On Analyzing the World Distribution of Income

Anthony B. Atkinson and Andrea Brandolini

Consideration of world inequality should cause reexamination of the key concepts underlying the welfare approach to measuring income inequality and its relation to measuring poverty. This reexamination leads to exploration of a new measure that allows poverty and inequality to be considered in the same framework, incorporates different approaches to measuring inequality, and allows varied expressions of the cost of inequality. Applied to the world distribution of income for 1820–1992, the new measure provides different perspectives on the evolution of global inequality.

JEL codes: D31, C80

There is currently a great deal of interest in the world distribution of income, as evidenced by the wide popular debate and by many academic articles (see the recent survey by Anand and Segal 2008). People are keen to know whether world inequality is growing or declining. They want to monitor progress toward eradicating world poverty, as in the UN Millennium Development Goals. The main argument of this article is that finding empirical answers to these questions requires first reconsidering the conceptual basis of the measurement of inequality and poverty. The move to a world canvas should be the

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occasion for a fundamental reexamination of underlying principles. While the issues raised apply at a national level as well, their heightened significance at a global level means that they can no longer be swept under the carpet. A critique of the standard inequality measures leads to an exploration of a new approach to measuring global inequality and poverty. This article is primarily about principles, but their application is illustrated by taking as a case study the data on the distribution of world income assembled by Bourguignon and Morrisson (2002).

There are three reasons why a reexamination is necessary. First, differences between incomes are much larger on a world scale than nationally. The Bourguignon and Morrisson data show the decile ratio (the ratio of the top to bottom decile groups) for all the world’s inhabitants in 1992 as 24.7 (available at www.delta.ens.fr/XIX). The figure given by Gottschalk and Smeeding (1997, figure 2) was 5.8 in 1991 for the United States (for a different income concept) and 2.8 for Sweden, almost an order of a magnitude less than the global figure. Measuring world inequality thus requires evaluating a much wider range of incomes than that found in a typical advanced high-income country. (The move to a global scale is the focus here, but there are countries where the within-country income differences are much wider than in the United States, and the argument made here may also be seen as questioning the use of standard inequality measures within those countries.) As is discussed in section II, standard inequality measures impose too tight a straitjacket for applying them both to differences within countries and across the world. More flexibility is needed than can be accommodated with a single parameter, which is why the new measure explored in section III has several parameters.1

The second reason is the need to consider the relationship between measuring income inequality and measuring poverty. People are interested in both world inequality and world poverty, but the two literatures are separate (see Atkinson and Bourguignon 1999), with an uneasy relationship between them. The same criticism applies to studies at the national level, but it is easier to avoid a confrontation between the two concepts when they are moving in the same direction. At a global level, however, the proportion of the world population living on less than $1 a day is falling while the world Gini coefficient remains stubbornly high (see figure 1 later in the article). Do we give priority to one of the indicators? Some people have a lexicographic approach, giving total priority to poverty reduction, but others believe that there is some trade-off between the two concerns. One possibility is to give both measures an independent role in a reduced-form social welfare function, as discussed by Fields (2006) and Kanbur (2008). The approach suggested here accommodates

1. A referee reasonably asked whether this argument is circular: that it suggests that the proper choice of inequality measure depends on how much inequality there is. It can be replied that the more flexible measure is appropriate in all circumstances but that, where income differences are sufficiently small, a single parameter measure may be a reasonable approximation.
differences in weighting of poverty and inequality in a social welfare function that can be tilted toward either concept by varying its parameters. More fundamentally, it goes to the heart of the difference between the two concepts by analyzing how society values an extra dollar at different places on the income distribution.

The third reason for a reexamination is that on a global scale, absolute as well as relative differences need to be considered. In 2005 the real per capita income of China was $4,091, or one-tenth the $41,674 of the United States (World Bank 2008, pp. 23–27). This means that China has to grow 10 times faster than the United States to achieve the same absolute increase in the production of goods and services per person. Even if China grows faster in relative terms, the absolute gap may widen. For example, with annual per capita growth rates of 5 percent in China and 2 percent in the United States, the absolute income gap between the two countries would widen for 49 years before starting to narrow, finally disappearing after 80 years. Concern for the absolute dimension of economic growth has far-reaching implications for assessing its distributive consequences, both between and within countries. As Livi Bacci (2001, p. 114) commented on Dollar and Kraay’s (2002) conclusions on the “pro-poor” effect of economic growth, “it is not much of a relief for somebody living on $1 day to see that his income, up by 3 cents, is growing as much as the income of the richest quintile” (authors’ translation).

At the empirical level, however, relative inequality measures predominate. Official publications do not report estimates of absolute inequality, and even academic studies are rare (one example is Del Río and Ruiz-Castillo 2001). Studies on global income inequality often take different routes, but they have in common a focus on relative measures of inequality (Chotikapanich, Rao, and Valenzuela 1997; Schultz 1998; Bhalla 2002; Bourguignon and Morrisson 2002; Milanovic 2002; Dowrick and Akmal 2005; and Sala-i-Martin 2006). Anand and Segal (2008) focus their survey on relative global inequality. Firebaugh (2003, pp. 72–3) briefly deals with the question to make it clear that “[i]nequality pertains to proportionate share of some item—not to size differences,” and to avoid confusion, he introduces the terms “widening and narrowing gaps” to refer to changing absolute differences.

Only in two recent contributions has attention been drawn to the absolute/relative issue. Ravallion (2004, p. 19) notes that “[w]hile relative inequality has been the preferred concept in empirical work in development economics, perceptions that inequality is rising may well be based on absolute disparities in living standards.” He shows how the “virtually zero correlation” between the relative Gini index and income growth becomes a “strong positive correlation” when an absolute Gini index is employed. Svedberg (2004, p. 28) highlights the importance of looking at the absolute distribution of income across countries and concludes that “[t]o pay more heed to the growing absolute
income gaps between rich and poor countries, and their consequences, seems an urgent task for future research into growth and distribution.”

Section I considers the application of the standard approaches to the world distribution of income and highlights the contrasting findings for trends in poverty, relative inequality, and the absolute cost of inequality. To understand this further, the “world social welfare function” underlying the exercises of measuring world income inequality and world poverty is made more explicit. The main tool in the analysis is the social marginal valuation of income, or the social value attached to an extra dollar received by people located at different points in the income distribution. Specifying how the social marginal valuation of income changes over the income scale is the first step in choosing an inequality measure, but expressing the cost of inequality relative to mean income is a second key step. These two steps underlie the construction of any inequality index.

Section II explains why the standard relative approaches to measuring inequality as well as the alternative, absolute approach proposed by Kolm (1976) fall short when applied over the whole range of world incomes. In effect, the existing measures excessively constrain how the social marginal valuation varies with income and provide no ready means to integrate the analyses of poverty and inequality. This leads to an exploration, in section III, of a new measure, grounded in an absolute approach but more flexible in form. The flexibility not only allows for a wider range of variation of income, as found on a global scale, but also shows how different measures can be obtained as limiting cases (and hence how the different approaches can be blended). The new measure, which differs in both of the key steps outlined above, is applied in section IV to the changes in the world distribution of income from 1820 to 1992. The data are not new—they are those of the Bourguignon and Morrisson (2002) dataset—but the new approach suggested here helps in understanding why people reach different conclusions about the evolution of world inequality and poverty. The main arguments are summarized in section V.

One important aspect should be clarified at the outset. Consideration of the world distribution as a whole, as in the studies cited above, assumes that there is a single world evaluation function. The main, but not the only, way in which inequality measures have been interpreted is in social welfare terms. In adopting such a welfarist perspective, this article posits the world social welfare function as a symmetric function \( W(y_1, \ldots, y_n) \) of the real (purchasing power–adjusted) incomes, \( y_i \), of the \( n \) people (households) in the world ranked by their income from lowest, \( y_1 \), to highest, \( y_n \). There are assumed to be no other relevant differences between people apart from income, which justifies the symmetry assumption. There is then a mapping from the properties of the

2. An important start has been made in studies of the global distribution combining average income and inequality measures; see Gruen and Klasen (2008).

3. The analysis is entirely static: it does not address the welfare evaluation of income changes, with which many people are concerned (see Bourguignon, Levin and Rosenblatt 2006).
world social welfare function to the properties of the inequality measure, and vice versa.

But there is an important difference between the world distribution and the distribution within a country. The people 1 to \( n \) are not all part of the same political entity. Redistributive mechanisms typically operate at the national level and are much more limited at the global level. The formulators of the social objective in a particular country may feel different degrees of responsibility for people who are citizens of that country and those who are citizens of other countries and so may treat them differently. This may, for example, lead to people with (real) income \( y \) being considered poor if they are citizens of country \( A \) but not if they are citizens of country \( B \). Such differential treatment would, however, be inconsistent with there being a single symmetric world social welfare function. Some people would, for this reason, simply reject the idea of a world welfare function and hence any calculations of global inequality or global poverty (see, for example, Bhagwati 2004). Here, the aim is to make sense of such calculations, which implicitly assume a symmetric world social welfare function, treating as irrelevant the country of which a person is a citizen. It is on this assumption that the analysis is based.

