Welfare Weights

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Abstract

We examine the application of welfare economics to the problem of determining distributional weights for project analysis. Both approaches based on individual utility and on social welfare functions are considered, we further analyse a link between the two approaches in the form of individual choice under uncertainty. Estimates from a variety of data sources of the elasticity of "marginal utility" or "marginal social utility" are critically examined.

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1 Introduction

The framework of welfare economics is regularly used to provide guidance on policy choice and on the assessment of economic performance in a number of contexts that relate to the well-being of individual consumers – price control, taxation, the provision of public goods, for example. In this paper we focus upon the main distributional issues in the welfare-economic approach to such normative questions and we examine the links between the abstract welfare-economic theory and practical judgments.

At the heart of the analysis is the following problem. Take the formula

$$\Delta W = \sum_{i=1}^{n} \omega_i \Delta y_i \tag{1}$$

which gives the change in welfare ΔW as a simple weighted average of changes in the incomes y of all n people in a particular community. What is involved in this simple statement?

- The justification for relating welfare to individuals' incomes
- The determination of the numbers ω_i to be used as weights.

We concentrate particularly on the second of these issues, although this cannot be divorced entirely from the first. In effect this amounts to an analysis of the problems of estimating the elasticity of welfare with respect to income, and of relating welfare weights to the concept of individual utility.

The principal difficulty with this is that estimating parameters of a welfare function or a utility function is not like estimating the parameters of the demand function for chocolate or cigarettes. There are important issues of principle (about what one is supposed to be measuring and why) and of implementation (how one can elicit information about what appears to be private and subjective valuations). So, in order to address the issues involved coherently, it is necessary first to examine the theoretical basis on which the analysis rests. Accordingly the paper is organised as follows: section 2 examines the basic theory, including the meaning to be given to utility and welfare weights; section 3 examines the issues involved in implementing these abstract ideas in practice; section 4 reviews and assesses the rather disparate collection of evidence that is available; section 5 summarises the main findings.

2 Theoretical issues

As noted in the introduction the essential first step is to consider the theoretical problems associated with the specification of welfare weights. We need to focus in particular on the meaning of individual utility, on the essential assumptions underlying social welfare analysis as conventionally understood, and the possible connection that there may be between these two bodies of theory in the context of welfare weights.

The central idea that is to be investigated is that the welfare weights that are of potential use in public policy analysis, including that related to consumer affairs policy, can be rooted in the economic analysis of the consumer. This approach involves a reconsideration of standard preference analysis and the way in which concepts of utility, income and resources are linked. We consider this in two major stages: the normative information that one may hope to recover from individual utility analysis (section 2.1) and the standard approach to formulating a social-welfare function (section 2.2).

2.1 Individual welfare

The idea of "utility" is central to our approach but should be used with care. It can be seen to perform two major roles in consumer theory and normative economic analysis (Harsanyi 1997):

1. It is a convenient device for representing consumers' preferences.
2. It can serve as a measure of the level of satisfaction or well-being arising from particular baskets of goods, or from a given money income.

The first of these is the standard interpretation of utility in microeconomics (Fishburn 1970) but we will begin with the second, because it appears to be so attractive for the present analysis.

2.1.1 Income and utility

If it were legitimate to assume the existence of an operationally useful concept of an individual's utility, then we have a direct approach to the welfare-economic system of appraisal and assessment that we mentioned in the introduction. A utilitarian approach to normative economics might then lead to the following formulation of social welfare

$$W = \sum_{i=1}^{n} v_i(y_i) \tag{2}$$

where v_i is individual i's utility expressed as a function of income; this begs the question of the appropriate definition of income, but that is beyond the scope of the present paper. Further note that the formulation of (2) requires that $v_i(\cdot)$ be a cardinal index and interpersonally comparable for welfare comparisons to make sense. Differentiation of (2) immediately gives:

$$dW = \sum_{i=1}^{n} v_i'(y_i) dy_i \tag{3}$$

where $v_i'(\cdot)$ is the first derivative of $v_i(\cdot)$. In other words the weight ω_i in (1) is exactly the marginal utility of income $v_i'(y_i)$. If we are prepared to adopt the ethical framework leading to the utilitarian framework (2) then the classical assumption of decreasing marginal utility implies that, other things being equal, a higher income for person i would lead to a lower weight ω_i. This observation appears to offer an attractive practical policy rule: "in evaluating the change in welfare associated with a policy assign lower weights to people on higher incomes". It is clear that this simple rule relies on two key assumptions:

- There are no significant differences between the utility scales of persons i and j
- The marginal satisfaction to be derived from income or wealth always fall as people get richer.

However neither assumption is as innocuous as it may seem and each requires careful justification. With reference to the first assumption, if individuals differ in relevant aspects of need then clearly the distributional weights ω_i cannot be determined by incomes alone: in some special cases one can get around the problem by "equivalising" individual's incomes (see the discussion in section 3.1.1); some have argued that what is required is a principle more fundamental than utilitarianism that appropriately takes these concerns into account, such as Sen's Weak Equity Axiom.[1] The second assumption requires one to rule out the possibility that "appetite grows with eating", but this property is problematic: it is not implied by the Weak Equity Axiom (Sen and Foster 1997, pp. 20-22), nor is it implied by risk aversion, except under special circumstances (see appendix B below).

Where does this leave the idea of using the "utility-as-satisfaction" concept as a foundation for distributional weights? Some would argue that, although there may not be overwhelming *a priori* reasons for adopting the two key assumptions discussed above, the issues involved could be resolved by use of empirical methods: for example the assumption of diminishing marginal utility could be subjected to formal test.[2] However it can also be persuasively argued that the whole approach

[1] "Let person i have a lower level of welfare than person j for each level of individual income. Then, in distributing a given total of income among n individuals including i and j, the optimal solution must give i a higher level of income than j." – (Sen and Foster 1997), page 18.

[2] This approach – relatively rarely employed in mainstream economics – requires an explicit model of the determination of individual satisfactions. An overview of the main ideas in the principal school of thought to pursue the approach is presented in section 3.1.3 below, although the particular model of utility-formation used there does *not* imply diminishing marginal utility.

is fundamentally flawed: that the interpretation of "utility" as satisfaction is just not an implementable line of approach. The reason for this emerges from the standard interpretation of utility as simply a representation of consumer preferences which may have no clear cardinal interpretation, discussed in section 2.1.2 immediately below.

2.1.2 Structure of preferences: limitations

Let us turn to the interpretation of utility that is most commonly adopted in applied work on consumer behaviour. A consumer's preferences are modelled in terms of his consumption of a basket of m commodities $\mathbf{q}$: under fairly weak conditions these preferences can be represented by a utility function which simply has the interpretation "if the utility of $\mathbf{q}^{\mathrm{B}}$ is higher than the utility of $\mathbf{q}^{\mathrm{A}}$ then the person prefers bundle B to bundle A". In some cases it is useful for the purposes of econometric application to restrict the form of this function to an additive structure across groups of commodities. This further restriction permits the interpretation of a consumer's objective function as the sum of utility components, and is usually especially relevant to variants of the optimisation problem that are naturally interpreted in the form of multistage budgeting.

Let us make the standard assumption that individuals are rational agents optimising the purchase of $\mathbf{q}$ given a fixed income y and conditions of perfect certainty which implies known prices $\mathbf{p}$. Then, from this maximising process one can establish a relationship between maximised utility and prices and money income and expenditure. This relationship is expressed as the *indirect utility function*

$$u = V(\mathbf{p}, y) \tag{4}$$

and its inverse, the consumer's cost function or expenditure function

$$y = C(\mathbf{p}, u) \tag{5}$$

which gives the monetary cost of hitting a particular utility level u given that consumer prices are $\mathbf{p}$. These two tools (4) and (5) are the standard equipment for evaluating the impact of price changes on individual consumers.[3] An individual's demand for good k can be written in terms of the slopes of the indirect utility function thus:

$$q_k = -\mu \frac{\partial V(\mathbf{p}, y)}{\partial p_k} \tag{6}$$

where

$$\mu := 1 \Bigg/ \frac{\partial V(\mathbf{p}, y)}{\partial y} \tag{7}$$

is the marginal utility of money. From (6) and (7) it is easy to illustrate a basic point about utility functions, as follows. Suppose an indirect utility function V

[3] See for example Vaughan (1999), pp. 7-21.

is considered to be a good representation of a consumer's preferences; consider another function $\tilde{V}$ that can be expressed as $\tilde{V} = \Phi(V)$ where Φ is an increasing function; then $\tilde{V}$ is just as good a representation of the consumer's preferences as $\tilde{V}$ but in general the relationship between the marginal utility of money and income will be changed by Φ.[4]

This well-known property of the utility function – the ordinal nature of utility – means that the concept of marginal utility is meaningless for the purposes of welfare comparisons unless we have some way of restricting attention to one particular cardinal representation of preferences. There may be ways of doing this if we go beyond the simplified model of individual choice in a riskless world. A prime example of this is the structure of individual choices under uncertainty.

