rule, can be seen as a reduced form of a richer signalling model. In such a model the authority would divide its audit resources among firms after observing their declarations. An appropriate information environment consistent with the assumption of an informational advantage of the firms would require that firms observe industry specific shocks, while the authority does not. One might want to add firm-specific shocks, which are private knowledge of the firms. If there is an equilibrium that is at least semi-separating then, in such an environment, the tax authority could extract some information about the industry-wide shocks from the firms’ declarations. Then the resulting equilibrium audit rule for a specific firm would be conditional on all declarations in the industry, just as our relative rule.\footnote{A mechanism-design approach, where the principal commits ex ante to an audit rule, would also yield a relative rule. Mookherjee (1984) shows that the principal’s optimal incentive scheme in a multi-agent production-problem conditions on actions of all agents if their types are correlated. An adaptation of this model to our problem would lead to the authority optimally choosing a relative audit rule.} We chose the reduced-form model for tractability. Within this simplified model we are able to separate the incentive effects of a relative rule (on declarations and production quantities) from other strategic effects arising from the signalling structure of a richer model.

3.2. The declaration stage

Begin with the stage where firms make the declaration to the tax authority: at this stage they treat gross profits as given. There is no incentive for a firm to declare more than the profit actually made and it is clear that the optimal declaration for a firm which incurs losses is to report truthfully. Take the case where gross profit is positive and the firm chooses to evade taxes. From (1) and (2),

\[
\beta \pi (d, q) - \pi_i (d, q) = f + \tau > 0. \tag{4}
\]

So, differentiating (3) the first- and second-order conditions for a unique interior solution are:

\[
\frac{\partial \beta_i (d)}{\partial d_i} [\pi_i - \pi] + \frac{\partial \beta_i (d)}{\partial d_i} [f + \tau] + \frac{\partial \beta_i (d)}{\partial d_i} [\pi_i - \pi] - \frac{\partial \pi_i (d, q)}{\partial d_i} = 0,
\]

\[
\Delta := \frac{\partial^2 \beta_i (d)}{\partial d_i^2} [\pi_i - \pi] + 2 \frac{\partial \beta_i (d)}{\partial d_i} [f + \tau] - \frac{\partial^2 \pi_i (d, q)}{\partial d_i} > 0. \tag{5}
\]

In view of Eq. (4) and the fact that \( \pi_i \leq \pi_i, \), \( \frac{\partial \beta_i (d)}{\partial d_i} < 0, \), \( \frac{\partial^2 \pi_i (d, q)}{\partial d_i} \), a sufficient condition for Eq. (6) is

\[
\frac{\partial^2 \beta_i (d)}{\partial d_i^2} \geq 0.
\]

To simplify matters we only consider audit rules that satisfy (D4).

Given positive profits and the global concavity of the objective function in firm’s own declaration there must be an interior solution whenever

\[
\lim_{d_i \to 0} \frac{\partial E_i (d, q)}{\partial d_i} > 0 \text{ and } \lim_{d_i \to \infty} \frac{\partial E_i (d, q)}{\partial d_i} = 0.
\]

Inspection shows that an interior solution is likely whenever

1. marginal concealment costs are high for extensive tax evasion, which prevents zero-declarations, and
2. there is a generally low level of detection probabilities and a low marginal evasion-cost for the first unit of profit concealed, which give incentives to evade.

From the point of view of the firm the first-order condition has a simple “marginal benefit = marginal cost” interpretation. Condition (5) rearranged yields:

\[
t - \beta_i (d) [f + \tau] + \frac{\partial \beta_i (d)}{\partial d_i} [\pi_i - \pi_i] = C_i (\Pi_i (q) - d_i, \frac{\partial \beta_i (d)}{\partial q_i} [\pi_i] - d_i) \tag{7}
\]

The first term on the left-hand side is the expected financial return to a small amount of evasion if the detection probability were fixed at \( \beta_i (d) \); the second term is the shading of this return that occurs when one takes account of the effect of a change in the declaration on the detection probability.

3.3. The market-response stage

Now consider the stage that determines the firms’ gross profits, the decision about output. Clearly the standard issue of the existence of an equilibrium in pure strategies will arise. Assuming that the strategy space is compact and non-empty and that the objective functions are continuous and strictly quasi-concave in the choice variable ensures a unique equilibrium in pure strategies: to avoid complications we assume for now that these conditions hold,\footnote{See, for example, Fudenberg and Tirole (1991), p.34. A compact strategy set results from restricting the feasible quantities to \( q_i \leq |d_q | \) VI, where \( d_q \) is the largest production quantity that is physically feasible. The objective functions are obviously continuous. Quasi-concavity can be achieved by choosing appropriate demand, cost and detection-probability functions.} but the assumption of uniqueness is dropped in the next section. Writing firm’s optimal declaration as a function of the output vector, \( d_i ^* (q) \) then the first-order condition for optimal output given that firms will file (and expect the competitors to file) an optimal tax return for any possible profit distribution can be written as:

\[
\frac{\partial \beta_i (d^* (q))}{\partial q_i} [\pi_i - \pi_i] + \frac{\partial \beta_i (d^* (q))}{\partial q_i} [1 - \beta_i (d)] \frac{\partial \pi_i (d, q)}{\partial q_i} = 0.
\]

Note that the change in the equilibrium audit-probability \( \frac{\partial \beta_i (d^* (q))}{\partial q_i} \) includes the effects of declaration changes of all firms due to the change in observed profits:

\[
\frac{\partial \beta_i (d^* (q))}{\partial q_i} := \sum_{j \neq i} \frac{\partial \beta_j (d^* (q))}{\partial q_j} \frac{\partial d_j^* (q)}{\partial q_i} + \frac{\partial \beta_i (d^* (q))}{\partial d_i} \frac{\partial d_i^* (q)}{\partial q_i}. \tag{9}
\]

Also note that players in a subgame perfect Nash equilibrium anticipate equilibrium play in future stages, which implies that the first-order condition for the declaration stage still has to hold. Then observing that

\[
\frac{\partial C_i (\Pi_i (q) - d_i ^*)}{\partial q_i} = C_i (\Pi_i (q) - d_i ^*) \left[ \frac{\partial \Pi_i (q)}{\partial q_i} - \frac{\partial d_i ^* (q)}{\partial q_i} \right].
\]
allows substituting $C$ from Eq. (7) into (8). Applying Eq. (9) and using Eq. (4) gives a simplified first-order condition for the optimal output choice:

$$
\frac{\partial \Pi_i(q)}{\partial q_i} \left[ \pi_i - \pi_j \right] + \left[ 1 - \frac{\partial \beta_i(d)}{\partial d_i} \right] \left[ \pi_i - \pi_j \right] = 0.
$$

The set of conditions (10), one for each firm, characterises the equilibrium. Using this characterisation of equilibrium we can now present the main results in three steps: Sections 4 to 6.