Finally, while the article focuses on the social welfare approach to measuring inequality and poverty, that is not the only approach that should be considered. It would be possible to start from a set of axioms; it would be possible to consider other spaces, such as those of capabilities (see Sen 1992).

I. Applying Standard Indices to the World Income Distribution

The most popular index applied to measuring inequality is the Gini coefficient (half the mean difference divided by the mean). Figure 1 shows its value for the world income distribution for 1820–1992 using the Bourguignon and Morrisson data. Bourguignon and Morrisson’s method is to use evidence on the national distribution (or the distribution for a group of countries) of the income shares of decile groups and of the top 5 percent. The groups are treated as homogeneous, which understates the degree of overall inequality. The distributional data are then combined with estimates of national GDP per capita, expressed in constant purchasing power parity (PPP) U.S. dollars at 1990 prices, which are derived from the historical time series constructed by Maddison (1995). The issues raised by this method and issues of data reliability are not considered here; the estimates are taken at face value.4

As Bourguignon and Morrisson (2002, p. 742) show, the Gini coefficient rose almost continuously from 1820 to 1950 and then more or less leveled off

between 1950 and 1992: “[T]he burst of world income inequality now seems to be over. There is comparatively little difference between the world distribution today and in 1950.” If there is a Kuznets inverse-U curve for the world as a whole, then the world is slow to enter the downward phase: see the Gini coefficient in figure 1. On the other hand, measures of world poverty based on a constant purchasing power poverty standard show a steady—indeed an accelerating—downward trend. Figure 1 shows the world poverty headcount calculated by Bourguignon and Morrisson applying a standard comparable to that of the $1 a day standard used by the World Bank.

“Relative” and “Absolute” Approaches

The poverty measure in figure 1 represents an “absolute” approach, in that the poverty line is fixed in terms of purchasing power; a “relative” approach would make it proportional to the median or the mean of the distribution. However, an absolute approach does not imply that the line must be kept constant over time, as discussed below. This suggests a need for care in the use of the word “absolute,” which may take on different meanings in the context of poverty measurement, as Foster (1998) shows.

A different use arises in measuring inequality. Following Kolm (1976), inequality measures are described as “relative” when they are invariant to proportional transformations (scale invariance) and “absolute” when they are invariant to additive transformations (translation invariance). The Gini coefficient described above is “relative.” If all incomes are doubled in purchasing power, the Gini coefficient is unchanged: it is the relative mean difference.

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**Figure 1.** Evolution of World Inequality and Poverty, Standard Measures, 1820–1992

Source: Authors’ elaboration on the Bourguignon and Morrisson (2002) database.
There are good reasons for considering absolute income levels. With a doubling of real incomes from their 2005 values, per capita income in the United States remains 10 times that of China, but the absolute difference increases from $37,583 to $75,166. The world would be getting richer, but the differences between countries would be becoming larger in absolute terms. One way in which this can be reflected is by taking the absolute mean difference, or the absolute Gini coefficient (see figure 1), rather than the relative mean difference. The absolute mean difference has increased throughout the period, accelerating upward after 1950. This alternative—rather neglected—measure of inequality gives a different perspective on the evolution of world income distribution. If the $1 a day poverty headcount is the optimistic view of recent decades of the world distribution, the absolute Gini is the pessimistic view.

Representing Different Social Values

Figure 1 helps explain why people may reach different conclusions about what is happening to world income distribution. People may look at poverty or inequality, and they may think of inequality in relative or absolute terms. This suggests that the functional form of the world social welfare function should reflect differences in social judgments. Indeed, Bourguignon and Morrisson (2002) show how alternative inequality indices may record different directions of change: the period 1980–92 saw the mean logarithmic deviation fall, the Theil index rise, and the Gini coefficient remain virtually unchanged. Different social values can be incorporated by using functional forms, such as those listed above, or by allowing a parameter to vary within a specific functional form. The analysis here uses the second approach, since it makes more transparent the underlying social values.

The constant elasticity index, \( I \), introduced by Kolm (1969) and Atkinson (1970) allows users to choose different values of the elasticity, reflecting different views about the weights to be applied to changes at different points in the income distribution. The index, which is based on the mean of order \( (1 - \varepsilon) \), is given by

\[
I = \begin{cases} 
1 - \left[ \frac{1}{n} \sum_{i=1}^{n} \left( \frac{y_i}{\mu} \right)^{1-\varepsilon} \right]^{1/(1-\varepsilon)} , & \varepsilon > 0, \varepsilon \neq 1 \\
1 - \prod_{i=1}^{n} \left( \frac{y_i}{\mu} \right)^{1/n} 
\end{cases}
\]

where \( y_i \) denotes the income of person \( i \) in a population of \( n \) people with mean income \( \mu \). People are assumed to be ranked by increasing income, so that \( i \) indicates their position in the income distribution. Here, and throughout the article, income is assumed to be strictly positive. As \( \varepsilon \) rises, inequality receives more weight. Where \( \varepsilon = 1 \), the second version of the formula applies, and \( I \) is
equal to 1 minus the ratio of the geometric mean to the arithmetic mean. Where $\varepsilon = 2$, the value of $I$ is higher since it is equal to 1 minus the ratio of the harmonic mean to the arithmetic mean.

The index $I$ can be interpreted as expressing the cost of inequality in terms of the proportionate amount of income that could be subtracted from the mean without affecting the level of social welfare: $I = 1 - \frac{y_e^I}{\mu}$, where $y_e^I$ is referred to as the equally distributed equivalent income, which can be written as $\mu(1 - I)$. This formulation involves two distinct steps, with choices to be made at each step, and this two-step distinction recurs throughout the article. The first step is to specify the function of individual incomes that is added across individuals. In effect, $y_i^{1-\varepsilon}/(1 - \varepsilon)$ is added across incomes, where division by $(1 - \varepsilon)$ ensures a nondecreasing function. (The degree of concavity of this function, captured by $\varepsilon$, embodies the chosen distributional values, as discussed further below.) This sum, divided by $n$, is denoted by $\Sigma$ and referred to below as the additive element of the social welfare function.

The second key step in the measurement of inequality is to take a function of $\Sigma$ and the mean income $\mu$ to arrive at an interpretable formulation. For the index $I$, the concave transformation is first reversed, to give $\left[(1 - \varepsilon)\Sigma\right]^{1/(1-\varepsilon)}$, and then divided by $\mu$ and subtracted from 1 to give $I$. The index $I$ thus expresses the cost of inequality as the proportionate shortfall of the equally distributed income from the mean.

This is, however, a choice. The cost could be expressed differently, as discussed below. The two-step process has been described for the constant elasticity index, but it applies generally, including to nonadditive forms of $\Sigma$, such as that embodied in the Gini coefficient, $G$. In that case, $\mu(1 - G)$ gives the equally distributed equivalent income, or what Sen (1976) called “real national income”: $\mu$ is a measure of aggregate economic performance, and $(1 - G)$ is the discount applied on account of the cost of inequality.

An increase in the income of person $i$ raises social welfare, and the social marginal valuation of income can be defined as the value placed on an additional dollar received by a particular person. For the constant elasticity index, $I$, social welfare is defined by its ordinally equivalent representation constituted by the additive element $\Sigma$ rather than by the equally distributed equivalent income $y_e^I = [(1 - \varepsilon)\Sigma]^{1/(1-\varepsilon)}$. The social marginal valuation of income, $y_i$, is hence equal to $y_i^{1-\varepsilon}$. The elasticity (defined positively) of the social marginal valuation of income is constant and equal to $\varepsilon$. For the index $I$, the marginal valuation tends to infinity as income goes to zero and to zero as income

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5. Throughout the article, the social welfare function is defined in per capita terms rather than in its aggregate form, which implies that the social marginal valuation of income is divided through by $n$. Since what matters are the relative valuations of incomes $i$ and $j$ rather than their absolute values, the division by $n$ is ignored in much of what follows—which affects all incomes equally—referring to the individual social marginal valuation of income. Note that the definition of social welfare in per capita terms has important implications for the interpretation of welfare changes when the population is growing. See footnote 15.
goes to infinity. For the Gini coefficient, $G$, the social marginal valuation of income is given by $[2 - (2i - 1)/n]$, where $i$ is the person’s rank in the income distribution and $n$ is the total number of people. For the poorest person, with $i = 1$, the social marginal valuation is $2 - 1/n$, which approaches 2 as $n$ becomes large; for the median person (with $n$ odd), it is 1; and for the richest person, it is $1/n$, which approaches zero as $n$ becomes large. For both indices $I$ and $G$, the social marginal valuation is nonnegative and nonincreasing.