2.1.3 Choice and welfare under uncertainty

The standard approach to modifying the basic consumer theory to allow for uncertainty is to take two steps:

1. One models "states of the world" as though they were separate distinguishable characteristics of commodities (so, an umbrella-if-it-rains-tomorrow is regarded as qualitatively different from an umbrella-if-it-doesn't-rain-tomorrow) and consider a world of N goods where $N := S \cdot m$, S is the number of possible states of the world and m is the number of physically distinct commodities.

2. One introduces a set of apparently reasonable assumptions which are used to characterise people's preferences across states of the world. (Cowell 1986), (Slovic and Tversky 1974), (Drèze 1974)

Given the usual assumptions we can express an individual's preferences in the face of uncertainty as the von-Neumann-Morgenstern utility function

$$\sum_{s=1}^{S} \pi_s \phi(y_s) \tag{8}$$

where y_s is the person's assumed income in state-of-the-world s (step 1 above), and π_s and $\phi(\cdot)$ are two key components of the representation of preferences (step 2 above), respectively the *subjective probability* that the person places on state-of-the-world s and the *felicity function* or *cardinal utility function* used for evaluating income in any state of the world.[5] Although the scale and origin of the function ϕ are arbitrary, the shape of ϕ is important: it characterises an individual's

[4]The first remark is true because the demands in (6) will remain unchanged by the transformation Φ; the marginal utility of money (7) changes if Φ is not a linear transformation.

[5]A way of interpreting this is to consider $v(y_i)$ as the counterpart of the indirect utility $V(\mathbf{p}, y)$ (see 4) evaluated in state-of-the-world i with the prices notation suppressed.

risk aversion. If an individual is risk averse the second derivative of ϕ is negative. Given that the first derivative is necessarily positive (the individual always prefers to have more income in any state of the world) then the following definitions of absolute risk aversion (A) and relative risk aversion (R), respectively, are useful ways of summarising individuals' attitudes:

$$A(y) := -\frac{\phi''(y)}{\phi'(y)} \tag{9}$$

$$R(y) := -y\frac{\phi''(y)}{\phi'(y)}. \tag{10}$$

Clearly the definitions (9) and (10) are independent of scale and origin changes of ϕ. We will find that the concept of risk aversion will play a central role in connecting the analysis of the individual with that of social welfare.

Notice three things about this formulation:

1. The concept of risk aversion appears to offer an alternative basis for assuming "diminishing marginal utility", but with reference to evaluation of payoffs contingent on a particular state of the world rather than "utility as satisfaction" , mentioned in section 2.1.1.[6] Whether and how risk aversion can help to inform the choice of welfare weights will be discussed in section 3.1.5 below.

2. The von-Neumann Morgenstern structure ties down the issue of cardinalisation for a particular person – only scale and origin changes to the felicity function ϕ in (8) are admissible as alternative ways of representing the person's preferences, not some arbitrary transformation as in section 2.1.2.[7] To see this note that replacing ϕ by $\tilde{\phi}$ where $\tilde{\phi} := a + b\phi$ leaves absolute and relative risk aversion in (9) and (10) unaltered whereas other nonlinear transformations of ϕ would alter $A(y)$ and $R(y)$.

[6]That the two concepts are different is illustrated in Appendix B. However it is quite common for the distinction between the two to be blurred – see for example H. M. Treasury (1991): "The elasticity of the marginal utility of consumption [...] visualised in terms of the relative pleasure given by £1 to people of different incomes [...] The empirical data on this is difficult to interpret. However the data available, and common observation, suggest that a value in the range of -1.5 is reasonable for the marginal utility of small variations in income which are not directly perceived (as distinct from gains or losses, such as theft, relative to a known, well-defined baseline which has wider impacts on utility) There is no persuasive evidence or *a priori* reason to suggest that this elasticity should vary significantly with income over the income range of the economy. (A constant value implies that people of different incomes would forgo the same percentage of their income to avoid a given small percentage variability in their income)" – page 82, footnote 2.

[7]However it is legitimate to consider arbitrary transformations of the sum $\sum_{s=1}^{S} \pi_s \phi(y_s)$, as in the example of Appendix B.

3. However, of course the von-Neumann-Morgenstern structure says nothing about the comparability of one person's marginal utility with another, given that – as we have just seen – the ϕ function is only defined up to arbitrary changes of scale. Each person i's function ϕ_i could be replaced by $a_i + b_i\phi_i$ where the constants a_i and b_i are arbitrary and differ from person to person. Some further restriction is needed to achieve comparability.

2.1.4 A preliminary assessment

It is important to understand the limitations of the first interpretation of utility on page 2 – that it is merely a device for the representation of a consumer's preference orderings – since this is the way the concept is normally understood by applied economists working with data about consumer behaviour. Without a coherent and compelling theoretical basis for restricting the form of the utility function no information relevant for welfare weights can be recovered from data on consumer behaviour. What can be achieved under certain restrictions is discussed in section 3.1.5 below. Of course one could *assume* that observable, measurable and interpersonally comparable utility exists, but it is not obvious how one goes about putting it into operation: see section 3.1.3.

2.2 Social welfare

Now let us consider the other principal theoretical strand of analysis – the direct approach to social welfare. At the core of this lies the idea of a *social-welfare function* (SWF) which is essentially a criterion function for assessing and comparing social states where social states may encompass a variety of information, but are usually interpreted in terms of peoples' levels of well-being. We begin with the assumptions about the SWF that are commonly made – often implicitly – in applied normative economics.

2.2.1 Basic assumptions

Assume that we have an observable, measurable, interpersonally comparable measure of individual well-being which we will denote by x (how this is related to the concept of "income" used in section 2.1 above will be discussed later in 3.1.1). The list of values of this for the n members of the community is given by $\mathbf{x} := (x_1, x_2, ..., x_n)$. Then we can introduce the following three basic properties.

Individualism The SWF is *individualistic* and *nondecreasing*, if the welfare level in any social state can be written:

$$W = W(x_1, x_2, ..., x_n) \tag{11}$$

and if $\mathbf{x}^{\mathrm{B}} \geq \mathbf{x}^{\mathrm{A}}$ implies, ceteris paribus, that $W(\mathbf{x}^{\mathrm{B}}) \geq W(\mathbf{x}^{\mathrm{A}})$, which in turn implies that social welfare in state B is at least as good as social welfare in state A. If the x-values completely represent individual well-being then non-decreasing property corresponds with the well-known Pareto principle.

Impartiality The SWF is *symmetric* if it is true that, for any social state,

$$W(x_1, x_2, ..., x_n) = W(x_2, x_1, ..., x_n) = ... = W(x_n, x_2, ..., x_1)$$

That is, the value of W does not depend on the particular assignment of labels to members of the population. It implies the acceptance of two important principles:

- The definition of individual well-being x appropriately takes into account interpersonal differences of needs or other relevant characteristics.
- The "end-state principle" of social welfare: the social state is completely described by the current "snapshot" $\mathbf{x}$ irrespective of how individuals' incomes or personal attributes may have been influenced by past events. (Nozick 1974) (Zajac 1996).