### 4. Compliance decisions

In this section we compare the different effects that fixed and relative audit rules have on the extent of tax evasion. If the declaration stage has an interior solution then, under weak conditions, a relative audit rule leads to less tax evasion than a comparable fixed audit rule.\(^{12}\)

The intuition for the result appears simple: in addition to the typical incentives provided by a fixed-detection probability and the corresponding fine, a relative audit rule provides a further incentive to increase the declaration, as this decreases the detection probability. If one replaces the relative audit rule in Eq. (5) with a fixed audit rule the first-order condition becomes

$$
t - \beta_i(f + \ell) = \frac{1}{\beta_i}(\Pi_i(q) - d_i).
$$

To compare the rules (7) and (11) we need a criterion that makes the two regimes comparable. Suppose we require the equilibrium detection probability in the relative-rule scenario to be equal to the fixed-detection probability. This ensures that the audit costs incurred by the authority are equal under both regimes. Setting $\beta_i(d^0) = \beta_i^0$ and keeping the profits the same in both cases it is clear that Eqs. (7) and (11) differ only by the "shading" term

$$
\frac{\partial \beta_i(d)}{\partial d_i} \left[ \pi_i - \pi_j \right].
$$

This is just the additional incentive to increase the declaration in order to reduce the detection probability and leads to higher declarations under a relative audit rule.

However, this intuitive argument neglects the dependence of firm’s decision upon the decisions of the others under a relative audit rule. More is required for a clear-cut result. We use a well known result on supermodularity to show the following — proofs of this and of other results are to be found in Appendix A.

**Lemma 1.** Denote the vector of interior equilibrium declarations that prevails under an arbitrary fixed rule $(\beta_i^0, \beta_2^0, ..., \beta_n^0)$ as $d^0 := (d_1^0, d_2^0, ..., d_n^0)$. Then for a relative rule $\beta_i(d)$ that induces a supermodular declaration game and satisfies

$$
\beta_i(d) = \beta_i^0 \quad \forall i \in \mathbb{N}
$$

an equilibrium exists with

$$
d_i^* > d_i^0 \quad \forall i \in \mathbb{N}.
$$

The supermodularity condition used in Lemma 1 is sufficient but not necessary for the result that an equilibrium with lower evasion exists under a relative rule. It ensures that all best responses are globally increasing in the declarations of the other firms and embodies the basic intuition of how a relative rule works. A relative rule designed to induce a firm to react with a higher declaration if another firm increases its declaration thereby creates an externality that is beneficial from the tax authority’s point of view: for given levels of gross profits there will be higher declarations than under a fixed rule.

There are many audit rules that satisfy the supermodularity condition; for example a simple rule that is linear and additively separable in the declarations — see Proposition 1 below. By contrast, consider what would be necessary for the condition not to hold: then there must be a positive cross-derivative of the detection probability that is large compared to the change of the probability in the declaration of the opponent. Under such circumstances the direct externality — the influence of the declaration of a competitor on the detection probability — is overcompensated by the rule-specific effect that an increased declaration of the other firm reduces the reactivity of the detection probability with respect to the own declaration. A rule which does not preserve supermodularity would have to be such that an increased declaration of a competitor reduced the marginal impact of a firm’s own declaration considerably. Such rules are not realistic, as this would mean that the authority puts more weight on the declarations of the competitors than on that of the firm in question when deciding on the audit intensity. However, even if the supermodularity condition is violated, the resulting equilibrium under a relative rule can still lead to less evasion. It is important to note that all the results on efficiency derived later in this paper do not depend on supermodularity.

The proposition below deals with two concerns: (a) the set of rules satisfying the supermodularity condition might be empty and (b) there might be multiple equilibria, of which some lead to higher evasion than the fixed rule. The proposition below shows that there exists a very simple rule which rules out both of these concerns.

**Proposition 1.** For given gross profits $\Pi$ and any fixed rule with detection probabilities $\beta_i^0$ leading to interior declarations $d^0$ there exists a simple relative rule $\beta_i(d)$ with $\sum_{i=1}^{n} \beta_i^0 = \sum_{i=1}^{n} \beta_i(d)$ that induces a unique equilibrium vector $d^*$ with $d_i^* > d_i^0 \quad \forall i \in \mathbb{N}$.

So, in a situation where there is competition among the few, switching to a more intelligent audit rule yields an immediate dividend — less evasion. Proposition 1 makes sure that such a rule exists for all initial conditions and that the authority does not need any additional information. The declarations under the old fixed rule are sufficient. Basically, any linear relative rule that assigns the same detection probability to the competitors than on that of the firm in question when deciding on the audit intensity. However, even if the supermodularity condition is violated, the resulting equilibrium under a relative rule can still lead to less evasion. It is important to note that all the results on efficiency derived later in this paper do not depend on supermodularity.

Again supermodularity is sufficient.

### 5. Output decisions

There is more to be said on behalf of the relative audit rule. As we noted (Section 2.1), in simple competitive and noncompetitive models the tax-enforcement parameters do not distort output, although this result depends on the way in which the audit probability and penalty rates are formulated (Lee, 1998). So too in our model: there is no effect on output if audit probabilities are independent of declarations. However, this is not true for oligopoly: here a switch to relative auditing will result in efficiency gains.

For this purpose we assume that the underlying market conditions and cost structure ensure that a Cournot game will have a unique and stable pure-strategy Nash equilibrium: we will refer to this as a regular Cournot oligopoly.\(^{13}\) We assume that the game has a compact strategy

\(^{12}\) The result is also valid if firms compete in prices rather than quantities.

\(^{13}\) Sufficient conditions for a regular Cournot oligopoly are set out in Vives (1999), chapter 4 and are derived from Novshek (1985). Oligopolies with multiple equilibria are excluded in order to have a definite reference for the comparative-statics analysis; but we do not exclude games where the resulting Cournot game with relative auditing has multiple equilibria as long as the underlying Cournot game has a unique equilibrium in pure strategies.
space, where the inverse demand function \( p(\cdot) \) is decreasing and log-concave and where, for every firm \( i \), production costs \( K_i(\cdot) \) are such that\(^{14}\)

\[
K_i'(q_i) - p'(Q) > 0, \quad Q := \sum_{i=1}^n q_i.
\]

These conditions are relatively mild and allow for oligopolies typically used in applied work (for example models with linear demand and cost). They are sufficient to establish the following result that guarantees uniqueness and stability of the equilibrium of the oligopoly game in the absence of relative auditing.

**Lemma 2.** In regular oligopolies the best-response functions of firms have non-positive slopes greater than \(-1\).

To examine the effects of audit rules on output we first establish that the best-response quantity of an individual firm under relative auditing is larger than under a fixed audit rule. As mentioned above under a fixed audit rule the quantity decision is equivalent to the decision in an oligopoly without taxation and tax enforcement.