The index $I$ has been criticized, like the Gini coefficient, for being a relative measure: measured inequality is unchanged when all incomes increase (or decrease) in the same proportion. As discussed, it is a matter of concern at the global level that equal rates of growth in all countries imply widening absolute gaps. Kolm (1976) introduced the absolute index

$$K = \frac{1}{\kappa} \ln \left[ \frac{1}{n} \sum_{i=1}^{n} e^{\kappa(\mu - y_i)} \right], \quad \kappa > 0.$$  

The index $K$ is absolute in the sense described earlier: inequality is unaffected by an equal addition to (or subtraction from) all incomes. With constant relative growth rates, inequality would increase.

As Kolm (1976, pp. 437–38) clearly recognized, the use of the index $K$ involves two distinct departures, corresponding to the two key steps in the formulation described earlier. The first involves the different functional form in the additive element $\Sigma$: exponential rather than isoelastic. The second involves expressing the cost of inequality in absolute rather than relative terms. The index $K$ represents the cost of inequality defined as the absolute amount of income that could be subtracted from the mean without affecting the level of social welfare: $K = \mu - y^K_e$, where $y^K_e$ is the equally distributed equivalent income, equal to $\mu - K$ (see also Blackorby and Donaldson 1980). Inequality is said to cost $X$ billion, rather than $x$ percent of total income. In this respect, the index $K$ is parallel to the absolute Gini coefficient. Equally, the measures $I$ can be expressed in absolute terms ($\mu I$), and the measures $K$ as a proportion of mean income. (The cost can be normalized in this way because an equally distributed equivalent distribution is being considered, and in this case absolute and proportional changes in the distribution are identical.)

The index $K$, like the index $I$, contains a free parameter $\kappa$ that captures inequality aversion. The larger $\kappa$, the higher is the weight attributed to the

6. This follows from writing the social welfare function as $\mu(1 - G)$ and $G$ as $\Sigma_i (2i - n - 1)y_i/n^2 \mu$. On the social welfare function implicit in the Gini coefficient, see Sen (1976) and Blackorby and Donaldson (1978).

7. The Kolm index, and more generally any nonrelative measure, is not unit invariant: a change in the unit of account of the incomes affects measured inequality, even if the underlying distribution is unaltered. Zheng (2007) proposes a new axiom of unit consistency requiring that income inequality rankings be preserved as the unit of account varies. The simpler approach adopted here takes account of the definition of units in the choice of $\kappa$. 


lowest incomes; when $\kappa$ tends to infinity, $K$ tends to the difference $(\mu - y_1)$ between the mean income and the lowest income, $y_1$. The individual social marginal valuation of income, as computed from the additive element of the social welfare function, is given by $\exp (-\kappa y_i)$, and its elasticity with respect to income, defined positively, is equal to $\kappa y_i$. The elasticity is increasing with income. Moreover, if the elasticity is specified at a particular value of income, then the value of $\kappa$ can be deduced. If, for example, the elasticity is set equal to 1 at the mean income, then $\kappa$ would equal the reciprocal of the mean.8

In empirical applications, the choice of the parameters $\varepsilon$ and $\kappa$ has to be considered. Researchers using the constant elasticity index $I$ have chosen a range of values. Mirrlees (1978) straightforwardly proposed the “inverse square law,” with a value of $\varepsilon = 2$. When used in official publications, however, the values tend to be lower. The study on high-income countries by Sawyer (1976) used values of 0.5 and 1.5. The U.S. Census Bureau (Jones, Weinberg, and U.S. Census Bureau 2000, p. 7, for example) publishes income distribution statistics taking values of 0.25, 0.5, and 0.75 (it also suggests that 1.0 is the maximum permissible value, although the expression for $I$ indicates that this is not the case).

One way to pin down these values is by resorting to estimates of the social preferences implicit in tax systems. Christiansen and Jansen (1978) estimated the elasticity of the social marginal value of income implicit in the Norwegian system of indirect taxation in 1975 to be equal to 1.7 or to 0.9, depending on the model specification. Stern (1977) found an elasticity of around 2 for the British income tax system of the early 1970s.

Today, political preferences may be for less redistribution, so that lower values should also be considered. This has been suggested by experimental evidence, which provides a second source. Amiel, Creedy, and Hurn (1999) found broad support for median values of the elasticity of around 0.2. Such experiments typically ask people to think about the elasticity in terms of Okun’s (1975) “leaky bucket.” Suppose that a transfer costing $1 to a person with double the mean income is made to a person with half the mean income, with 50 cents being lost in the transfer, so that the recipient receives only 50 cents. Whether this “leaky” transfer increases social welfare depends on the relative valuation of marginal changes in income. An elasticity of 1 means that, compared to the $1 cost to the person with double mean income, four times the weight is attached to the 50 cents received by the person on half average income. So the transfer would raise social welfare. If the elasticity were 0.5, then the weight would only be twice, and the cost and the benefit would be equal. Put more generally, a loss $\ell$ is socially acceptable up to the point at

8. The aim of this procedure is to fix the magnitude of $\kappa$. Once chosen, the value of $\kappa$ is kept constant over time. This implies that, as real income grows, the actual elasticity of the social marginal value of income must also rise. To keep the elasticity constant over time, $\kappa$ would have to be inversely proportional to the mean. However, this would change the nature of the index $K$, which would no longer be translation invariant.
which \( z^\varepsilon (1 - \ell) = 1 \), where \( z \) is the ratio of the income of the donor to that of the recipient. This mental experiment is helpful in thinking about the implications of different values of the elasticity of the social marginal value of income, and it is considered again in the next section.

Applying Parameterized Measures to the World Income Distribution

In applying these measures to the world income distribution, values were taken for the elasticity in the interval \([0.125, 2.0]\), which should cover a wide range of social preferences. As is clear from figure 2, adopting different values for \( \varepsilon \) gives very different measures of the cost of world inequality, varying in 1992 from 10 percent with \( \varepsilon = 0.125 \) to 74 percent with \( \varepsilon = 2 \). But the time trend does not differ much from that of the Gini coefficient, shown without markers. For the index \( K \), figure 3 assumes that the values of the elasticity apply at the world median income in 1992, estimated from the Bourguignon and Morrisson data to be $1,712 at 1990 PPP. Here the cost of inequality is expressed absolutely, and the comparator is the absolute Gini coefficient, again shown without markers. The time path of the \( K \) index for elasticities of 1 and 2 is similar to that for the absolute Gini, and there is no great difference between the \( K \) index and the corresponding absolute version of the \( I \) index. The time paths for the elasticity of 0.125 show more difference.

These findings suggest that the major difference between the inequality indices \( I \) and \( K \) applied at a world scale lies in expressing the cost of inequality in absolute terms. Of the two key stages identified earlier, the expression of cost is crucial. The individual functional form plays less of a role.\(^9\) But this is not necessarily the case when considering a wider range of functional forms, as examined next.

II. Sensitivity to Different Transfers

The functional forms considered so far do not allow sufficient flexibility when considering the world distribution. This may be seen by returning to the hypothetical leaky bucket experiment and the effect of transfers of income at different points in the world distribution. The essential problem is that of devising a path for the social marginal valuation of income that treats appropriately both transfers within a rich country, such as the United States, and transfers between people in rich countries and the poor in poor countries.

Table 1 shows the means for decile groups in a selection of countries (or groups of countries), according to the Bourguignon and Morrisson data for 1992, with income expressed relative to the 1992 world median ($1,712 at 1990 PPP). Thus, the first row in table 1 shows that the mean income for the first (lowest) decile group for 46 African countries (total population of 357

\(^9\) The same considerations apply to Kolm’s (1976) “centrist” index and Bossert and Pfingsten’s (1990) intermediate indices. These alternatives are discussed in Atkinson and Brandolini (2004).
million) is 0.15 of the world median. The average income for the tenth (highest) decile group in the United States in 1992 is some 40 times the world median.
### Table 1. World Incomes in 1992 Expressed Relative to the World Median and Social Marginal Valuation of Income