Additive structure The SWF is *additive* if it can be written

$$W(x_1, x_2, ..., x_n) = \sum_{i=1}^{n} U_i(x_i) \tag{12}$$

where U_i is a function of x_i alone. We may seek a rationalisation of additivity in a decomposition argument. Consider a policy which transfers a small amount of x from person i to person j without any impact on the living standards of any other person. Should the impact of this transfer on social welfare depend on the personal information of anyone other than i and j?

In short we have the property that ω_i is independent of x_j. This is an important assumption which greatly simplifies the analysis and is often used. However we should recognise that this apparently attractive property is not possessed by many analytical tools that are commonly used for welfare judgments on income distribution – for example the use of this assumption is inconsistent with the use of the Gini coefficient to rank income distributions (Newbery 1970).

2.2.2 The standard model

If these three properties are all satisfied then we can write the SWF as

$$W(x_1, x_2, ..., x_n) = \sum_{i=1}^{n} U(x_i) \tag{13}$$

where U is the same function for each person and where $U(x_i)$ increases with x_i. This is the basic structure that we shall employ in the present context.[8]

2.2.3 The shape of the U-function

If we restrict attention to the form (13) then further assumptions about the SWF can be simplified, since they may be expressed in terms of the function U alone. The number $U(x_i)$ can be interpreted as the *social utility* of, or the *welfare index* for, person i, and the function U is the *social utility function* or *evaluation function.* The rate at which this index increases is

$$U'(x_i) = \frac{dU(x_i)}{dx_i} \tag{14}$$

which can be thought of as the *social marginal utility* of, or the *welfare weight* of person i. Notice that, because of the first property, none of the welfare weights (14) can be negative.

Concavity The SWF is *strictly concave* if the welfare weight always decreases as x_i increases. The reason for invoking the assumption of concavity derives from the desire to make the welfare function inequality-regarding. It is an implication of the general property of the principle of transfers for the special case of additive social-welfare functions (13). The principle of transfers requires that if £1 is transferred from a poorer person to a richer person, without anyone else's income being changed, then social welfare must decrease.

Constant Elasticity In some circumstances it is argued that a further restriction might reasonably be imposed: the SWF has *constant elasticity*, or *constant relative inequality* aversion if $U(x_i)$ can be written

$$U(x_i) = \frac{x_i^{1-\varepsilon} - 1}{1-\varepsilon} \tag{15}$$

(or in a cardinally equivalent form), where ε is the *inequality aversion parameter*, which is non-negative: relative inequality aversion is the exact counterpart of the concept $R(y)$ introduced in (10) to discuss consumer attitudes to risk. The

[8]Given that W is an ordinal concept one may use transformations of (12) where appropriate. For example it is possible to find an index of social welfare that is denominated in the same units as individual wellbeing or "income" This is known as equally-distributed-equivalent income (Atkinson 1970) or *representative income* x^* (Champernowne and Cowell 1998) defined by

$$x^* := U^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} U(x_i)\right)$$

implications of this type of function (15) – commonly attributed to Atkinson – are discussed in section 3.1.2.

2.2.4 A preliminary assessment

The standard approach to social-welfare functions requires comparatively few assumptions to yield a practical tool that can be applied to the basic question on page 1. It is conventional to characterise the SWF as inequality-regarding and this has an obvious counterpart – risk aversion – in the von-Neumann-Morgenstern preference structure used for the analysis of risky prospects. In fact the two types of analysis have the same structure and it is one of the tasks of section 3 to show how the two may be usefully connected.

3 Implementation

The theoretical discussions so far have been necessarily abstract. We now look at the ways in which one might expect to be able to introduce them into practical analysis. This involves interpreting the building blocks of each of the two main strands of section 2, examining the meaning of inequality aversion and considering some practical difficulties of representation and measurement.

3.1 Utility, Inequality and Social Welfare

In order to connect social welfare analysis in section 2.2 with individual utility in section 2.1 two major steps are required:

- Establish the relationship between x (individual well-being used in social welfare analysis) and y (personal income): this typically requires information on the entity x and that one make an appropriate allowance for needs and family size.
- Link the issues involving social values for comparing *income* distributions – such as inequality aversion – with methods of comparing *probability* distributions. This requires an interpretation of the form of the social evaluation function U.

We will briefly consider the intellectual basis for the assumptions that are conventionally made about each of these steps in the mainstream economics literature: the first is dealt with in 3.1.1, and several complementary approaches to the second step are discussed in 3.1.2 to 3.1.5 below.

3.1.1 Households, individuals and needs

The connection between the income y used in the analysis of consumer behaviour and the index of individual well-being x used in welfare theory is one of a set of interrelated issues concerning the heterogeneity of relevant personal and household circumstances other than income. For example, sensible comparisons of income distributions should take into account relevant aspects of "need" that may be related to age, disability or health status. Furthermore, in most practical cases we have to deal with the fact that individuals live in multi-person units – households or families. In any distributional analysis based on households or families a judgment has to be made as to whether the unit of analysis – as opposed to the level of aggregation of income – should be individuals or households. However, assessments of whether consumer-price adjustments are beneficial, or of whether or not a particular tax is regressive, are implicitly concerned with *individuals*' well-being or living standards: analysis at an individual level is appropriate.

This raises three separate issues that have implications for welfare comparisons – pooling, equivalisation and household weighting.

Pooling The position of a household in the household income distribution may be different from the positions of the individuals who make up that household in an individual income distribution. For example, a couple consisting of a well-paid earner and a partner who does not work may appear in the middle of the household income distribution, but in an individual income distribution the earner would appear near the top and the non-working partner at the bottom. Using household income (whether equivalised or not) as a measure of individuals' living standards or well-being implies an assumption of equal sharing of resources within the household; using individual income implies no sharing. Sutherland (Sutherland 1997) explores the implications of this choice, particularly with respect to hiding gender inequalities. However we cannot know the sharing arrangements adopted within each household, and it is usual practice to make the "pooling" or "equal sharing" assumption, i.e. to impute the same proportion of household income to all household members.

Equivalisation Because a given level of household money income at given prices will imply differing standards of living for persons with different needs or who live in different types of household, income comparisons involving different family or household types are usually handled with the use of an equivalence scale. This takes the general form

$$x_i = \chi(y_i, a_i) \qquad (16)$$

where a_i is a list of personal, family or household attributes. In the present case we assume that the information a_i refers to *household* i, that y_i is the total income of all members of the household and x_i is the *equivalised income* of any particular member of that household. There is no single "correct" specification of an equivalence scale (Coulter et al. 1992). It is possible to use a cost-function approach to specify a particular form of equivalence scale once a specific judgment has been made on individual welfare comparisons (Lewbel 1997), but analysis of demand patterns does not uniquely identify a particular set of equivalence scales (see (Deaton 1997), page 248). Adoption of any one particular version will involve making some arbitrary assumptions about the comparison of one person's needs with another, about the welfare costs of children and about economies of scale.

Household weighting Because social welfare is about the living-standards of *individuals*, but observations may be on households, observational units may need to be weighted by the number of persons living in each household (h_i) in order to ensure that the persons living in them are correctly represented in the distribution of individual living standards.

A standard form Using the principles of within-household pooling, equivalisation and household weighting outlined above means that the standard representation of the SWF in (13) can be restated in terms of household incomes and attributes as follows:

$$W^*((y_1, a_1), (y_2, a_2), ...) = \sum_{i=1}^{n_h} h_i U(\chi(y_i, a_i)) \tag{17}$$

where again h_i denotes the number of persons in household i. The change in social welfare resulting from a change in personal incomes (as in equation 1) is then given by

$$\Delta W = \sum_{i=1}^{n_h} \omega_i \Delta y_i \tag{18}$$

where Δy_i is now the change in *household* income, and

$$\omega_i = h_i \cdot \frac{\partial}{\partial y_i} \chi(y_i, a_i) \cdot U'(x_i) \tag{19}$$

Now it is clear that there are three components to the general distributional weight ω_i in (19):

1. the household weight h_i,
2. the responsiveness of equivalised income x_i to nominal income y_i,
3. the social marginal utility or welfare weight $U'(x_i)$.