**Proposition 2.** Under an independent audit rule output is independent of the evasion decision and equals the Cournot quantities.\(^{15}\)

Now consider relative auditing. Denote the best response of firm \( i \) in the regular Cournot game as \( BR_i^1(q_{-i}) \), while \( BR_i^2(q_{-i}) \) gives the best-response correspondence under a relative audit rule.

**Lemma 3.** Assume that the market organisation has the form of a regular Cournot oligopoly. If \( q_{-i} \) implies \( BR_i^2(q_{-i}) > 0 \) then \( BR_i^1(q_{-i}) > BR_i^2(q_{-i}) \).

This result might seem surprising. After all, what does a firm gain from extending its output beyond the Cournot best-response level? By increasing output in this way it would seem that a firm reduces its own gross profit and the gross profits of the competitors – a loss rather than a gain. However, recall the role of the informational externality here. As \( q_i \) increases the profits of other firms fall, so the optimal declarations of the other firms fall. Therefore the probability of audit of firm \( i \) decreases, which in turn gives firm \( i \) more scope for evasion. By increasing output beyond the Cournot quantity a firm intends to trade some gross profit for a better environment for evasion. This externality can be characterised by the differences in the first-order conditions for the quantity choices under the different rules. Comparing the first-order conditions shows that the externality under relative auditing is given by the term

\[
\sum_{j \neq i}^{n} \frac{\partial \hat{\alpha}_j (\hat{d}^s)}{\partial q_j} \frac{\partial q_i}{\partial q_i} [\hat{\alpha}_i - \hat{\alpha}_j].
\]

which describes the influence of firm \( i \)'s quantity on \( i \)'s expected payoff by the indirect effect on the other firms’ declarations and hence firm \( i \)'s audit probability. As the other firms will decrease their declaration if their profit is decreased, which decreases firm \( i \)'s audit probability, firm \( i \) has an incentive to sabotage the other firms’ profits by producing more than under the equilibrium of the underlying Cournot oligopoly.

The result from **Lemma 3** is sufficient to ensure that the aggregate quantity under a relative auditing rule is larger if (a) the oligopoly is regular and symmetric and (b) the equilibrium under relative auditing is also symmetric. However, we can use the properties of a regular oligopoly to generalise it:

**Proposition 3.** If the underlying Cournot oligopoly is regular then any interior equilibrium outcome under a relative audit rule yields higher aggregate equilibrium quantity than under a fixed rule.

The intuition is illustrated for a duopoly in Fig. 1. The solid straight lines represent the best-response functions of firms 1 and 2 in a Cournot duopoly: so the intersection shows the equilibrium quantities \( q^* := (q_1^*, q_2^*) \) in the case where a relative audit rule is not employed. The best-response quantities for a given quantity of the competitor are greater under a relative than under a fixed rule (Lemma 3). Therefore in the case of the relative audit rule the best-response function for firm 1 must lie to the right of firm 1’s Cournot response function and that for firm 2 must lie above firm 2’s Cournot response function. Essentially the same reasoning holds if the firm’s best response is characterised by a multivalued correspondence. Without the need to specify the reaction functions in detail it is clear that an equilibrium must lie in the shaded area of Fig. 1: the broken lines illustrate the reaction functions for one such case. Given that iso-quantity contours are lines with slope \(-1\) the equilibrium must lie on a higher contour than the one passing through \( q^* \); total output under the relative audit rule must be greater than under a fixed rule.

In other words, increasing individual output reduces the impact of the externality resulting from the relative audit rule. A lower gross profit helps to close the gap in the net profits between the situation with an audit (\( \Pi \)) and the situation without (\( \bar{\Pi} \)). Since all firms have the same incentives this does not really work: they all try to reduce the impact of the externalities by increasing quantity, so they are all worse off than if they had just produced the Cournot quantity at the first stage. The externalities imposed by the authority on the profit declaration spill over into the output-decision stage.

6. Collusion

Does the relative audit rule still produce desirable results if firms do not behave competitively? We analyse this using three scenarios based on the two-stage nature of firms’ decisions. First (Section 6.1),...
one might think of a situation where firms are able to cooperate when they file their tax returns; the repeated nature of preparing tax returns might allow for tacit collusion by using simple trigger strategies, and tax benchmarking might be used as a monitoring device to detect cheating. Second (Section 6.2), firms may be able to collude at the production stage in the classic fashion. Finally (Section 6.3) we consider the case of firms colluding on both declarations and production quantities.

In all three scenarios we only consider firms implementing strategy profiles that maximise joint profit. This can be seen as the most conservative test for the usefulness of a relative rule, as this assumption gives firms the best chance of jointly eliminating the externality imposed among them by a relative rule. Moreover perfect collusion might be a rare event for two reasons. First, the conditions necessary for full collusion to be sustained might not be satisfied in a particular industry. For collusion to be sustainable among other things the detection of cheating needs to be possible (or at least actual cheating must increase the believed probability that cheating has taken place) and firms must be sufficiently patient. Secondly, it is not clear that firms actually collude perfectly even if collusion is sustainable as an equilibrium, as there are also many other, more competitive equilibria (including some implementing the competitive stage-game Nash equilibrium analysed above).

6.1. Collusion: declaration stage

One concern with a relative rule is that firms might collude when filing their tax returns in order to overcome the externality imposed by the rule. Collusion in a one-shot game can only be an equilibrium if binding contracts on declarations can be written. This is not the case for tax returns in the real world. A more compelling rationale for potential collusion lies in the repeated nature of the process of declaring profits. For collusion at the declaration stage in the repeated-game context it is necessary that firms observe a signal of the previous declarations of other firms. This may even be facilitated by official policy if the tax agency requires public disclosure of information such as use of tax shelters and book-tax differences in an attempt at “moral suasion” on corporations. Slemrod (2004) has argued that poor implementation of this policy may actually lead to a “race to the bottom” (p. 895) — i.e. effective collusive behaviour amongst supposed competitors.

Let us analyse this part of the problem by assuming that firms are able to sustain perfect collusion at the declaration stage, while Cournot competition takes place at the production stage. The important question here is whether colluding firms are able to turn the relative rule to their advantage and achieve better results than under a fixed rule. Does a relative rule perform worse or better than a fixed rule if firms collude at the declaration stage?

Under a relative rule without collusion the equilibrium outcome at the declaration stage is a vector $d^R(I)$, giving optimal declarations as a function of all firms’ gross profits. These equilibrium declarations implement a vector of expected net profits $(E_{11}(d^R, q), ..., E_n(d^R, q))$.