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<tr>
<th>Country and decile groups</th>
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<th>Social marginal valuation of income</th>
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<tr>
<td></td>
<td></td>
<td>Constant elasticity, ( e = 2 )</td>
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<td>---------------------------</td>
<td>---------------------------------</td>
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<tr>
<td>0.15 46 African countries, decile group 1</td>
<td>0.15 46 African countries, decile group 1</td>
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<td>1.11 Turkey, decile group 4</td>
<td>1.11 Turkey, decile group 4</td>
<td>0.812</td>
</tr>
<tr>
<td>1.27 37 Latin American countries, decile group 7</td>
<td>1.27 37 Latin American countries, decile group 7</td>
<td>0.620</td>
</tr>
<tr>
<td>1.40 45 Asian countries, decile group 6</td>
<td>1.40 45 Asian countries, decile group 6</td>
<td>0.510</td>
</tr>
<tr>
<td>1.49 Mexico, decile group 5</td>
<td>1.49 Mexico, decile group 5</td>
<td>0.450</td>
</tr>
<tr>
<td>1.57 Portugal-Spain, decile group 1</td>
<td>1.57 Portugal-Spain, decile group 1</td>
<td>0.406</td>
</tr>
<tr>
<td>1.68 Poland, decile group 4</td>
<td>1.68 Poland, decile group 4</td>
<td>0.354</td>
</tr>
<tr>
<td>1.76 United States, decile group 1</td>
<td>1.76 United States, decile group 1</td>
<td>0.323</td>
</tr>
<tr>
<td>2.00 Brazil, decile group 7</td>
<td>2.00 Brazil, decile group 7</td>
<td>0.250</td>
</tr>
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<td>2.36 Germany, decile group 1</td>
<td>2.36 Germany, decile group 1</td>
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</tr>
<tr>
<td>2.77 United States, decile group 2</td>
<td>2.77 United States, decile group 2</td>
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<td>3.03 Italy, decile group 2</td>
<td>3.03 Italy, decile group 2</td>
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</tr>
<tr>
<td>3.44 Germany, decile group 2</td>
<td>3.44 Germany, decile group 2</td>
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</tr>
<tr>
<td>7.02 Italy, decile group 5</td>
<td>7.02 Italy, decile group 5</td>
<td>0.020</td>
</tr>
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<td>9.19 United States, decile group 5</td>
<td>9.19 United States, decile group 5</td>
<td>0.012</td>
</tr>
<tr>
<td>10.01 Germany, decile group 7</td>
<td>10.01 Germany, decile group 7</td>
<td>0.010</td>
</tr>
<tr>
<td>Income relative to world median</td>
<td>Country and decile groups</td>
<td>Social marginal valuation of income</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>---------------------------</td>
<td>-----------------------------------</td>
</tr>
<tr>
<td></td>
<td>Constant elasticity, ( e = 2 )</td>
<td>Constant elasticity, ( e = 1 )</td>
</tr>
<tr>
<td>11.08 United States, decile group 6</td>
<td>0.008</td>
<td>0.090</td>
</tr>
<tr>
<td>14.79 France, decile group 9</td>
<td>0.005</td>
<td>0.068</td>
</tr>
<tr>
<td>20.66 United States, decile group 9</td>
<td>0.002</td>
<td>0.048</td>
</tr>
<tr>
<td>38.79 United States, decile group 10</td>
<td>0.001</td>
<td>0.026</td>
</tr>
</tbody>
</table>

**Note:** Decile group 1 is the lowest and decile group 10, the highest.

a. As income refers to the mean income of each decile group (as a ratio to the world median), in the expression for the social marginal valuation of income, the term \((2i - 1)/n\) represents the mean rank of all people in the decile group and is calculated as the sum of the cumulative share of all groups poorer than the one indicated and half the population share of the group itself.

**Source:** Authors’ elaboration on the Bourguignon and Morrisson (2002) database.
Now consider the individual social marginal valuation of income, expressed initially as an isoelastic function of income, $y^{-\varepsilon}$, so that the social valuation of an extra dollar accruing to a person with income $y$ is $2^\varepsilon$ times that of an extra dollar accruing to a person with income $2y$. The implied social marginal valuations of income, expressed as a ratio to the social marginal valuation of the median income, are shown for three values of $\varepsilon$ in table 1. As envisaged in the leaky bucket experiment, the value of $\varepsilon$ determines the degree of loss that people are willing to accept when making a redistributive transfer. For domestic redistribution in the United States, the mean for decile group 6 is four times the mean for decile group 2, according to the Bourguignon and Morrission data. Then $\varepsilon = 2$ implies that a transfer of $1$ from decile group 6 to decile group 2 would raise social welfare if all but $1/4^2 = 1/16$ leaked away before reaching decile group 2, or that a loss of up to almost 94 cents would be acceptable. This degree of leakage might appear too high. Put another way, the implied social marginal valuation for a person in decile group 2 in the United States would be $16 (=4^2)$ times that for a person in decile group 6, and the implied marginal valuation for a person in decile group 2 would be $196 (=14^2)$ times that of a person in decile group 10 (the mean income of decile group 10 being 14 times that of decile group 2). If $\varepsilon = 1$, then for a transfer of $1$ from decile group 6 to decile group 2, the maximum acceptable leakage is 75 cents, and the marginal valuation for a person in decile group 2 would be 14 times that for a person in decile group 10. If $\varepsilon = 0.5$, the central value used by the U.S. Census Bureau, the maximum acceptable leakage for a transfer of $1$ from decile group 6 to decile group 2 would fall to 50 cents, and the marginal valuation for a person in decile group 2 would be 3.75 times that for a person in decile group 10.

How does this extend to the world scale? Table 1 shows that the average income of the top 10 percent in the United States is some 140 times that of the bottom 10 percent in India. A value of $\varepsilon = 0.5$ implies that a transfer of $1$ from U.S. decile group 10 to India decile group 1 would be acceptable if the loss is 92 cents or less (if 8 cents are received). Would such a level of loss be acceptable? The social marginal valuation of income accruing to decile group 1 in India is, at $\varepsilon = 0.5$, nearly 12 times that of a person in decile group 10 in the United States.

Some might believe that a lower value of $\varepsilon$ should be applied. A value of $\varepsilon = 0.25$ implies that the social marginal valuation of income for a person in the bottom decile group in India is 3.44 times that of a person in the top decile group in the United States; a value of $\varepsilon = 0.125$ implies that the marginal valuation would be 1.85 times that of a person in decile group 10 in the United States and that a loss of up to 46 cents would be acceptable. However,

10. It should be noted that issues of agency are not considered here, in particular the fact that the United States has less control over the leakages with an international transfer than it has with a domestic transfer.
what are the implications of low values of $e$ for the evaluation of transfers from other countries to a person in decile group 1 in India? Table 1 shows that a relatively low-income person in Western Europe, say a person in decile group 2 in Germany, might have an income 12.5 times that of a person in decile group 1 in India. A value of $e = 0.125$ implies that the marginal valuation of income for a person in decile group 1 in India is only 1.37 times that for a person in decile group 2 in Germany. This will strike many people as too low.

Moreover, reducing $e$ to such low values would have implications for transfers within the United States. With $e = 0.125$, for example, a transfer would be made from decile group 10 to decile group 2 only if the leakage was less than 28 cents, which seems a limiting requirement. (A considerable fraction of those in decile group 2 are below the official U.S. poverty line.) The marginal value of $1$ to a person in decile group 2 would be treated as worth only 1.4 times $1$ to a person in decile group 10. Adjusting the parameter to fit the world distribution is, in effect, squeezing the range of distributional weights applied within the United States. Adopting values more appropriate to the within-country situation instead, however, implies a very wide range of marginal valuations on the global scale. With the inverse square law ($e = 2$), for example, the marginal value of income to a person in the bottom decile group in India is almost 20,000 times that to a person in the top decile group in the United States.

These difficulties arise from the straitjacket imposed by the assumption of a constant elasticity. To quote Little and Mirrlees (1974, p. 240), “there is no particular reason why [the social marginal valuation] should fall at the same proportional rate at all consumption levels. Why should twice as much consumption deserve a quarter of the weight, whether consumption is low or high?”

Anand and Sen (2000) make a case for a variable elasticity function in which elasticity increases with income. As they note, this can be achieved by adopting the Kolm absolute index, $K$. Table 1 shows the marginal valuation of income implied by the Kolm index with an elasticity of 0.125 at the world median. This has a large effect on the marginal valuations within the United States: the marginal value of $1$ to a person in decile group 2 rises to 90 times that to a person in decile group 10. But it would have little effect on the marginal valuations of income for the person in decile group 1 in India relative to that of a low-income person in Western Europe, rising from 1.37 to 1.48. The use of the Kolm index relaxes the constant elasticity assumption, but it does not reconcile both ends of the world distribution. The same consideration would apply if the social welfare function proposed by Anand and Sen (2000) were used, which combines the constant relative and constant absolute inequality versions.

The Gini coefficient, possibly the most used among inequality indices, provides an insightful alternative. As seen above, the social marginal valuation implicit in the Gini coefficient depends on the income rank order and is
bounded above by 2 and below by zero. (In Table 1, this is approximated by the mean rank of all people in each decile group, calculated as the sum of the cumulative share of all groups poorer than the one indicated and half the population share of the group itself.) The Gini coefficient has another appealing property, which may be seen in figure 4 (corresponding to table 1). With the Gini index, the social marginal valuation of income declines above the 1992 world median in a fashion similar to the constant elasticity $\varepsilon = 1$ but differs at lower values. Initially, the marginal valuation falls slowly with income, but then the decline accelerates up to the mode. Finding a functional form that has this “slow, quick, slow” property would enable, at least in part, differentiating between incomes received within poor and rich countries, while also bounding the differentiation between poor and rich countries. The widespread use of the Gini coefficient in studies of the world distribution can be seen as an implicit revelation of preference for such a pattern.