The first of these requires no further discussion since it is simply the number of individuals grouped into a particular household. The second component is usually greatly simplified in empirical analysis by the – perhaps questionable – assumption that, for any given household, equivalised income is always proportional to nominal income. Under this assumption

$$\frac{\partial}{\partial y_i}\chi(y_i, a_i) = \frac{1}{\nu(a_i)} \tag{20}$$

where $\nu(a_i)$ is interpreted as the number of equivalent adults in the ith household. Then, for any pair of households i and j we have the attractive property relating the distributional weights in (19) with the welfare weights of section 2.2:

$$\frac{\omega_i}{\omega_j} = K \cdot \frac{U'(x_i)}{U'(x_j)} \tag{21}$$

where K is a constant[9] that is independent of any household's income. The estimation of equivalence scales implied in $\nu(\cdot)$ is standard and goes beyond the scope of this paper. However, the third component of ω_i requires special consideration: it is closely related to the concept of inequality aversion.

3.1.2 Inequality aversion

Let us focus upon the "inequality-regarding" property of the SWF that was mentioned on page 9. The key issues which are relevant to the determination of distributional weights are: how is the aversion to inequality to be interpreted? On what is it supposed to be based?

The interpretation of inequality aversion There are at least two ways of interpreting the idea of inequality aversion – or two types of inequality aversion – which may be summarised in the questions:

1. "How should transfers from the rich to the quite-well-off be ranked against transfers from the quite-well-off to the poor?"

2. "At what rate should society be prepared to trade off equality against mean income?"

Question 1 picks up on two complementary ways of showing concern for extremes in a distribution of a fixed-size cake; question 2 is a fundamental issue of political economy highlighted by Okun (1975) and others. The two questions are, in general, not identical (Cowell 1985) although sometimes the specification of the SWF obscures this point. In particular the assumptions about SWFs in 2.2.1

[9] Equal to $\frac{h_i \nu(a_j)}{\nu(a_i) h_j}$.

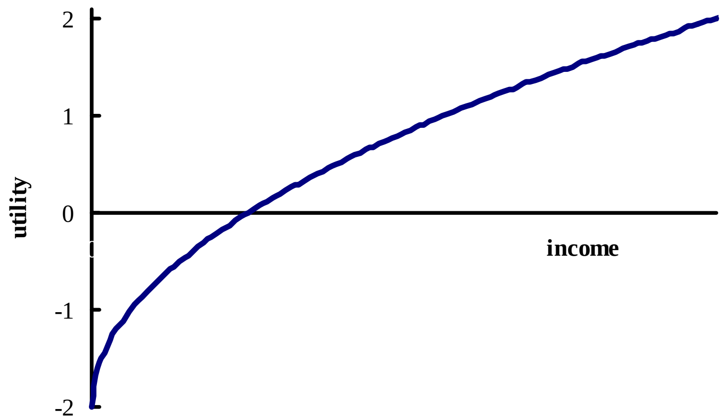

Figure 1: The Atkinson Social Utility Function ($\epsilon = 0.5$)

conflate the two questions into one: given the SWF structure in (13) a specific answer to question 1 will predetermine the answer to question 2.

However if we are prepared to make the assumptions in 2.2.1 and 2.2.3, and if we set aside for the moment questions of interpersonal differences of need, then we have a very appealing characterisation of the inequality-regarding property of the SWF, and the parameter ε introduced in (15) is central to its interpretation. For example one can use (15) to represent social welfare attitudes in the form of an inequality index conditional on ε. Given that the assumptions on structure and shape in 2.2.1 and 2.2.3 are adopted then the *Atkinson inequality index* can be written:

$$1 - \frac{1}{\bar{x}} \left[\frac{1}{n} \sum_{i=1}^{n} x_i^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \qquad (22)$$

where $\bar{x}$ denotes mean (equivalised) income $\frac{1}{n}\sum_{i=1}^{n} x_i$.

The social-utility function for one particular member of the family of Atkinson-type SWFs (the case $\varepsilon = 0.5$) is illustrated in Figure 1; the corresponding welfare weights are depicted in Figure 2, where it is evident that they decrease monotonically with income.[10] However, the shape characteristics of Figures 1 and 2 are fairly general for any SWF of this type. We know that if a person's income increases then (from the strict concavity property) his welfare weight necessarily decreases: the parameter ε indicates by how much. The constant-elasticity as-

[10]The U-functions for other members of this class of SWF have a similar shape: for $0 < \epsilon < 1$ they are bounded below; for $\epsilon > 1$ they are bounded above (Cowell 1995)

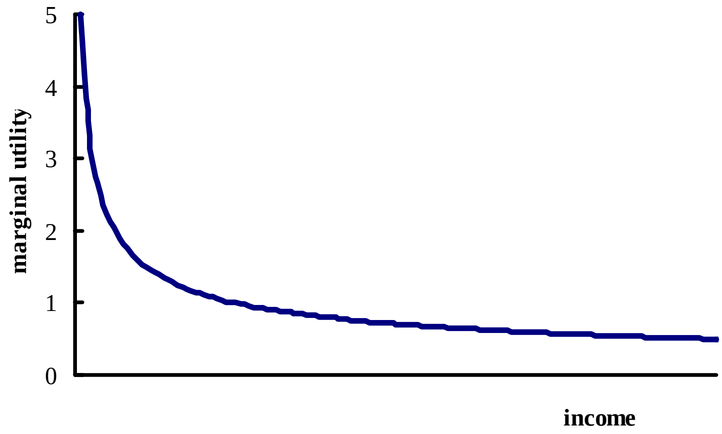

Figure 2: The Atkinson Welfare Weight Function ($\epsilon = 0.5$)

sumption (page 9) states that the proportional decrease in the weight U' for a given proportional increase in income should be the same at any income level. So if a person's income increases by 1% the welfare weight drops by ε% of its former value. The higher is ε, the faster is the rate of proportional decline in welfare weight to proportional increase in income – hence its name as the "inequality aversion parameter". The number ε describes the strength of society's desire for equality *vis à vis* uniformly higher income for all.

Consider as an example a rich person R with five times the income of poor person P (Cowell 1995). Inequality aversion or inequality neutrality ($\varepsilon = 0$) would imply approval of a redistribution of exactly £1 from R to P – a transfer

value of ε	*maximum amount of sacrifice by R*
0	£1.00
$\frac{1}{2}$	£2.24
1	£5.00
2	£25.00
3	£125.00
5	£3 125.00
...	...

Table 1: How much should R give up to finance a one pound bonus for P?

with no net loss of income. But if $\varepsilon > 0$ we might also approve of the transfer even if it were going to cost R more than £1 in order to give £1 to P – in the process of filling up the bucket with some of Mr. R's income and carrying it over to Ms. P we might be quite prepared for some of the income to leak out from the bucket along the way. In the case where $\varepsilon = 1$ we are in fact prepared to allow a sacrifice of up to £5 by R to make a transfer of £1 to P (£4 leaks out). The trade-off of social-values against maximum sacrifice is illustrated in Table 1. If we were to consider $\varepsilon \to \infty$, we would in effect give total priority to equality over any objective of raising incomes generally. Social welfare would then be determined simply by the position of the least advantaged in society (Hammond 1975).

The foundation of inequality aversion As far as the basis of inequality aversion is concerned we could consider it to be rooted in individual distributional judgments or in the imputation of some public values attributed to the State. If it is the case that the basis for social inequality-aversion is to be rooted in individual judgments then there are, broadly, three possible approaches open for inquiry which are developed in the next three subsections:

- Basing concern for inequality upon individual's cardinal utility of income interpreted as "satisfaction", (3.1.3),
- Treating it as a form of "externality" (3.1.4),
- Basing it upon peoples' attitude to risk (3.1.5).