Now assume that under collusion all the declaration vectors $d^*$ that at least weakly increase all individual payoffs are feasible. Denote the set of all declaration vectors that are feasible as

$$D = \{d \in \mathbb{R}_+^n : E_{11}(d, q) \geq E_{11}(d^*, q)\}.$$

Consider the case where firms are able to sustain perfect collusion by maximising joint profit. A collusion agreement is characterised by a vector $d^C = D^C$ where

$$D^C = \arg \max_{d \in D} \sum_{i=1}^n E_i(d, q).$$

(13)

Note that the minimum payoff condition can always be satisfied if side payments are allowed. The first-order condition for an interior maximum for firm $i$ is $\frac{\partial E_i}{\partial d_i}(d^C, q) = 0$, which can be written as

$$\sum_{j=1}^n \frac{\partial \beta_j(d)}{\partial d_i} (\Pi_j - \Pi) + \beta_j(d) \left( \frac{\partial \Pi_j}{\partial d_i} - \frac{\partial \Pi}{\partial d_i} \right) + \frac{\partial \Pi}{\partial d_i} = C_i(I_j(q) - d_i) = 0.$$  (14)

This gives us:

**Lemma 4.** Any jointly optimal declaration vector satisfies

$$n_1 \left( t - \bar{P}(d) / f + t \right) = \sum_{j=1}^n C_j \left( I_j - d_j \right).$$

(15)

where $\bar{\beta}_j(d)$ is the average audit probability

$$\bar{\beta}_j(d) := \frac{1}{n} \sum_{i=1}^n \beta_j(d).$$

(16)

Applying Lemma 4 in a symmetric environment where the gross profits of all firms are equal and the relative audit rule is symmetric shows that under these conditions the relative audit rule leads to the same declaration as a uniform fixed audit rule if firms collude on declaration.

**Proposition 4.** Let $d_{-i}$ denote the declarations of all firms other than $i$ and $j$. If $I_i(11) = l$ and $d_i = d_{-i} \Rightarrow \beta_i(d_{-i}d_{-i}) = \beta_i(d_{-i}d_{-i}) \forall i \neq j, d_{-i}$, then the symmetric profile of jointly optimal declarations coincides with $d^C$, the declarations resulting from a uniform fixed audit rule.

**Proposition 4** gives a guideline of how a relative rule compares in terms of tax revenue to a fixed rule if firms collude at the declaration stage. When cooperating on tax returns firms eliminate the externalities imposed by a relative rule. In this case the relative rule and the fixed rule lead to the same outcome if the gross-profit situation is symmetrical.

Some of the intuition underlying this result also applies to asymmetric industries, where gross profits differ across firms: by Lemma 4 colluding firms will always internalise the externalities created by the relative rule. But in the asymmetric case firms can do more because they can exploit the relative rule to save some evasion costs. It is jointly optimal to distort the declaration of firms with higher profits somewhat upwards, while distorting the declarations of low-profit firms somewhat downwards, compared to the individually optimal declaration under a fixed rule. This effect is driven by the increasing marginal evasion cost; so the shape of the evasion-cost curve is the principal factor determining whether collusion leads to higher or lower aggregate declarations. However, the precise result depends on the shape of the relative audit rule. The role of evasion costs is most

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16 Tax benchmarking can be carried out by independent observers from publicly available information. It “allows corporations to identify how much they are paying compared to other companies in the industry. It may highlight a high tax rate which could be due to lack of efficient control and management of the tax function. Conversely, it could highlight a low tax rate compared to the peer group which could be due to a tax strategy which, if successfully challenged, could harm the company’s reputation...” (Burak et al., 2007).

17 See Mailath and Samuelson (2006) for details of what is implementable as an equilibrium depending on the informational structure.

18 Greater transparency in reporting could have other effects such as fine-tuning of “evasion incentives” to corporate tax managers (Slemrod, 2004, page 886) and enhancing political pressure for good tax policy (Lentler et al., 2003). These topics go beyond the scope of the present paper.
clearly seen in the case of a linear-symmetric rule — the simplest rule satisfying assumptions (D1)–(D4). In this case
\[
\frac{\partial \Pi_i(d_i)}{\partial d_i} = \frac{\partial \Pi_j(d_j)}{\partial d_j} = c \quad \forall d_i \neq d_j \tag{D5}
\]
and the following result is available:

**Proposition 5.** For any \( d^0 \in D^0 \) a linear-symmetric evasion rule satisfying (D5) implies
\[
\sum_{i=1}^{n} d_i^{bs} \leq \sum_{i=1}^{n} d_i^{cf} \quad \text{if} \quad C_i^s(I_i - d_i) \geq 0 \quad \forall d_i \in [0, I_i],
\]
\[
\sum_{i=1}^{n} d_i^{bs} \geq \sum_{i=1}^{n} d_i^{co} \quad \text{if} \quad C_i^r(I_i - d_i) \leq 0 \quad \forall d_i \in [0, I_i],
\]
where \( d_i^{b} \) stands for the declaration of firm \( i \) under collusion.

In the case of the simple relative rule (D5), if marginal evasion costs increase at an increasing rate then joint expected net profit maximisation leads to a higher aggregate declarations than under a fixed rule.

How is production influenced by a relative rule if collusive declarations are possible? The declaration vector that maximises joint profits at the second stage depends on the vector of gross profits resulting from the production stage. In the non-collusive setting (Section 5) we know that there is an incentive to produce more than the Cournot quantity in order to increase the evasion opportunity; the main channel for this is the fact that increasing your quantity reduces the profit and declarations of your rivals, which in turn reduces your own detection probability. The following lemma confirms this effect under cooperative declaration behaviour:

**Lemma 5.** Increasing the own quantity under cooperative declaration decreases the equilibrium declarations of the competitors at the collusive declaration stage:
\[
\frac{\partial d_i^{co}}{\partial q_i} < 0 \quad \forall j \neq i.
\]

So there is still an incentive to increase production beyond the Cournot quantity in order to reduce competitors’ profits, with the intention of decreasing one’s own equilibrium detection probability. Then the absence of other effects (such as cooperative saving of evasion cost in the asymmetric case) is a sufficient condition for a relative rule to increase output and hence to yield efficiency gains. A sufficient condition is symmetry.

**Proposition 6.** If identical firms collude at the declaration stage under a symmetric relative rule then the aggregate production quantity exceeds the quantity under a fixed rule with the same equilibrium detection probabilities.

The incentive to sabotage the gross profit of competitors by choosing a quantity higher than the Cournot quantity is still present even when collusion takes place at the declaration stage. Symmetry is sufficient – but not necessary – for this effect to translate into efficiency gains. In a symmetric industry where ex-ante identical firms collude on declarations, a relative rule leads to the same declaration behaviour as a fixed rule while the second benefit of a relative rule, which is greater allocative efficiency, still obtains.

6.2. Collusion: production stage

Now focus on the question whether the relative rule (at the second stage) has any impact on the cartel quotas chosen at the first stage. Does the relative rule fare better than the fixed rule with respect to economic efficiency?