At the same time, despite its popularity, the Gini coefficient has two features that are open to challenge. The first is that, unlike the I and K indices, it is not additively separable in incomes. It lacks the property that the ratio of the social marginal valuations of income for person $i$ and person $j$ depends only on their incomes. Consider an example. Suppose that the European Union is contemplating a switch from a policy transferring $1$ to a person in decile group 4 in Turkey (under a program for countries applying for EU membership) to a policy transferring $1$ to a person in decile group 1 in India (under its development program). With Gini weights, the social marginal valuation for decile

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**Figure 4. Social Marginal Valuation of Income**

![Graph showing social marginal valuation of income](image)

*Note: All values of the social marginal valuation of income are normalized by its value at the world median.*

*Source: Authors’ elaboration on the Bourguignon and Morrisson (2002) database.*

11. The kernel estimates of the world distribution of income by Bourguignon and Morrisson (2002, figure 1) have a secondary mode, but the broad shapes are consistent with the statement in the text.
group 1 in India is 1.97 times that for decile group 4 in Turkey (see table 1). Between these two groups lie the bottom six decile income groups in China. If rapid development in China were to shift these decile groups above decile group 4 in Turkey, the fall in the income ranking in the world population would cause the social marginal valuation for the Turkish decile group to rise from 0.962 to 1.218. As a result, the social marginal valuation for decile group 1 in India relative to decile group 4 in Turkey would fall by more than a fifth, to 1.55. Incomes in India and Turkey would have remained the same, but the attractiveness of the switch in policy would have been affected by development elsewhere. This is the argument for assuming additive separability (although there may be circumstances in which separability might not be an appropriate assumption).

The second problem with the Gini coefficient arises from its treatment of high incomes. It is going too far to say that it involves “spiteful egalitarianism” (Feldstein 2005, p. 12), but it is true that the Gini weights do tend to zero very fast at the top of the income scale, as can be seen from table 1. It is not clear that the social marginal valuation for a person in decile group 9 in France should be 2.14 times that for a person in decile group 9 in the United States. It might be desirable to allow for the possibility that the social marginal valuation remains strictly above a positive value as income tends to infinity.

III. Toward a New Approach

The previous discussion provides the rationale for exploring a new measure. The objective is to design a measure that combines the “slow, quick, slow” empirical property of the Gini coefficient with additive separability, while allowing for a strictly positive social marginal valuation of income at all income levels. The second motivation for devising a new measure goes back to the objective of measuring poverty and inequality within a common framework. This can be achieved by assigning the role of a poverty line to a particular income level, a feature not part of any of the measures considered so far. Identifying a poverty threshold within the social welfare function helps to show that concern about poverty may arise because incomes are unequally distributed and some people fall below the poverty line or because mean income is below the poverty line (or both). Put differently, poverty may occur even if everyone has the same income, if a society is globally poor. Clearly this depends on how the poverty line is defined. A society could not be globally poor if the poverty line were taken as some percentage (less than 100 percent) of the mean income.

Several approaches are considered here. That just described, often referred to as a “relative” poverty line, may be contrasted with “absolute” poverty lines that are independent of mean income, although it should be noted that “absolute” poverty lines are not necessarily constant over time. As Sen (1983) has
stressed, a standard fixed in one evaluative space, such as that of capabilities, might imply a poverty line in terms of income that varies over time.

Achieving the objectives of increasing flexibility and integrating poverty and inequality requires introducing several parameters governing the form of the social welfare function. Understandably, there may be resistance to being asked to consider a measure of inequality that requires thinking first about the values of different parameters. The popularity of the Gini coefficient is in part due to the fact that it does not require specifying any parameters. However, this does not mean that there are no implicit value judgments underlying the Gini coefficient; as already shown, its properties can be challenged. The virtue of parameterization, as argued in Atkinson (1970), is that it forces the user to make explicit choices about the instrument of evaluation and it allows readily for different views about the importance of redistribution. At the same time, guidance on the choice of parameter values may be welcomed. One aim of the numerical application in the next section is to give a flavor of the consequences of different choices.

Consider the following four-parameter measure of global social welfare:

\[
\Sigma = \frac{1}{n} \sum_i W_i = \frac{1}{n} \sum_i \left\{ y_i - \frac{\lambda}{\beta} e^{\beta(y_i - \delta)} \ln \left[ 1 + e^{\beta(y_i - \delta)} \right] \right\}
\]

where \( \beta \) is positive and has the dimension of 1/income, \( \delta_0 \) and \( \delta \) have the dimension of income, and \( \lambda \) is a non-negative pure number. As a consequence, the expression \( \Sigma \) used to evaluate the total world welfare has the dimension of income.\(^{12}\)

Specification (3) embodies the two key steps described earlier—the shape of the individual function and the calculation of the welfare cost—both of which involve assumptions. The first step is to adopt an exponential form that tilts the measure in the direction of index \( K \) rather than index \( I \). Indeed, as shown below, the Kolm measure may be seen as a limiting case of specification (3). In this sense, \( \Sigma \) is an absolute measure.\(^{13}\) The first element in specification (3) is mean income, from which the second term, which captures the unequal distribution of income weighted by the parameter \( \lambda \), is subtracted. In this sense, too, \( \Sigma \) is an absolute measure. Note that \( \Sigma \) can be negative.

How can the different parameters be interpreted? It is useful to begin with the first derivative:

\[
W_i' = \frac{1}{n} \left[ 1 + \frac{\lambda e^{\beta(y_i - \delta)}}{1 + e^{\beta(y_i - \delta)}} \right]
\]

12. The social welfare function is assumed to be defined over incomes, not individual utilities. This is not to assert that there exists a well-behaved utility function such that the private marginal valuation of income can be written in this form.

13. It would be an interesting extension to consider a version closer to the \( I \) measure. The authors are grateful to Peter Hammond for suggesting the derivation of the \( K \) or \( I \) indices as a limiting case.
As before, the divisor \( n \) is ignored, and the term in brackets is referred to as the individual social marginal valuation of income for person \( i \). There are four parameters in specifications (3) and (4), but \( \delta_0 \) plays only an instrumental role; unless explicitly signaled, it is assumed that \( \delta_0 = \delta \), reducing to three the parameters that need to be chosen: \( \lambda \), \( \beta \), and \( \delta \).

The parameter \( \lambda \) captures the varying importance attached to distributional concerns. If no weight is attached to distribution, then one simply sets \( \lambda = 0 \), the social marginal valuation is everywhere 1, and that is the end of the story. The implications of different, nonzero, choices of \( \lambda \) may be seen from considering the fact that (with \( \delta_0 = \delta \)) the social marginal valuation falls monotonically from \( [1 + \lambda/(1 + e^{-\beta \delta})] \) when \( y_i \) is 0 to 1 as \( y_i \) tends to infinity. The social marginal valuation of a person with zero income is at most \((1 + \lambda)\) times that for the richest person, so that \( \lambda = 4 \) corresponds to a maximum ratio of five, which implies a maximum socially acceptable loss of 80 percent from a transfer from the richest person to the poorest. This value of \( \lambda \) is applied in the illustrations below, although in the light of the large world income differences, this may be regarded as a conservative choice.

The two remaining parameters, \( \beta \) and \( \delta \), determine the nature of the concern for inequality and poverty. Specification (3) gives a special status to the income level \( \delta \), and one interpretation, taken up below, is that of a poverty line. Other interpretations can be given as well, however, and variations in \( \delta \) allow the measure to adopt either a Kolm-like form or a Gini-like form. The parameter \( \beta \) determines the “angularity” of the measure, which has a natural interpretation in each of the cases, now discussed in turn. Because the discussion below focuses not on incomes but on their ratios to the median \( m \), the actual values of the parameters in the income space are \( \delta m \), \( \delta_0 m \) and \( \beta/m \).