3.1.3 Individual welfare: the "Dutch school"

What could be the basis for determining an individual welfare function – a cardinal utility function – on which to base utilitarian welfare judgments? The research programme based on the work of Van Praag (1977, 1978)[11] focuses precisely on this issue. Van Praag suggests the use of a specific functional form for individual welfare functions that is based on a particular view of the way in which consumers derive benefit from commodities. The main idea is that each consumer derives utility from a set of m characteristics represented in a group of commodities. The utility from each of these characteristics enters multiplicatively into the consumer's preferences. In effect van Praag's assumptions involve assuming that the consumer believes that by spending a money amount y any combination of characteristics can be attained such that

$$\kappa_1 \cdot \kappa_2 \cdot \ldots \cdot \kappa_m \leq y$$

[11] See also Van Herwaarden et al. (1977), Van Batenburg and Van Praag (1980)

where each κ_j is an expression that is an increasing function of the quantity of characteristic j. This implies

$$\sum_{j=1}^{m} \ln(\kappa_j) \leq \ln y \tag{23}$$

The utility that the person derives from income y is then given by

$$v(y) = \int ... \int_{\sum_{j=1}^{m} \ln(\kappa_j) \leq \ln y} \mathrm{d}u(\kappa_1, \kappa_2, ..., \kappa_m) \tag{24}$$

where u is the satisfaction to be obtained from a particular bundle of characteristics represented by the numbers $\kappa_1, \kappa_2, ..., \kappa_m$ (Kapteyn and van Wansbeek 1985). If the consumer's perceptions about each of these characteristics is independently distributed then the sum on the left-hand side of (23) is asymptotically normally distributed , which means that the utility of income (24) must take the lognormal form

$$v(y) = \int_0^y \frac{1}{\sqrt{2\pi}\sigma \ln y} \mathrm{e}^{-\frac{1}{2\sigma^2}[\ln y - \mu]^2} \mathrm{d}y \tag{25}$$

where μ and σ are parameters relating to the particular group of commodities over which the consumer is exercising choice. One of the difficulties with this approach is that it implies that the marginal utility of income (welfare weight) is *not* everywhere decreasing in income: see the portion to the left of the dotted line in Figure 3 and also Figure 4: contrast this with Figures 1 and 2. This result on the shape of the welfare weight function clearly has major consequences for the assumptions that lead to its shape and its implications for the assessment of income distributions.

3.1.4 The externality approach

In formulating the utilitarian form of the SWF (2) an important assumption was slipped in – that each person's utility depends only upon his income or his own basket of consumption goods. Clearly this makes the problem tidier and simpler but it may be ruling out an important motivation for social concern about income inequality. This could take the form of the individual valuation of an externality involving other people's incomes or living standards (Hochman and Rodgers 1969).

A basic model of this can be found by modifying (2) as follows:

$$W = \sum_{i=1}^{n} v_i(y_i, E_i(\mathbf{y})) \tag{26}$$

where $\mathbf{y}$ denotes the list of incomes of the whole population and E_i is individual i's evaluation of the income distribution represented by $\mathbf{y}$. Note that the form

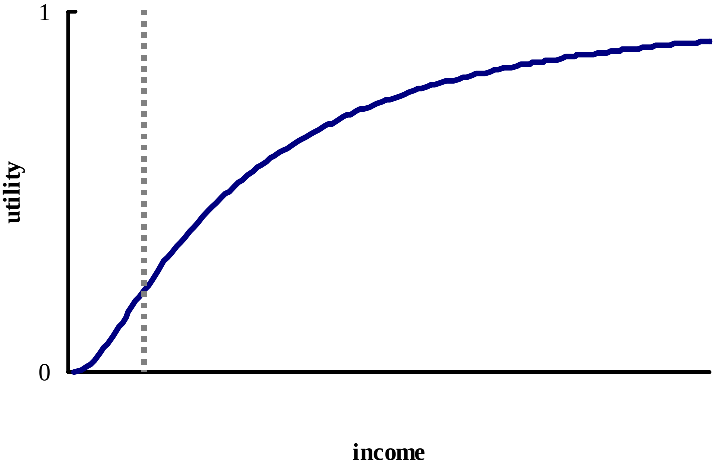

Figure 3: The Dutch School utility function

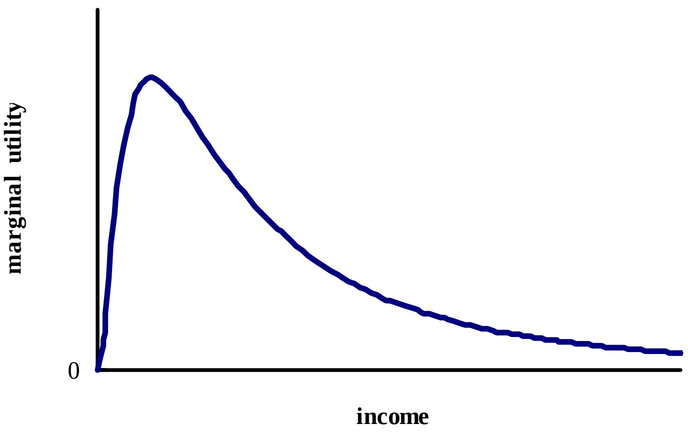

Figure 4: Welfare weight for the Dutch School utility function

(26) is still utilitarian, but we have respecified what people's utility depends on. The idea is that individuals may care about inequality or poverty in the same way as they may care about air quality, noise, traffic congestion, all of which are directly determined by the actions of others but all of which may have spillover effects on a person's utility Kolm (1964, 1969), (Thurow 1971), (Van Praag 1977). Irrespective of the supposed relationship between a person's utility and his income – whether or not v_i in (26) is concave in its first argument, for example – social values as represented by W "inherit" the property of inequality aversion from the private values represented by the dependence of v_i on $E_i(\mathbf{y})$. Inequality aversion is determined by the marginal utility of the externality.

3.1.5 Risk-attitudes and Social welfare

A comparison of the formulae (8) and (12) reveals a remarkable similarity. If we further make allowance for the relationship between x and y in the equivalisation formula (16) then the connection appears to be complete. Is there anything more to this than happy coincidence?

As we discussed in section 2.1.2 a common assumption in applied econometrics is that the individual preferences are additive. Apart from the simplicity of structure this has nothing to do with the assumption of additivity made for the social-welfare function. However, it can be argued that there is an important ethical connection between the presentation of social welfare in terms of "social utility" and the structure of consumer preferences in the face of uncertainty. To see this note that expression (13) can be written equivalently as

$$\begin{aligned} & \sum_{i=1}^{n} \pi_i U(x_i) \\ \pi_i &= \frac{1}{n},\ i = 1, 2, ..., n \end{aligned} \tag{27}$$

which has the interpretation of "expected utility" of a special type of lottery . The equivalised incomes $(x_1, x_2, ..., x_n)$ can be seen as a set of n prizes associated with the identities $(1, 2, ..., n)$; a representative person in the population regards it as equally likely that he should receive any of the prizes in the income distribution (Harsanyi 1955).[12]

In this approach social attitudes to inequality are evidently linked to the attitudes of an individual to risk. Inequality aversion is determined by risk aversion and inequality measures can be interpreted as measures of riskiness of an income distribution (Dahlby 1987).

[12] In the language of section 2.1.3 the set of n identities becomes for the representative person the set of S possible states of the world; each state-of-the-world s corresponds to an individual identity i. For general discussion of the lottery approach to the social-welfare function see Broome (1991), Kolm (1996a, 1996b), Roemer (1996), Zajac (1996).

3.2 Estimation issues

In the light of the several distinct ethical bases for interpreting welfare weights in terms of utility what are the prospects for adducing evidence that would enable policy-makers to attach meaningful weights to individuals at different points in the income distribution? Let us briefly consider the principal problems in an empirical implementation of each of the three main theoretical strands of thought 3.1.3 to 3.1.5.

- The individual cardinal utility approach is limited by the terms of the specification of the cardinalisation. Even if one does not follow the exact specification in 3.1.3 there are considerable difficulties implementing a utility function of this form.[13]
- Broader evidence on inequality attitudes (in sections 3.1.4 or 3.1.5) is available from questionnaire and experimental evidence (Amiel and Cowell 1999). However the precise experimental design is likely to influence the quality and nature of the results.
- Furthermore evidence specifically on risk attitudes (section 3.1.5) is potentially available from a variety of sources. As with inequality attitudes one could focus directly on personal judgments in the form of risk perceptions (Amiel and Cowell 1998)[14] (but this would run into the same design problems as with experimental evidence on inequality) or one could look at evidence drawn from individuals' real-life market behaviour in the face of risk.