Suppose firms can jointly maximise expected profits by choosing quantities, anticipating individually optimal choices at the declaration stage; but suppose they cannot cooperate at the declaration stage. Then under a relative rule firm \( i \) will declare \( d_i^r \) as defined by the condition (7) depending on the vector of gross profits realised at the production stage; under a fixed rule the relevant condition is (11). If firms can collude on production only then they will choose a jointly optimal vector of quantities given the anticipated declarations. For simplicity consider identical firms and concentrate on a symmetric equilibrium. An equilibrium with a production vector requiring identical quantities for all firms implicitly assumes that the cartel of identical firms restricts all feasible allocations to the ones where firms produce identical quotas.19

First consider the case of a fixed audit rule. Denote the gross profit per firm by \( \Pi \) (so that the aggregate gross profit is \( n\Pi \)) and the resulting average declaration by \( d \). The expected joint net profit for equal quotas and consecutive optimal individual declaration becomes:
\[
\sum_{i=1}^{n} E_i \left[ 1 - \beta^0 \Pi_i - \beta \Pi_i (1 - t - f) \Pi_i + \beta \Pi_i \right] - nC(\Pi - d),
\]
where \( \beta^0 \) is the fixed audit probability. Maximising this joint profit with respect to symmetrical quotas (taking into account the declaration behaviour at the declaration stage) leads to the expected result. Firms choose the quotas that maximise joint gross profit.

**Proposition 7.** Under a fixed rule collusion at the production stage leads to monopoly output if firms consider only symmetric cartel agreements.

We now turn to collusion at the first stage under a relative rule. We again restrict our analysis to ex-ante identical firms, which are restricted to symmetric cartel agreements. At the second stage firms will independently declare their profits for a given vector of production quotas they agreed upon. The declarations are determined by the first-order conditions (5). At the production stage firms agree on the set of symmetric quotas that maximise the expected joint net profit given the anticipated declaration behaviour.

Recall that property (D3) of the relative rule implies that the sum of the individual audit probabilities is constant. Substituting \( \beta(d^0) \), the average audit probability, (see Eq. (16)) into the expected joint profit gives
\[
\sum_{i=1}^{n} E_i \left[ n(\Pi_i - d_i^r) - \beta(d^0) n(\Pi_i - d_i^r) + \beta(d^0)(1 - t - f) \Pi_i + d_i^r \right]
+ nC(\Pi - d_i^r).
\]

Comparing the expression above to the expression for the case with a fixed rule shows that they are qualitatively equivalent. The difference is that \( \beta^0 \) is now replaced by \( \beta(d^0) \). Not surprisingly, joint profit maximisation leads to the monopoly outcome under a relative rule too.

**Proposition 8.** Under a relative rule collusion at the first stage leads to the monopoly output if firms only consider symmetric cartel agreements.

Our analysis above shows that the relative audit rule loses its bite with respect to the second dividend when firms are able to form a

19 In some cases firms might be able to increase the expected joint net profit by using an asymmetric allocation of quotas. But there are good reasons why firms in a cartel would refuse to produce less than other firms. If identical firms bargaining over the collusion surplus have equal bargaining power they should end up with the same surplus. An allocation that implements different quotas can only be sustained if side payments are possible. In the more likely case that side payments are not possible due to the oversight of a control agency then the best firms can do is to choose the best cartel agreement where firms have identical quotas. Even if side payments are possible firms still might refuse to produce less than other identical firms, as they do not want to lose market share.
cartel at the production stage. The relative rule does not prevent the firms from restricting the aggregate output to the monopoly quantity. With respect to allocative efficiency the relative rule does not fare worse than the fixed rule, though, as also under a fixed rule only the monopoly quantity is produced. It is noteworthy that the first dividend of a relative audit rule is not affected by the cartel at the production stage. Tax declarations are still higher under the relative rule than under the fixed rule.

6.3. Collusion: both stages

Here we will briefly comment on the case where firms are able to collude perfectly on both the production and the declaration stage. We can draw on the results derived in the previous two sections. Let us start with the declaration stage, where a vector of gross profits will have emerged from the collusive production stage. At that point the problem of the firms colluding at both stages is identical to that of firms only cooperating at the declaration stage, with the difference that now the gross profits were determined by the collusive vector of declarations. The jointly optimal declarations are now determined by

\[
D^o = \arg \max_{\beta, d} \sum_{i=1}^n E_i(d, q^o) .
\]

This maximisation problem only differs from the one considered above by a predetermined variable. So all the results on the performance of the audit rule from above (Lemma 4 and Propositions 4 and 5) still go through. As expected, firms that are able to sustain collusion at both stages eliminate the negative externality at the declaration stage, which leads to the same declarations as the corresponding fixed rule if the gross profits from the collusive production stage are symmetric. In the asymmetric profit situation the performance of the relative rule depends (as above) on both the specific formulation of the rule and some properties of the concealment technology. Under a linear-asymmetric rule the curvature of the marginal concealment cost determines whether the joint saving of cost leads to higher or lower declarations than under a fixed rule.

We now turn to the production stage. There we have seen that restricting attention to the symmetric case yields the result that firms eliminate the production externality imposed by the relative rule if they collude only at the production stage. This is also the case if identical firms can collude at both stages. The ex-ante expected joint profit in this case follows Eq. (20), with the slight difference that the anticipated declaration vector is now jointly optimal instead of being the result of a Nash equilibrium in the continuation subgames:

\[
\sum_{i=1}^n E_i = n[I - t\beta] - \beta(d^o) n[I - t\beta] + \beta(d^o) n(1 - t - f) II + f\beta] - nC_i(I - \beta). \tag{22}
\]

From Proposition 4 we know that collusion at the declaration stage leads to the same declarations as under a fixed rule. This means that it is straightforward to show that the aggregate production of the cartel is the monopoly quantity: the proof is identical to the one proving Proposition 7.

6.4. Collusion: summary

Collusion at the declaration stage leads to firms eliminating the externality imposed by the relative rule at the declaration stage if they collude. The resulting aggregate profit declaration is the same as under a fixed rule if firms are identical. But if firms differ with respect to their gross profits then the outcome (in terms of the total profit declared) depends on the evasion-cost structure. If marginal evasion cost increase sufficiently strongly with the tax evaded then firms will declare more under the relative rule than under a fixed rule. The effect that a relative rule has in terms of efficiency gains (by increasing the quantities produced) still remains. The relative rule still does better than the fixed rule.

If there is collusion at the production stage the relative rule’s positive effect on allocative efficiency vanishes. Under either audit rule firms will jointly produce the monopoly quantity. However, the relative rule’s positive effect on the profit declaration remains. Collusion at the production stage does not spill over into the declaration stage. Firms are not able to eliminate the externality imposed on them if they can only collude at the production stage. If firms can collude at both stages then both dividends of a relative rule vanish, as then firms completely internalise the externality created by a relative rule. In a symmetric environment the collusive firms generate outcomes that are just the same as under a fixed rule. In asymmetric environments, as in the case of collusion at the declaration stage only, the performance of the relative rule with respect to revenue collection (compared to a fixed rule) depends on the evasion technology.