**The Poverty Gap**

Some people believe that poverty is a concern, but not inequality. This position is exemplified by Feldstein (2005, p. 12): “I have no doubt about the appropriateness of transferring income to the very poor... the emphasis should be on eliminating poverty and not on the overall distribution of income or the general extent of inequality.” This position has been called “charitable conservatism” (Atkinson 1990). An attraction of the measure explored here is that it encapsulates the poverty gap if \( \delta \) is set as the poverty line, with \( \delta_0 = \delta \), and \( \beta \) tends to infinity. Under these assumptions, the social welfare function (3) becomes:

\[
(3a) \quad \sum_{\beta \to \infty, \delta_0 = \delta} = \frac{1}{n} \sum_i y_i - \lambda \frac{1}{n} \sum_i \max[0, (\delta - y_i)]
\]

14. As \( \beta \) goes to infinity, if \( y_i \geq \delta \) the term \((1/\beta \ln[1 + \exp(\beta (\delta - y_i))]\) in equation (3) tends to zero; if \( y_i < \delta \), application of L’Hôpital’s Rule allows the limit to be calculated as \((\delta - y_i)\).
Thus, world welfare is evaluated as the mean minus $\lambda$ times the aggregate poverty gap per person of the total population. As may be seen from equation (4), as $\beta$ tends to infinity, with $\delta_0 = \delta$, the social marginal valuation equals $(1 + \lambda)$ where income is below $\delta$ and 1 where it is above $\delta$. Distributional concern is concentrated below the poverty line, to an extent that depends on $\lambda$. Where $\lambda = 4$, four times the poverty gap is subtracted from national income: multiplying by $\lambda$ allows for the concerns of those who feel that the small size of the poverty gap, expressed per person of the total population, understates its significance.

**A Less “Angular” Version**

With the poverty gap, the social marginal valuation is constant as a function of income when income is below the poverty line, falls like a stone at $y = \delta$, and is again constant for all incomes above the poverty line. For some people, this is too abrupt. They might well want to taper the marginal valuation as income approaches the poverty line and to recognize that the needs of the “near-poor,” just above the poverty line, are greater than those of people comfortably above. The 1991 modification to the Human Development Index (HDI) was based on the argument that “the idea of diminishing returns to income is now better captured by giving a progressively lower weight to income beyond the poverty cut-off point, rather than the zero weight previously given” (United Nations Development Programme 1991, p. 15). The HDI modification took the form of a fractional weight above the poverty line, but such a less “angular” version can also be achieved using specification (3) by retaining $\delta (= \delta_0)$ as the poverty line and taking a finite value of $\beta$. With $\beta$ finite, the social marginal valuation changes more smoothly around $\delta$. This may be seen from the second derivative:

\[
W''_i = -\frac{1}{n} \frac{\beta \lambda e^{\beta (\delta_0 - \delta)}}{[1 + e^{-\beta (y_i - \delta)}][1 + e^{\beta (y_i - \delta)}]}
\]

which has its minimum value (the steepest downward slope for the marginal

15. Formulation of the social welfare function in per capita terms implies that world welfare goes up, *ceteris paribus*, whenever the aggregate poverty gap grows less than the population. However, one could argue that what matters in assessing poverty is the amount of resources necessary to eliminate poverty—the absolute aggregate poverty gap, not its value per person. This corresponds to viewing the world poverty as measured by the absolute number of the poor rather than by their number relative to the total population. Which of these two conceptions of poverty is chosen has important consequences for interpreting the evolution of poverty and welfare, as the absolute and per person aggregate poverty gaps need not move in the same direction. Chakravarty, Kanbur, and Mukherjee (2006) attempt to unite these two conceptions of poverty by developing a family of poverty measures that avoid the population replication axiom.

16. This quotation is drawn from Anand and Sen (2000), who present an extensive (and sympathetic) critique of the treatment of the social marginal valuation of income in successive versions of the HDI.
valuation) at \( y_i = \delta \). Both before and after \( y_i = \delta \) the slope is less steep. The value of \( \beta \) determines how sharply the social marginal valuation changes around the point of inflexion. This is illustrated in figure 5, where the poverty line is 0.5. The marginal valuation at the poverty line is \((1 + \lambda/2)\), independent of \( \beta \). All the curves relating to the new measure in figure 5 pass through this point since \( \lambda \) has a common value of 4. (To ease comparison, the curves for the Kolm and the constant elasticity measures are rescaled to go through this point as well.) With \( \beta = 12 \), the function is a “smoothed” version of the poverty gap, giving some additional weight to people above the poverty line, but the weight falls rapidly away: at the world median, the social marginal valuation is indistinguishable from that with the poverty gap. With \( \beta = 4 \), on the other hand, less significance is attached to the poverty line. Those with incomes up to three times the poverty line receive a perceptible additional weight, which is similar to that assigned to them by the (rescaled) constant elasticity index \( I \) with \( \varepsilon = 1 \); for higher incomes, the social marginal valuation stabilizes at 1, the lower bound for the new measure, while it keeps declining for the index \( I \). With \( \beta = 4 \), those below the poverty line get lower weight, relative not only to the poverty gap version and the function with higher values of \( \beta \) but also to the constant elasticity measure.

**Toward an Inequality Measure**

If \( \delta \) is no longer regarded as the poverty line, the new measure can represent the views of people who are concerned with overall inequality. With \( \delta = 0 \)

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**Figure 5. Social Marginal Valuation of Income with New Measure, Poverty Line Version**

![Figure 5](image-url)

*Source: Authors’ elaboration.*
(and \( \delta_0 = 0 \)), there is no interior point of inflexion, and (with \( \lambda = 4 \)) the social marginal valuation of income has the form shown in figure 6 by the three curves starting from the same value 3 (the social valuation at zero income is \( 1 + \lambda / 2 \)). The three curves are based on different values of \( \beta \) and illustrate different speeds of approach to the limiting value of 1: the greater the value of \( \beta \), the more rapidly the weight attributed to higher incomes converges to 1. For the range of incomes shown in figure 6, the curve with the lowest value of \( \beta \) (0.5) has some similarity with the Kolm index with an elasticity of 0.2 at the median (after rescaling so that it also starts from 3 when income is nil).

There is indeed a close relationship with the Kolm index. If \( \delta_0 \) is held to zero, but \( \delta \) is allowed to tend to minus infinity, the individual social marginal valuation of income becomes \( 1 + \lambda \ e^{-\beta y_i} \), which for large \( \lambda \) approaches the Kolm form with \( \beta \) corresponding to \( \kappa \) in equation (2). As \( \delta \) tends to minus infinity, equation (3) becomes:

\[
(3b) \quad \Sigma_{\delta \rightarrow -\infty, \delta_0 = 0} = \frac{1}{n} \sum_i y_i - \frac{1}{\beta n} \sum_i e^{-\beta y_i}
\]

The separation of \( \delta_0 \) and \( \delta \) is introduced to allow this limit to be taken. (The limit may be seen from equation (3) by regarding \( \exp(\beta \delta) \) as the denominator and applying L’Hôpital’s Rule.) Arrival at a form similar to the Kolm index underlines the absolute rather than relative nature of the generalization, but the difference remaining where \( \lambda \) is finite should be stressed: as income goes to

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**Figure 6.** Social Marginal Valuation of Income with New Measure, Inequality Version

![Figure 6](image-url)

*Source: Authors’ elaboration.*
infinity, the social marginal valuation of income goes to zero in the case of the
Kolm index while it approaches one with specification (3b). Thus, the social
evaluation of an extra dollar accruing to the poorest person relative to an extra
dollar accruing to the richest person approaches infinity with the Kolm index,
while it is at most \((1 + \lambda)\) with this formulation.

The similarity with the Kolm index is illustrated by the curves in the upper
part of figure 6. The two curves virtually coincide within the shown income
range, with the continuous curves that correspond to the Kolm indices having
the same elasticity at the median, rescaled to start from the same value at zero
income. (The two curves would, however, depart from their Kolm counterparts
at some higher level of income.)

**Slow, Quick, Slow**

So far, \(\delta\) has been allowed to vary downward. If \(\delta\) is allowed to be positive, a
measure is obtained with the “slow, quick, slow” property. This is illustrated
in figure 6 by setting \(\delta = 1\) (once again equal to \(\delta_0\)) and \(\beta = 6\). The key
element is the behavior of the second derivative of the social welfare function.
From equation (5), it may be seen that the third derivative of the social welfare
function explored here is first negative (for \(y_i < \delta\)) and then positive (for \(y_i > \delta\)). The literature on transfer “sensitivity” (Atkinson 1973; Kolm 1976; Davies
and Hoy 1985) shows that the assumption that the third derivative is positive
is equivalent to the “principle of diminishing transfers,” or third order stochas-
tic dominance.

Suppose that the two people in the earlier leaky bucket experiment (person
1 poorer than person 2) are now each joined by a friend with income \(h\) above
their, and that $1 is simultaneously transferred from person 1’s friend to
person 1 and $1 from person 2 to person 2’s friend. In other words, there are
two mean-preserving transfers of the same size, but in opposite directions.
Then, the principle of diminishing transfers means that more weight is attached
to the transfer affecting the poorer person and that social welfare increases (see
Shorrocks and Foster 1987, for a more general treatment). With the social
welfare function explored here, this principle is assumed to apply at middle
and higher incomes, above the point of inflexion \(\delta\). In contrast, over the initial
range of incomes, up to \(\delta\), there is increasing sensitivity to transfers.