3.2.1 Coherence and stability

A basic question which one might hope to address is whether perceptions of distributions are coherent. For example do people's inequality perceptions match risk perceptions, as one might expect from the approach of section 3.1.5? (Amiel and Cowell 1998). Are perceptions of inequality stable over time, and do they differ significantly between different communities? As we will see there is some important qualitative evidence on these important points.

[13]Typically one codes viewpoint questions about satisfaction from different income levels constrained within a scale of (0,100) – see Kapteyn and van Wansbeek (1985) for an overview. This virtually guarantees that one has a nonconcave utility function of the form of Figure 3 where welfare weights fall with income over a certain range; however the precise shape is going to depend on the arbitrary numerical constraints within which one forces the verbal responses. Other studies that attempt to get a personal evaluation of income make use of an Economic Ladder Question (nine steps on the ladder ranging from the poorest to the richest: where do you, the respondent stand today?) (Mangahas 1995) (Ravallion and Loshkin 1998). However these personal ELQs are not usually employed to give estimates of marginal utility of income.

[14]In fact both types of view could be addressed simultaneously (Kroll and Davidovitz 1999).

3.2.2 Evidence on structure

A second basic question that should be addressed concerns the validity of the structures introduced in sections 2.1 and 2.2 on which we have built a specific framework for estimating distributional weights.

Unfortunately some of the fundamental assumptions required for the implementation of the von-Neumann-Morgenstern utility function (8) – additive separability and concavity (risk aversion) – have been called into question by experimental evidence (Kahneman et al. 1982, Tversky and Kahneman 1974, 1981, 1988). A similar problem exists with the basic assumptions of social welfare and inequality: the Pareto principle is often rejected (Amiel and Cowell 1994) (McClelland and Rohrbaugh 1978) as too is the transfer principle (Amiel and Cowell 1992).[15] In the light of this, one has to approach with considerable caution either the direct route of evidence about attitudes to inequality, or the indirect route – via evidence about attitudes to risk.

3.2.3 Types of evidence

Let us briefly summarise the issues of implementation by reconsidering the viable ways in obtaining information about distributional welfare weights. Suppose we take on board the assumptions required to embody distributional judgments in the standard form (12) or (27). We may then identify three possible ways of capturing the values implicit in U.

- Direct enquiry
- Inference from private behaviour
- Inference from public actions.

The first of these is particularly appropriate to direct evidence on attitudes to income distribution, although it has also been applied to evidence about risk attitudes; the second is relevant to risk attitudes; the third has sometimes been used to infer how governments have appeared to incorporate distributional issues into their decision-making.

4 Evidence

Because there is no single body of work on which to draw, nor even a single established set of analytical techniques that are relevant to the present study, we will review the patchwork of evidence that bears on the subject of welfare weights in three broad categories that encompass the types identified in 3.2.3 above:

[15]For a more general discussion of the extent to which people's distributional judgments conform to standard axiom systems see Amiel and Cowell (1999)

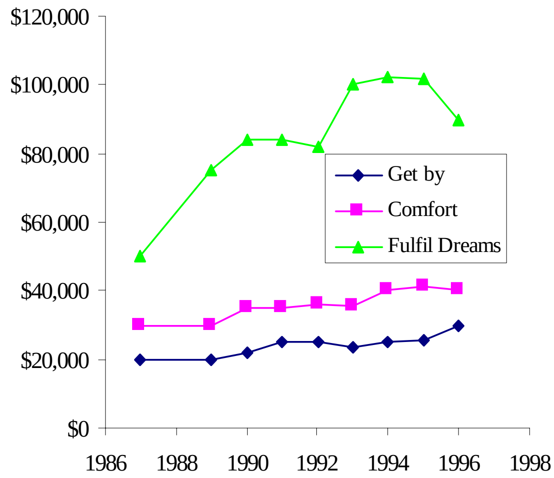

Figure 5: "How much income would you need to Get by/Live in comfort/Fulfil all your dreams?". Source: Ladd and Bowman (1998).

direct evidence about social values concerning inequality and income distribution (4.1.1 and 4.1.2), direct evidence on attitudes to risk from which one may infer something about distributional weights (4.2.1), and indirect evidence on attitudes to risk that may be inferred from other aspects of consumer behaviour (4.2.2). We begin with a discussion of whether it makes sense to attribute a coherent system of distributional values to society.

4.1 Direct evidence on social values

Of course social scientists other than economists have a keen interest in attitudes to income distribution and from the sociological and political science literature there is some evidence on the pattern of social attitudes to inequality broadly interpreted. This is principally drawn from the studies based upon opinion poll surveys. In brief, attitudes to inequality differ by country but are relatively stable over time. (Ladd and Bowman 1998) Figures 5 and 6 show the responses to two key studies in the area, based on Gallup poll surveys that are comparable over time and between the USA (Figures 5 and 6) and the UK (Figure 6).

It is striking that, while there is some evidence of variability, views are broadly consistent, both over time and by country. Of course, despite this reassurance, the types of question in these studies are not suitable for estimating welfare weights. Let us turn to the specific sources that are available.

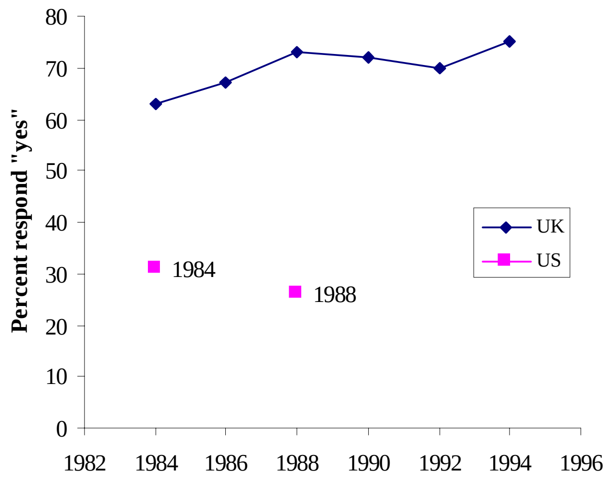

Figure 6: Do you think of [UK]/[US] as divided into haves and have-nots...? Source: Ladd and Bowman (1998).

4.1.1 Empirical measurement of inequality aversion

Whether inequality aversion is interpreted as externality or as the reflection of risk aversion in an "income-distribution lottery" it is clear that social welfare can, in some sense, be taken as an embodiment of personal views. So it may be illuminating to investigate the strength of, and factors determining, inequality aversion.

Gevers et al. (1979) and Glejser et al. (1977) pioneered the use of experimental studies on groups of students and suggested a very high degree of inequality aversion. However, their results are subject to two important qualifications (1) they adopted a non-conventional definition of inequality aversion that is difficult to translate unambiguously into the criterion that has been conventionally used (for example in section 3.1.2 above); (2) the extent of inequality aversion depended on the context of the experiment.

By contrast Amiel et al. (1998) find very low values of implied inequality aversion ε in (15). In experiments that investigated SWFs of the constant-elasticity form (15) their estimates of the elasticity of marginal social utility range from 0.2 to 0.79, depending on the background of the student and the precise specification of the experimental question. For other specifications the implied value of the elasticity of marginal social utility is also relatively low: for example the constant absolute inequality aversion[16] specification implies estimates ranging from 0.09

[16]For details see A.1 in the Appendix.

to 0.89, again depending on student background and type of experiment.

4.1.2 Revealed social values

An alternative approach is to suppose that social values, including inequality aversion, will be revealed by public policy decisions (Christiansen and Jansen 1978), (Stern 1977), although this may run into the problems of falsely assuming coherence and rationality on the part of governments and their agents, as well as problems of specification of the SWF. The trick is to find some unseen relationship which can plausibly be assumed to transform the unknown social values into observable phenomena or structures.

For example Guerrero (1987) suggests that, given the standard isoelastic social utility function (15), and given the observed skewed distribution of income, one might use a Box-Cox (1964) method of estimating ε on the assumption that "utility" is normally distributed. However, it is difficult to see why this data-driven statistical procedure should be appropriate to the selection of an essentially normative parameter.