7. Conclusions

This paper has focused on a relatively neglected aspect of tax compliance. It has shown that market structure matters in tax enforcement, a result that is in sharp contrast to the neutrality results that are typical in the literature. We have seen that, in standard models of industrial organisation, the enforcement policy affects firms’ behaviour in two dimensions — their market behaviour as well as their compliance behaviour. Appropriate design of the enforcement policy can thus have a “double dividend” in the absence of collusion, (1) through improved tax compliance and (2) through greater allocative efficiency in the economy. Even where there is collusion by firms — either in production or in declaration behaviour — the relative rule can be shown to have advantages over the fixed rule.

The relative audit rule has an advantage over the independent rule and even over a system without taxation if one is concerned with efficiency in the goods market. A relative audit rule creates externalities in the declaration of profits, which spill over to the quantity decision. In the quantity-competition model this audit rule leads to higher outputs than in a pure Cournot oligopoly. Because an audit regime which treats each firm independently does not impose those externalities, under such a regime the quantity choice is not influenced by the evasion decision and the Cournot quantities are produced.

Appendix A. Proofs

A.1. Lemma 1

Proof. Supermodularity requires

\[
\frac{\partial^2 \Pi_i(d, q)}{\partial d_i \partial d_j} \geq 0 \quad \text{forall } i \neq j.
\]

Using Eq. (3), condition (23) becomes

\[
[f + t] \frac{\partial \beta_i(d)}{\partial d_i} = [I - d_i] \frac{\partial^2 \beta_i(d)}{\partial d_i \partial d_j} \geq 0 \quad \text{forall } i \neq j.
\]

or

\[
\frac{\partial \beta_i(d)}{\partial d_i} = [I - d] \frac{\partial^2 \beta_i(d)}{\partial d_i \partial d_j} \geq 0 \quad \text{forall } i \neq j.
\]

The set of rules satisfying supermodularity is nonempty since Eq. (24) is trivially satisfied by an additively separable rule where \(\frac{\partial \beta_i(d)}{\partial d_i} = 0\) and \(\frac{\partial^2 \beta_i(d)}{\partial d_i \partial d_j} > 0\).
The fixed-rule equilibrium declaration $d_i^0$ of firm $i$ solves condition (11). Using Eq. (5) the marginal benefit of the declaration for firm $i$ under a relative rule at the point $d = d_i^0$ with $\beta_i(d_i^0) = \beta_i^0$ gives
\[ \frac{\partial \beta_i(d_i, q)}{\partial d_i} = -t + \beta_i^0 (f + t) - \frac{\partial \beta_i^0(d_i^0)}{\partial d_i} \left[ \Pi_i - \Pi \right] + C_i (\Pi(q) - d_i^0). \] (25)

Substituting in from Eq. (11) the RHS of Eq. (25) becomes
\[ -\frac{\partial \beta_i^0(d_i^0)}{\partial d_i} \left[ \Pi_i - \Pi \right] > 0. \]

which implies that $BR_i(d_i^0, q_i)$, firm $i$'s best response to $d_i^0$, is greater than $d_i^0$. Supermodularity implies non-decreasing best responses. It follows that
\[ BR_i(d_i, q_i) > d_i^0 \quad \forall d_{-i}, d_i \geq d_i^0, \forall i. \] (26)

Also note that $sup\{d_i; d_i \in BR_i(d_{-i})\} = \Pi_i$ as declaring more profit than the true profit is a dominated strategy. This implies that best replies to declaration vectors $d_{-i}$ with $d_{-i} \in [d_i^0, \Pi_i]$ lie in the interval $[d_i, \Pi_i]$. Consequently, any Nash equilibrium of the truncated game with strategy space $S = [d_i, \Pi_i] \times [d_j, \Pi_j] \times \ldots \times [d_n, \Pi_n]$ is also a Nash equilibrium of the original game. By the Tarski (1955) fixed-point theorem supermodularity and compactness of the strategy space imply that the truncated game has at least one pure-strategy Nash equilibrium. Eq. (26) implies that in this equilibrium (or in these equilibria) all firms declare higher profits than under the corresponding fixed rule. \(\square\)

A.2. Proposition 1

Proof. If the relative rule induces a supermodular declaration game then, by Lemma 1, such an equilibrium exists. By construction the sum of detection probabilities is equal under the fixed rule and the relative rule for all $d_i$ if $\beta_i(d_i^0) = \beta_i^0$ and if the rule satisfies condition (D3). Now consider an additive separable rule, linear in the declarations:
\[ \beta_i(d) := a_i + \sum_{j=1}^{n} b_{ij} d_j, \quad b_{ij} = 0, \quad b_i < 0, \quad b_{i \neq i} > 0. \] (27)

By construction rule (27) satisfies conditions (D1)–(D4) and induces a supermodular game, since condition (24) requires $b_{i \neq i} > 0$.

Sufficient conditions for a unique equilibrium are a compact strategy space, continuous payoffs, which are quasi-concave in the own action and a best-response map which is a contraction (Vives, 1999, p. 47). The strategy space is compact, payoffs are continuous and, by (D4), concave in own actions. For the best-response map to be a contraction we require
\[ \frac{\partial^2 \Pi_i(q)}{\partial d_i^2} + \sum_{j=1}^{n} \left| \frac{\partial^2 \Pi_i(q)}{\partial d_i \partial d_j} \right| < 0 \quad \forall i \in N. \]

Under rule (27) the condition becomes
\[ 2b_i (f + t) - C_i (\Pi_i - d_i) + \sum_{j \neq i} b_j < 0, \]
which, because rule (27) requires $\sum_{j \neq i} b_j = -b_i$, implies $b_i (f + t) - C_i (\Pi_i - d_i) < 0$. \(\square\)

Condition (28) must hold, since $b_i < 0$ and $C_i (\Pi_i - d_i) > 0$.

A.3. Lemma 2

Proof. We have to prove that $d_i^0/dQ_{-i} \equiv (-1, 0)$ where $Q_{-i}$ denotes the output of all firms other than $i$. First, observe that the slope of the best-response is given by:
\[ \frac{d_i^0}{dQ_{-i}} = \frac{-\partial^2 \Pi_i(q) / \partial q_i / \partial d_i}{\partial^2 \Pi_i(q) / \partial q_i / \partial d_i} = -\frac{p_i + p_i q_i}{2p_i + p_i q_i - K_i} \] (29)

In what follows we prove by contradiction that $p_i + p_i q_i \leq 0$.

Log-concavity of the inverse demand function requires that:
\[ \frac{d^2 \ln[p_i(q)]}{dQ^2} \leq 0, \]
which is equivalent to
\[ p_i p_i - p_i^2 \leq 0 \] (31)

Now suppose that $p_i + p_i q_i > 0$.