As before, the parameters can be calibrated by considering the elasticity of
the social marginal valuation. As shown by equation (6), below, this varies
with income. With \(\delta = \delta_0\), the elasticity at the point of inflexion \(y_i = \delta\) is equal
to \(\lambda \beta \delta/[2(2 + \lambda)]\). In figure 6, \(\lambda = 4\) and \(\beta = 6\), so that the elasticity at \(\delta = 1\) is
2. In the example below, a Gini-like measure is constructed by taking the point
of inflexion at twice the world median income, \(\delta = 2\), and setting \(\beta = 3\); this
leaves the elasticity at the point of inflexion unchanged at 2, but gives a much
lower elasticity of 0.11 at the median income. With higher values of \(\delta\), the
flatter, initial section applies over a wider range. Indeed, by letting \(\delta\) go to
infinity, distributional indifference becomes a limiting case.
On the Interpretation of the New Measure

The new measure explored here has been constructed to embody a desired pattern of change in the social marginal valuation of income. But how is the new measure to be interpreted? Its interpretation may be aided by re-arranging the expression for social welfare. By adding and subtracting from equation (3) the term \( \frac{\lambda}{\beta} e^{\beta(\delta_0 - \delta)} \ln \left[ 1 + e^{\beta(\delta - \mu)} \right] \), social welfare can be treated as being made up of two components:

\[
\Sigma = \Sigma(\mu) - \sigma = \Sigma(\mu) - \left\{ \frac{\lambda}{\beta} e^{\beta(\delta_0 - \delta)} \frac{1}{n} \sum_i \ln \left[ 1 + e^{\beta(\delta - y_i)} \right] \right\}
\]

The first term on the right side of equation (6), \( \Sigma(\mu) \), is the level of social welfare attained if everyone has an income equal to the mean, \( \mu \). In general, this level of social welfare is less than \( \mu \), although it approaches \( \mu \) as the mean tends to infinity. (With the poverty gap, it is equal to \( \mu \) once the mean passes the poverty line.) This reflects the fact that it is a welfare measure and that there are diminishing returns in the transformation of income into welfare. The second term, denoted by \( \sigma \), represents the costs of income differences. The term reduces to zero if all incomes are equal to the mean.

How this new measure differs from earlier approaches can be illustrated in the simple example in figure 7. If there are two people with incomes (measured on the horizontal axis) as shown and mean \( \mu \), the achieved level of social welfare is given by point C (the midpoint). Welfare is measured on the vertical axis. The I and K measures proceed by calculating the equally distributed equivalent income, \( y_e \) (obtained by reading across horizontally from C to D), and the cost of inequality is the loss CD. Unlike the I and K social welfare functions, however, the new measure has the same dimension as income. This implies that the level of welfare, \( \Sigma \), can be directly compared with the mean income, \( \mu \), and that there is no need to introduce the equally distributed equivalent income. GC, the overall difference between \( \mu \) and \( \Sigma \), is made up of two components, GF and FC: GF reflects the diminishing returns in the transformation of income into welfare and shrinks as income grows; FC measures the cost of inequality, the second term in equation (6).

In the case of the poverty gap, the curve in figure 7 becomes a kinked line, coinciding with the 45\(^\circ\) line from the level \( \delta \) of the poverty income onward. The distance GC is \( \lambda \) times the aggregate poverty gap per person. This gap consists, potentially, of two components, either of which may be zero. Where the mean income is above the poverty line, poverty is entirely due to the unequal distribution of income. If the mean income is below the poverty line, there can be both aggregate poverty (corresponding to the difference between \( \mu \) and \( \Sigma (\mu) \)) and distributional poverty. Aggregate poverty can remain even if incomes are equalized. Indeed, if everyone has an income below the poverty line, then the component FC disappears even though income differences remain (since
the poverty gap is unaffected by transfers of income among people below the poverty line). The problem of poverty can therefore be seen as a problem of distribution or a problem of the overall level of income.

These observations highlight the crucial role of \( d \) when it is seen as the poverty line. The parameter \( d \) (and \( d_0 \)) could be defined as a fraction of mean income, a purely relative approach that is not explored here. On an absolute approach, \( d \) (and \( d_0 \)) is independent of mean income, but, as noted, this does not imply that it should be kept constant over a long period. Where the underlying concern relates to a more fundamental space, such as the achievement of a level of functionings, the necessary level of income may be changing as a result of economic and social progress. This issue is taken up again in the next section.

In the general case, the concern is not with the GF component but only with the FC component, the costs of inequality. It is instructive to see how the new measure departs from Kolm’s absolute approach on the costs of inequality. The Kolm index \( K \) measures the costs of inequality as the absolute difference between the mean and the equally distributed equivalent income: \( K = \mu - y^* \).

Equation (6) expresses the cost of inequality as the difference between the social welfare at the mean, \( \Sigma(\mu) \), and the social welfare for the actual income
distribution, $\Sigma$. As by definition social welfare at the equally distributed equivalent income equals $\Sigma$, the term $\Sigma(\mu) - \Sigma(y_e)$ rather than $\mu - y_e$ is being taken as the cost of the unequal distribution of income. For the Kolm-like measure defined by (3b), this term equals $e^{-\beta \mu} (e^{\beta K} - 1) \lambda / \beta$, where $K$ is the Kolm index with $\kappa = \beta$. For given mean income, the two measures generate the same ranking, but the cost of inequality defined here is smoothed out by a rise in mean income. Raising all incomes by $1$ leaves the Kolm index unchanged by construction but reduces the costs of inequality with the Kolm-like measure, and it does so at a decreasing rate as mean income rises: the richer the economy, the less an extra $1$ is worth.

The new inequality measure $\sigma$ is decomposable by population subgroups (see Cowell 1980; Shorrocks 1980):

\[
\sigma = \sum_j w_j \sigma_j + \frac{\lambda}{\beta} e^{\beta (\delta_0 - \delta)} \sum_j w_j \ln \left[ \frac{1 + e^{\beta (\delta - \mu)}}{1 + e^{\beta (\delta - \mu)}} \right],
\]

where subscript $j$ refers to the $J$ mutually exclusive subgroups of the population, and $w_j$ is the population share of subgroup $j$. The first term on the right side of equation (7) is the population-weighted average of within-group inequalities; the second term is between-group inequality, calculated after atributing the group mean income to each member in a group. For the poverty gap measure (equation 3a), the decomposition is:

\[
\sigma_{\beta \rightarrow \infty, \delta_0 = \delta} = \sum_j w_j \sigma_j + \lambda \sum_j w_j \max[0, (\delta - \mu_j)] - \lambda \max[0, (\delta - \mu)]
\]

When the overall mean is above the poverty line, the between-group component is $\lambda$ times the weighted average of the aggregate poverty indicator, that is, the difference $(\delta - \mu_j)$ if positive; when the overall mean falls short of the poverty line, the aggregate poverty indicator must be subtracted from this sum.

IV. THE NEW APPROACH APPLIED TO WORLD INEQUALITY

The alternative measure suggested above is now applied to the evidence on world inequality. The pattern of the social marginal valuation of income is illustrated in the final four columns of table 1 (alternatives 1–4), where the maximum acceptable leakage is taken to be 80 percent ($\lambda = 4$). The first two alternatives take $\delta$ as the poverty line (assumed to be half world median income in 1992), and set $\delta_0$ equal to $\delta$. With alternative 1, $\beta$ has a high value, reflecting concern about poverty but not about inequality (in the direction of the poverty gap version). The social marginal valuation of income falls sharply once the poverty line is reached and is essentially constant above world median income. Alternative 2, with a smaller value of $\beta$ ($=4$), corresponds to a less angular position. Below the poverty line, the social marginal valuation is lower...
than with alternative 1, but it crosses at the poverty line. For incomes up to the world median, the weight attached to marginal income is at least 40 percent higher than that attached to marginal income in the United States.

In contrast, alternatives 3 and 4 break the link between $d$ and the poverty line and lean toward measures of inequality. Letting $d = -4$ (and $d_0 = 0$) moves in the direction of the Kolm index, $K$. Alternative 4 goes in the opposite direction, setting $d = d_0 = 2$ with $\beta = 3$, which generates a Gini-like inequality measure (but with additive separability and decomposability by population subgroups). The social marginal valuation first falls slowly and then more quickly, exhibiting increasing and then decreasing sensitivity to transfers. The difference in transfer sensitivity is particularly important when considering the world scale of incomes. Individual countries may lie largely within the increasing or the decreasing phase (see table 1). Even so, alternative 4 is consistent with substantial redistribution within the United States: the social marginal valuation for decile group 1 is more than three times that for the U.S. median.