A more promising approach to finding ways in which social attitudes might be revealed is to examine overtly redistributive instruments such as the income tax.[17] There are two basic prerequisites in order to implement this approach:

- An assumed principle on which the tax has – perhaps unconsciously – been constructed. A standard principle, which seems to have some empirical justification is that of equal absolute sacrifice. This means that if y is taxable income and $v(\cdot)$ is the assumed utility function, then the tax function $T(\cdot)$ must satisfy:
$$v(y) - v\left(y - T(y)\right) = \text{constant} \tag{28}$$
(Richter 1983), (Vitaliano 1977), (Young 1987).

- An assumption about the structure of utility to be imputed to citizens. It seems appropriate to use the individual-utility counterpart to the constant-elasticity form (15), namely
$$v(y) = \frac{y^{1-\varepsilon} - 1}{1 - \varepsilon} \tag{29}$$

[17] Other areas of public decision making could be used in similar manner to infer values for social weighting. For example, given (1) an independent estimate of the statistical value of a life, (2) a specific algorithm for relating the value of life to individuals' utility expressed as a function of income, and (3) a functional form for individual utility, one could recover values of the parameters of the specified utility function (prerequisite 2) and then use this to impute distributional weights. However this procedure is more roundabout than the tax-structure method which focuses *directly* on distributional issues and social values.

	1998/1999	*1999/2000*
Tax only	1.4306	1.4136
	(0.0736)	(0.0684)
Tax and NIC	1.2947	1.2838
	(0.0729)	(0.0690)

Table 2: Estimated utility-elasticities from UK Personal Income Tax and National Insurance Contributions

Combining the relationships (28) and (29) it is clear that the income tax should satisfy the relationship[18]

$$-\ln\left(1 - T'(y)\right) = \varepsilon \cdot \ln\left(\frac{y}{y - T(y)}\right) \tag{30}$$

Using (30) Stern (1977) estimated the elasticity of marginal utility as 1.97. By contrast the results for the UK in the late 1990s are in Table 2. Notice that for income tax alone one obtains an estimate that is considerably lower than that of Stern (1977) for the 1970s and that the results are substantially lower when employee National Insurance Contributions are aggregated together: these results are broadly in line with what one would expect given the changes in the tax structure away from high progressivity during the 1980s and 1990s, and the generally regressive nature of NIC.[19]

4.2 Personal attitudes to risk

Let us now consider the indirect evidence on attitudes to income distribution that might be inferred from an examination of people's attitudes to risk. There are three broad types of evidence that are potentially useful:

- Estimates of the cost of risk to individuals at different income and wealth levels derived from market behaviour.
- Studies on the perception of risk in probability distributions
- Inferences drawn from related aspects of the structure of individual preferences.

However the first source of information is unlikely to be particularly useful because of problems with the data. The limited coverage of financial asset-holding

[18]For derivations see Appendix A.2.

[19]Tax rates are computed for a single non-elderly man with no special reliefs. The numbers in parentheses in Table 2 are standard errors.

	Income	Net Worth
1st	0.2556	0.2601
2nd	0.2366	0.2381
3rd	0.2310	0.2318
4th	0.2312	0.2319
5th	0.2511	0.2435

Source: Barsky et al. Tables 3-Q, 3-R

Table 3: Mean Risk Tolerance by Family Income and Net Worth Quintile Groups

by individuals and credit constraints mean that samples are unlikely to be representative.

4.2.1 Direct evidence on risk

As we have noted, direct evidence on risk aversion from market behaviour is difficult to come by because of poor data quality.[20] However, there is good quality experimental and questionnaire evidence. For example Barsky et al. (1995) used survey questions from the Health and Retirement Survey to elicit information on risk aversion, subjective rate of time preference, and willingness to substitute intertemporally.[21] The questions involved choice in hypothetical situations about willingness to gamble on lifetime income. Their principal evidence concerns the degree of relative risk tolerance (the inverse of $R(y)$ in equation 10) by individuals at different points in the income distribution – see Table 3.

The implications of these estimates for relative risk aversion by income and wealth groups group is shown in Figure 7. Notice that, whatever income- or wealth-group one examines, the implied estimates of the elasticity of marginal utility (the vertical axis in Figure 7) are much higher than for the direct inequality experiments cited in section 4.1.1.

Secondly consider the experimental study by Binswanger in the context of semi-arid rural India.[22] The study took the form of an experiment involving over 300 individuals randomly selected from six villages. The evidence is consistent with the hypothesis that individuals maximise expected utilities defined on outcomes. The attitude to income risk is well approximated by the constant partial

[20]It is interesting to note that commonly cited US study (Friend and Blume 1975) tentatively suggested that the estimate of relative of risk aversion is “likely to be in excess of two” (p. 920).

[21]This is a panel survey of a nationally representative sample of the US population aged 51 to 61 in 1992 and their spouses. The sample is 12,600 and respondents are reinterviewed every two years.

[22]See Newbery and Stiglitz (1981), Chapter 7 for a description, summary and further discussion of the results.

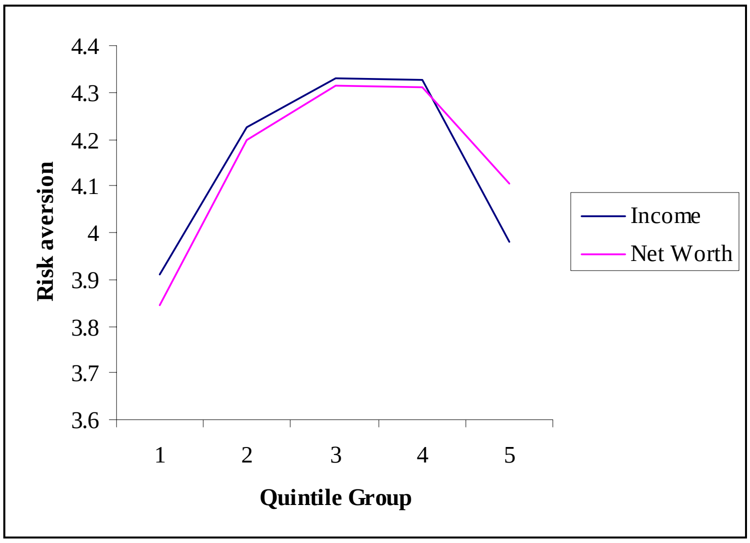

Figure 7: Estimates of RRA by quintiles from Barsky et al. (1995)

risk aversion utility function, where partial risk aversion is defined as

$$P = \frac{y}{w+y} R(w+y),$$

and, as before, $R(\cdot)$ is the coefficient of relative risk aversion, in this case defined on total wealth w and income y combined. The experiment suggests P lying between 1.74 and 0.32. Binswanger rejects the Asset Integration Hypothesis and the hypothesis of constant relative risk aversion: for a modal individual with wealth of 10,000 Rs relative risk aversion falls from 1000 at the lowest game level to between 1 and 2 at the highest game level.[23]

This therefore shows that the decision is not seen in the context of the overall asset position – only current outcomes seem relevant to current choices. The coefficient of partial risk aversion increases from about 0.5 for small fluctuations in income (standard deviation of about one month's wage) to about 1.2 for large fluctuations (standard deviation of about 50% of annual income).

[23]This is in contradistinction with the statement in H. M. Treasury (1991), page 82 Footnote 2: "There is no persuasive evidence or *a priori* reason to suggest that this elasticity should vary significantly with income over the income range of the economy." Note also that the experimental evidence of Levy (1994) suggested that relative risk aversion could be constant or falling, and that, although Friend and Blume (1975) argue that constant relative risk aversion is constant using their preferred definition of net worth, they acknowledge that their conclusion does not hold for other definitions of net worth.

quantile point:	*0.10*	*0.25*	*0.50*	*0.75*	*0.90*
Model 1	-0.85	-0.80	-0.77	-0.74	-0.72
Model 2	-2.9	-1.9	-1.4	-1.13	-0.96

Table 4: Blundell et al (1994): Distribution of elasticities at average household characteristics

4.2.2 Indirect behavioural evidence

Under certain circumstances we can also draw inferences about risk aversion on evidence from estimates of lifetime consumption behaviour. In a simplified representation a person's preferences over time can be modelled as:

$$\phi(c_t) + \frac{1}{1+\delta}\phi(c_{t+1}) \tag{31}$$

where c_t represents "consumption today", c_{t+1} represents "consumption tomorrow" and δ is a personal discount rate. The similarity of structure with (8) is evident.