From the first-order condition we know that $\frac{d_i^0}{dQ_{-i}} \equiv (-1, 0)$ iff $p_i + p_i q_i \leq 0$.

Comparing this with the condition for log-concavity from (31) leads to a contradiction, as $K_i$ is positive. \(\square\)

A.4. Proposition 2

Proof. The first-order condition (FOC) for optimal output under independent audit is similar to the corresponding condition under a relative audit rule but does not contain any indirect influences via changes in the competitors’ declarations due to a changed output. Setting $\frac{\partial \Pi_i(d)}{\partial q_i} = 0$ in Eq. (10) gives the condition
\[ 1 - t + \frac{\partial \beta_i(d)}{\partial d_i} \left[ \Pi_i - \Pi \right] \frac{\partial \Pi_i(q)}{\partial q_i} = 0, \] (33)

which only holds if $\frac{\partial \Pi_i(q)}{\partial q_i} = 0$. This implies that the oligopolists choose the Cournot quantity. \(\square\)

A.5. Lemma 3

Proof. From condition (10) for optimal quantities we have:
\[ \frac{\partial \Pi_i(q)}{\partial q_i} = -\sum_{j \neq i} \frac{\partial \beta_i(q)}{\partial d_j} \left[ \Pi_i - \Pi \right] / \sum_{j \neq i} \frac{\partial \beta_i(q)}{\partial d_j}, \] (34)

Given that $t < 1$, $\Pi_i - \Pi$ and $\frac{\partial \beta_i(d_i)}{\partial d_j}$ is positive (negative) if $i \neq j$ ($i = j$), the sign of $\frac{\partial \Pi_i(q)}{\partial q_i}$ has the same sign as $\frac{\partial \beta_i(q)}{\partial d_j} - \frac{\partial \beta_i(q)}{\partial d_j}$.
competitors have qualitatively identical first-order effects. Firm j’s declaration is influenced by a change of \( q \) through the change in its own profit and the reactions of other firms due to their changed profits. Differentiating (7) and using Eq. (6), firm j’s subgame-perfect reaction to firm i changing its quantity can be written as:

\[
\frac{\partial q_i}{\partial I_i} = \frac{1}{2} \left[ C_i - f + t \frac{\partial \beta_j(d^*)}{\partial d_j} \right] \frac{\partial I_j(q)}{\partial q_i} \tag{35}
\]

where \( \Delta = \partial^2 E_i / \partial q_i^2 \). Since \( \Delta < 0 \), \( C_j > 0 \), \( \partial \beta_j(d^*) / \partial d_j < 0 \), and \( \partial I_j(q) / \partial q_i > 0 \) the right-hand side of Eq. (35) is negative. So any optimal quantity is higher than the best response under Cournot for the same output vector of the other firms

\[
BR^B_i(q_{-i}) > BR^C_i(q_{-i}) \quad \forall i, q_{-i}.
\tag{36}
\]

A.6. Proposition 3

**Proof.** Fix the quantities of firms \( i = 3, \ldots, n \). Inequality (36) from the proof of Lemma 3 ensures that for, every given quantity vector of the \( n - 2 \) remaining firms:

\[
BR^B_i(q_3, \ldots, q_n) > BR^C_i(q_3, \ldots, q_n).
\]

So the candidates \( q_1 \) and \( q_2 \) for mutual best responses given the others’ quantities have to jointly satisfy:

\[
q_1 > BR^C_i(q_3, \ldots, q_n), \tag{37}
\]

\[
q_2 > BR^C_i(q_3, \ldots, q_n). \tag{38}
\]

As \( BR^C_i(\cdot) \) is strictly monotonic for a regular oligopoly in the neighbourhood of an interior equilibrium, condition (38) may be inverted to give

\[
q_1 > BR^C_i(q_3, \ldots, q_n)
\]

which, combined with (37), gives

\[
q_1 > \max \{BR^B_i(q_3, \ldots, q_n), BR^C_i(q_3, \ldots, q_n)\}.
\]

where \( BR^C_i(q_3, \ldots, q_n) \) denotes the inverse best-response function of firm 2. From Lemma 2 we can conclude for a regular Cournot game that:

\[
\frac{d}{dq_2} BR^C_i(q_2) < 0 \quad \forall q_2.
\]

We have \( BR^C_i(q_2^*) = BR^B_i(q_2^*) \), where \( q_2^* \) is firm 2’s quantity in the equilibrium of the reduced two-firm game for given quantities of the others. Because the slopes are different this implies:

\[
q_1 > \begin{cases} BR^C_i(q_2^*) & \text{for } q_2 < q_2^* \\
BR^B_i(q_2^*) & \text{for } q_2 \geq q_2^* \end{cases}
\]

Given that the isouquant for aggregate output \( q_1(q_2) \) for the Cournot equilibrium of the reduced game has slope \(-1\) and satisfies \( q_1(q_2^*) = BR^C_i(q_2^*) = BR^B_i(q_2^*) \) it follows that:

\[
q_1 > q_1(q_2) \quad \forall q_2.
\]

So the aggregate quantity of two firms with relative auditing is larger than without, for any given quantity of the others. Since this is true for any two firms and any given quantity of the other firms we may conclude that any equilibrium output vector \( q^{e} \) is such that \( \sum_{i=1}^{n-1} q_i^{e} > \sum_{i=1}^{n-1} q_i^{e} \).

A.7. Lemma 4

**Proof.** Summing the \( n \) FOCs from Eq. (14) gives:

\[
\sum_{j=1}^{n} \sum_{f=1}^{n} \frac{\partial \beta_j(d)}{\partial d_j} \left[ q_f - \pi_j \right] + \sum_{j=1}^{n} \left[ \beta_j(d^*)f + t - t \right] + \sum_{j=1}^{n} \left[ I_j(q) - d_j \right] = 0. \tag{39}
\]

Substituting (D3) and (16) in Eq. (39) gives the result.

A.8. Proposition 4

**Proof.** Given the uniform fixed audit rule \( f^0 = \beta(d) \), \( \forall i \in N \), condition (11) implies that \( d^0 \) solves

\[
t - \beta(d^*)f + t = C_i(I_j(q) - d_j). \tag{40}
\]

Now take Eq. (14) and invoke symmetry by replacing all \( d_i \) with \( d_j \), which gives

\[
\sum_{j=1}^{n} \frac{\partial \beta_j(d)}{\partial d_j} \left[ q_f - \pi_j \right] + \beta_j(d^*)f + t - t + C_j(I_j(q) - d_j) = 0. \tag{41}
\]

Using property (D3) for a relative rule immediately proves the claim.