These four alternative measures are applied to the world income estimates of the Bourguignon and Morrisson database. Figure 8 shows the evolution of world social welfare from 1820 to 1992, where welfare has the dimension of per capita income and is expressed as a percentage of world median income in 1992. As noted earlier, welfare may be negative, as was the case for all four alternatives until the beginning of the 20th century. For the poverty line measures, it is scarcely surprising that the earlier values are so low since a contemporary (1992) standard is being applied, but the inequality measures are

**Figure 8.** Evolution of World Social Welfare, Alternatives 1–4, 1820–1992

Source: Authors’ elaboration on the Bourguignon and Morrisson (2002) database.
also absolute in the sense described in the previous section. This applies not only to alternative 3, approaching the Kolm index, but also to the Gini-like alternative 4. Indeed the Gini-like measure is initially off the scale.

All four measures indicate a considerable improvement over the period, driven by the growth of mean income (the thick top line). However, while the upward tendency was similar, the rates of increase in social welfare differed from those in mean income. For example, the (absolute) annual increase in mean income between 1980 and 1992 was double that between 1890 and 1910, but the rise in social welfare using the Gini-like index was only a quarter higher. Distributionally adjusted income, as with the new social welfare measure, may give rather different pictures of different historical periods.

The absolute cost of inequality, $\sigma$, is given in figure 9, again expressed as a percentage of the 1992 world median, so that a value of 100 corresponds to a cost of U.S. $1,712 per person. Figure 9 has a panel for each of the four alternatives and one for the poverty gap measures defined in (3a) with $\delta$ set at 0.5 and one with it set at 1 (alternatives 5 and 6). These last two values roughly correspond to the $1 a day and $2 a day poverty lines, as defined by Bourguignon and Morrisson (2002). With alternative 1, whose parameters lead in the direction of the poverty gap, the cost due to inequality rises until 1890 and then declines, accelerating after the Second World War. The time path with the less angular version in alternative 2 and the Kolm-like version in alternative 3 also have an inverse-U shape, but the peak cost of inequality is reached much later, in 1950. In contrast, with the Gini-like measure, alternative 4, the cost due to inequality increases steadily, then very rapidly between 1950 and 1970 before reaching a peak in 1980. (Recall that the Gini-like measure is not the same as the Gini coefficient: the social marginal valuation of income received by one person does not depend on what is happening elsewhere in the distribution.)

Thus, the two inequality versions of the new measure, the Kolm-like and the Gini-like, move in opposite directions after 1950. If the two poverty gap measures represented by alternatives 5 and 6 are compared with alternatives 1 and 2, all are found to share the same inverse-U shape, in particular the steep downward trend after 1950, though they differ in the time of the turning point and in the size of the change. With the $1 a day poverty line, the turning point is 1870; with the $2 a day line, it is in the 20th century.

For all six alternatives, within-country inequality, the population-weighted average of inequality calculated within countries or groups of countries, is far more stable than the total, suggesting that the secular variation in the total cost is driven largely by changing income differences across countries. Notice, however, how a significant rise of within-country inequality from 1970 to 1992 offsets the international convergence in mean incomes with alternative 4. The less angular poverty measure and the Kolm-like inequality measure level off, and the $2 a day poverty gap and the Gini-like inequality measure show a rise after 1950.

These results assume that the cost is measured in absolute terms. Figure 10 shows that some differences arise if the cost is measured as a proportion of
**Figure 9.** Evolution of the Absolute Cost of World Inequality, Alternatives 1–6, 1820–1992

*Note:* Within-country inequality is the population-weighted average of inequality calculated within the 33 countries or groups of countries included in the Bourguignon and Morrisson (2002) database.

*Source:* Authors’ elaboration on the Bourguignon and Morrisson (2002) database.
FIGURE 10. Evolution of the Relative Cost of World Inequality, Alternatives 1–6, 1820–1992

Note: Within-country inequality is the population-weighted average of inequality calculated within the 33 countries or groups of countries included in the Bourguignon and Morrisson (2002) database.

Source: Authors’ elaboration on the Bourguignon and Morrisson (2002) database.
Figure 11. Evolution of the Absolute Cost of World Inequality, Alternatives 1–6, Time-variable δ, 1820–1992

Note: Within-country inequality is the population-weighted average of inequality calculated within the 33 countries or groups of countries included in the Bourguignon and Morrisson (2002) database.

Source: Authors’ elaboration on the Bourguignon and Morrisson (2002) database.
mean income. The peaks with the less angular version (alternative 2) and with the Kolm-like measure (alternative 3) come earlier—toward the end of the 19th century. The relative cost due to inequality with the Gini-like measure also peaks earlier, in 1960, and then falls thereafter. But the differences are not nearly as striking as those found for the standard measures presented at the opening of this article.

These evaluations are based on a value of $\delta$ that is kept constant across the period 1820–1992. For the two poverty lines, $\delta$ is assumed to be half the world median income in 1992. This value sets a very high standard: in 1820 only Western European countries, the United States, and Argentina-Chile enjoyed a mean income greater than $\delta$. It is reasonable to wonder how the results would change if this extreme absolutist hypothesis were relaxed by varying the poverty standard over time in step with economic and social progress.

Figure 11 shows the consequences of recomputing the measure retaining the value of $\delta$ for 1992 but assuming that it grew over time along with median world income. Doing so amounts to applying the values of $\delta$ from table 1 to the median income in each year rather than to the median income in 1992, taking the increase in median income as a reference point. It should be stressed that this does not assume that the poverty line is proportionate to median, or mean, incomes. The (externally derived) time variation in $\delta$ may involve a faster or slower rate of growth. All other parameters are kept unchanged. As shown in figure 11, under a time-varying $\delta$, the secular pattern of world income inequality looks considerably different from the one reported earlier for all poverty-type measures (alternatives 1 and 2) and alternatives 5 and 6: the inverse-U shape turns into a steadily ascending trend, which flattens out only after 1980. The impact is barely visible on the two inequality-type measures, except for the upward trend of the Gini-like measure, now continuing even after 1970. Assuming a time-varying $\delta$ also affects the within-country component, which tends to account for a much larger share of the overall inequality: in alternatives 1 and 5, it almost wipes out the between-country component.

To sum up, contrary to what is suggested by the earlier analysis using the standard inequality indices, the conclusions depend very much on distributional judgments.

**V. Conclusions**

The effects of globalization on world income inequality have been much debated in recent years. In the literature, as noted by Anand and Segal (2008, p. 61), “no consensus emerges concerning the direction of change in global inequality in the last 20–30 years.” Some commentators have stressed the impressive growth performance of emerging economies such as China, India, and other countries in Southeast Asia and have concluded that world inequality...
and poverty must have decreased. Others have countered that these impressive rates of growth have not yet translated into absolute increases in income comparable with those of developed economies, given the very different levels of GDP per capita. Thus, world income gaps must have risen. This article argued that—before such judgments can be made—the foundations of inequality measurement need to be reexamined. The sheer scale of global income differences means that the tools applied to inequality measurement at the domestic level cannot simply be carried over. In the discourse about global justice, both poverty and inequality and their interrelation have to be considered, as do the different meanings of “absolute” and “relative.”

Differences of view about the evolution of world inequality and poverty stem in part from differences in how to measure them. In seeking to provide a framework for considering the cost of world inequality and poverty that encompasses different types of concern, this article adopts a welfarist approach (without endorsing it as the only possible approach). Its first findings, in section I, suggested that the differences in conclusions about changes in world inequality could be attributed largely to how the cost is expressed. However, section II argued that existing measures of inequality impose too tight constraints on how social marginal valuation varies with income and provide no ready means to integrate the analysis of poverty and inequality.

To encompass the global income differences and allow for concerns about poverty as well as inequality, section 3 explored a new parameterized measure of global social welfare. This measure has, in a sense explained in the article, an absolute structure (and it would be interesting to consider the parallel, relative measure), but it is sufficiently flexible to include different value systems and to incorporate a poverty line. Including several approaches within a single measure helps not only to better understand their interrelation but also to obtain measures that blend different concerns. People differ, for example, in the relative importance they attach to poverty and inequality. This difference appears fundamental, but it can be embedded within the new measure explored here. Letting one of the key parameters increase allows the measure to take on a poverty gap form, whereas lower values permit a less angular version of the poverty gap, tapering the measure for the near-poor. If other parameters are allowed to vary, more general concerns about inequality can be introduced. These may follow the pattern of standard welfare-based measures, with declining sensitivity to transfers with movement up the income scale. Or they may exhibit increasing and then decreasing sensitivity to transfers, mimicking the Gini coefficient but with the property of additive separability (and subgroup decomposability).

The new measure can accommodate a constant poverty line or one varying over time with economic and social development, an alternative with considerable consequences for interpreting the evolution of the world income distribution. Finally, the overall weight attached to distributional issues can be varied according to individual interests. For example, one person may be
concerned about poverty but not attach much weight to this consideration, relative to total income. Another person might feel that, in the context of world poverty, little weight should be attached to additional income received by those at the top of the distribution. Stated more pragmatically, the new measure can exhibit a willingness to redistribute within rich countries without magnifying to an implausible degree the willingness to make transfers across the whole spectrum of world incomes.

REFERENCES