A.9. Proposition 5

**Proof.** The set \( D^{\omega} \) given by Eq. (13) is a contour in \( \mathbb{R}^n \) each point of which satisfies Eq. (15). Clearly \( d^0 \in D^{\omega} \) because individual profit maximisation under a fixed rule requires (11) and summing over \( i = N \) gives condition (15). In view of the assumed properties of \( C(\cdot) \) and \( \beta(\cdot) \) the contour can be expressed as

\[
\Phi(d) := n[t - \beta(d)f + t] - \sum_{j=1}^{n} C_j(I_j(q) - d_j) = 0. \tag{42}
\]

where \( \Phi(\cdot) \) is a differentiable function. Substituting in Eq. (42) for \( \partial C_j / \partial d_j \) from Eq. (11) and for \( \beta(\cdot) \) from Eq. (16) it is clear that the slope of \( \Phi \) at \( d^0 \) is given by:

\[
\frac{dd_d}{dd_j} = -\frac{\partial \beta_j(d)}{\partial d_j} / \partial \beta_j(d) / \partial d_j = -1. \tag{43}
\]

by the linear-symmetric rule. Differentiating (42) twice it is clear that \( \Phi \) is convex for \( C_j'(I_j - d_j) \leq 0 \) and concave for \( C_j'(I_j - d_j) \geq 0 \). Now consider the hyperplane representing the vectors \( d \) that have the same aggregate value as \( d^0 \). By Eq. (43) this cannot intersect the locus of potential equilibrium candidates elsewhere if \( \Phi \) is convex or concave. By the supporting hyperplane theorem either Eq. (17) or Eq. (18) must hold.
A.10. Lemma 5

Proof. Taking the FOC for the jointly optimal declarations from Eq. (14) and applying the implicit function theorem we find that the effect is similar to that in the non-cooperative case:
\[ \frac{\partial d_q^o}{\partial q} = \frac{\partial d_q^o}{\partial \Pi_I(q)} \frac{\partial \Pi_I(q)}{\partial q} = - \frac{1}{\Delta} \left[ C_i \left[ f + t \left( \frac{\partial d_q^o}{\partial d_q} \right) \right] \frac{\partial \Pi_I(q)}{\partial q}, \right. \]
where \( \Delta = \partial^2 E_n/\partial d_q^2 \). Since \( \Delta > 0, C_i(\cdot) > 0, \partial d_q^o/\partial d_q < 0, \) and \( \partial \Pi_I(q)/\partial q < 0 \) the result is established. \( \square \)

A.11. Proposition 6

Proof. The FOC at the production stage is given by Eq. (8). Using Eq. (14) gives
\[ \sum_{j=1}^{n} \frac{\partial \beta_j(d_q^o)}{\partial d_q^o} \frac{\partial d_q^o}{\partial q} \left[ \tau_j - \tau_i \right] + \frac{\partial \Pi_I(q)}{\partial q} \left[ 1 - f \right] + \left[ \frac{\partial \Pi_I(q)}{\partial q} - \frac{\partial d_q^o}{\partial q} \right] \times \sum_{j=1}^{n} \frac{\partial \beta_j(d_q)}{\partial d_q} \left[ \tau_j - \tau_i \right] = 0. \quad (44) \]
Contrast this with Eq. (10) in the non-collusive case. Invoking symmetry - i.e. \( \tau_j - \tau_i = 0 \forall j \in \tau \) - using (D3) and rearranging reduces Eq. (44) to
\[ \frac{\partial \Pi_I(q)}{\partial q} = - \sum_{j=1}^{n} \frac{\partial d_q^o}{\partial d_q^o} \left[ \frac{\partial \beta_j(d_q^o)}{\partial d_q^o} \right] \left[ \tau_j - \tau_i \right] \times \left[ 1 - \frac{\partial \Pi_I(q)}{\partial q} \right] \left[ \tau_j - \tau_i \right] = 0. \quad (45) \]
Contrast this with Eq. (34) in the non-collusive case. For the same reasons as in the proof of Lemma 3 we must have \( \partial \Pi_I(q)/\partial q = 0 \forall i \in N \) in Eq. (45), which implies \( q_i^o > q_i^o \), \( \forall \), where \( q_i^o \) stands for firm \( i \)'s Cournot quantity. \( \square \)

A.12. Proposition 7

Proof. Maximising Eq. (19) with respect to \( Q \) gives the FOC:
\[ n \left[ \beta_t^o(f + t) - t \right] \frac{\partial Q}{\partial Q} + n \left[ 1 - \beta_t^o(f + t) \right] \frac{\partial \Pi}{\partial Q} \quad (46) \]
\[ - n \left[ \frac{\partial \Pi}{\partial Q} - \frac{\partial Q}{\partial Q} \right] C_i \left( I - \beta_t \right) = 0. \]
Substituting \( C_i \) from the FOC (11) for optimal declaration into Eq. (46) yields
\[ \left[ 1 - t \right] \frac{\partial \Pi}{\partial Q} = 0, \]
which, given that \( t < 1 \), requires \( \partial \Pi/\partial Q = 0 \). This implies that under the stated assumptions made a fixed rule leads to the monopoly solution. \( \square \)

A.13. Proposition 8

Proof. First note that symmetry ensures that all firms increase their declaration by the same amount if they all earn an additional dollar of gross profit. Therefore increasing the profit for all by the same amount does not influence the individual ex post detection probability, and an individual dollar of gross profit will never lead to an additional declaration of more than one dollar if the detection probability does not change. Therefore
\[ \frac{\partial Q}{\partial Q} = \frac{\partial Q}{\partial \Pi} \frac{\partial \Pi}{\partial Q} \leq 1. \quad (47) \]
Differentiating Eq. (20) with respect to \( Q \) gives the FOC
\[ n \left[ \beta_t^o(f + t) - t \right] \frac{\partial Q}{\partial Q} + n \left[ 1 - \beta_t^o(f + t) \right] \frac{\partial \Pi}{\partial Q} \quad (48) \]
\[ - n \left[ \frac{\partial \Pi}{\partial Q} - \frac{\partial Q}{\partial Q} \right] C_i \left( I - \beta_t \right) = 0. \]
Substituting the FOC for an optimal declaration from Eq. (5) into (48) and simplifying yields
\[ \left[ 1 - \beta_t^o(f + t) \right] \frac{\partial \Pi}{\partial Q} + \left[ \frac{\partial \Pi}{\partial Q} - \frac{\partial Q}{\partial Q} \right] \left[ 0. \quad (49) \right. \]
which further simplifies to
\[ \left[ 1 - t - \beta_t^o(f + t) \right] \left[ I - \beta_t \right] \left[ 1 - \frac{\partial Q}{\partial Q} \right] \left[ 0. \quad (50) \right. \]
Given that \( t < 1 \), conditions (D1) and Eq. (47) imply that Eq. (50) holds if and only if \( \partial \Pi/\partial Q = 0 \). \( \square \)

References