Blundell et al. (1994) used Family Expenditure Survey data over the period 1970-1986 to estimate a model of lifetime consumption based on a more general preference structure that reduces to (31) given some fairly strong assumptions about the way in which within-period preferences are represented in the intertemporal utility function.[24] They estimated the intertemporal substitution elasticity in different parts of the income distribution: a summary of results for their two principal models is given in Table 4,[25] and the implied value of relative risk aversion is in Figure 8. Note that the implied estimate of risk aversion *rises* with the economic status of the households.

Furthermore Attanasio and Browning (1995) also find that the intertemporal substitution elasticity increases with the level of consumption: hence the implied coefficient of relative risk aversion will also increase with consumption.

Finally Besley and Meghir report estimates from a number studies on developed countries of the intertemporal elasticity of substitution of consumption, with most showing it to be below unity (Besley and Meghir 1998).

The indirect evidence on relative risk aversion – if one is prepared to accept the analytical trick that allows one to connect this to the intertemporal substitution

[24]For the definition of intertemporal substitution elasticity and its relationship to relative risk aversion under certain constraints of the model see Appendix A.3.

[25]In model 1 of the Euler equation for consumption, the dependent variables are head of work in paid employment, wife in paid employment, number of children 0-2 years of age, total number of children, owner occupier, single adult household, multiple (>2) adult household and the rate of time preference.

In model 2, a dummy variable is added which takes the value one prior to 1981 to capture the effect of high real interest rates in the 1980s.

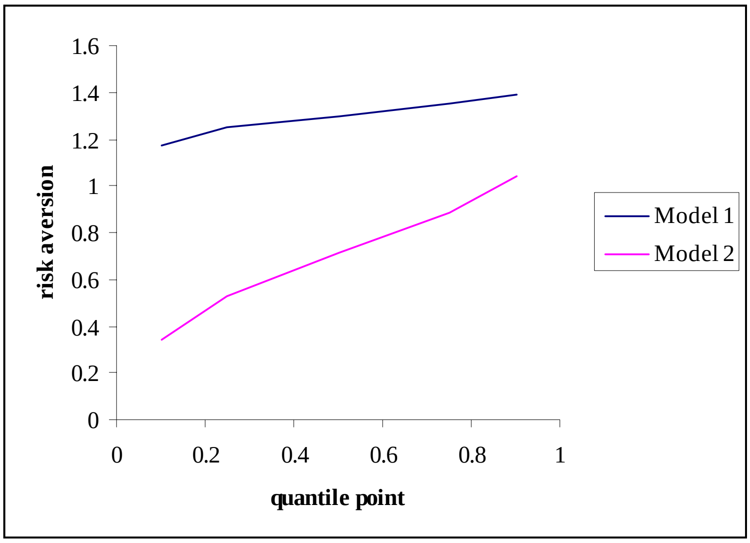

Figure 8: Estimates of RRA by income quantiles from Blundell et al. (1994)

elasticity – indicates a rather modest value for the coefficient of relative risk aversion, in contrast to the high levels elicited from the Barsky et al. (1995) direct experimental study. This in turn suggests welfare weights that have a relatively low elasticity with respect to income.[26]

5 Conclusions

As has become evident from the discussion of sections 2 and 3 a quantitative approach to distributional weights in applied welfare economics cannot neglect the key theoretical questions underlying the approach. So we shall briefly review the main points of these before summarising what is known from empirical studies.

5.1 Theoretical basis

It should be emphasised that this is an area where there is not – and perhaps cannot be – one unified analytical approach which would command acceptance by mainstream economists. The principal theoretical strands are as follows.

[26]It is interesting to note that the low values of the elasticity that emerge from these studies and the high values that emerge from experimental work bracket, but do not approximate, the value of 1.5 suggested in H. M. Treasury (1991). However the status of the 1.5 value is left unclear since H. M. Treasury (1991) only refer to "the data available, and common observation" – see our note 1 above.

Utility as satisfaction Some would argue that utility as consumer satisfaction is simply not observable. If one assumes that it is observable, then the measurement of marginal utility has to be within the context of a carefully specified model of how such satisfactions are generated. The principal exponents of this approach (the research programme discussed in section 3.1.3) adopt a model that many would regard as idiosyncratic and that does not imply diminishing marginal utility of income.

Utility and preference analysis The marginal utility of money derived from conventional preference theory and its applications provides no information for welfare analysis unless strong additional assumptions are made. These assumptions are typically justified on the basis of mathematical convenience rather than an appeal to economic realism.

The von-Neumann-Morgenstern (vNM) approach The vNM structure incorporating use of a "felicity" or cardinal utility function is a useful approach that rationalises choice under uncertainty. It may be *assumed* as a model for utility as satisfaction, but without this explicit assumption risk aversion does not imply diminishing marginal utility in terms of satisfaction. However, although this structure is often invoked in both theoretical and applied work, experimental evidence on consumer behaviour under uncertainty is not encouraging from the point of view of the vNM approach. So the adoption of the vNM structure as an explicit model of satisfaction is a questionable assumption.

The social-welfare function (SWF) approach One could argue for social-welfare weights based on "society's" evaluation of inequality, but the nature of the axioms or assumptions conventionally adopted in the SWF approach are to some extent arbitrary and the artefact of academic convention rather than any systematic representation of the social will. In principle these social values can be established from surveys, experimental evidence or from other social choices that achieve electoral support. However, the available evidence again suggests that people's values and perceptions of distributional questions are often at variance with the standard assumptions made by welfare economists.

The Harsanyi lottery In general the values incorporated in the SWF approach (and the weights that derive from them) need have no direct connection with individual consumer preferences or utility. One argument for making a connection between the SWF approach and individual values is the Harsanyi lottery-of-identity logical exercise. However this again relies on the validity of the vNM structure.

Under these circumstances, what constitutes "appropriate" theory is a matter for nice judgment by the applied economist or policy maker. We have tried to give some guidance of what could reasonably be done.

5.2 Numerical results

A casual reader might become rather depressed by the considerable range of estimates for the elasticity of marginal utility that have emerged in the various parts of section 4. However we feel that there is good reason for this and something to be learned for future work in the area. The extreme values – as indicated by the very low levels of relative inequality aversion on page 23 and the very high levels of relative risk aversion on page 26 – both relate to experimental work. This is not to suggest that there is anything suspect about experimental work in general or these studies in particular, but that experiments by their very nature may pick up on information that is special to the circumstances of the experiment or the respondent group.

By contrast the estimates obtained for relative risk aversion by the indirect route of inference from the intertemporal substitution elasticity are fairly consistent. Most imply values for the elasticity of marginal utility of just below or just above one.

Furthermore there is a good policy argument for taking the estimates from tax schedules seriously. It could be argued that the elasticity of marginal social utility is a "necessary intangible" and, although it might defy direct observation by an economist or anyone else, evidence on the appropriate value for this intangible can be obtained from other public-sector decisions that may have taken it into account. In this respect the argument is comparable to the problem of the valuation of life in cost-benefit analysis and other applications (Broome 1978): again there is no clear-cut analytical approach that would be accepted by everybody, and it is hard to make serious headway on the subject without recourse to some fairly abstract theorising. However in both cases one might reasonably take the view that policy decisions ought at least to be consistent in the value that is imputed to this intangible and that one can infer something about the appropriate value from other reasoned decisions that appear to have some measure of public support. So, in the case of the value-of-life analogy, in deciding to apportion the marginal million pounds in airport safety measures one takes into account the revealed social values from decisions taken about crash-barriers on motorways.

In the present case the analogy runs as follows. It is arguable that one of the major policy instruments which in some sense embodies social values about income distribution and redistribution is personal income tax. That being so it is reasonable to look at the progressivity and level of the tax to infer what values appear to be implicit in the structure. Presumably these values have been defended before an electorate and could be consistently applied to other areas where distributional considerations are important, such as the determination of welfare weights.

If this argument is accepted it is clear that the appropriate value for the elasticity of the social marginal utility of income can be expected to change over time as national priorities and resources change. it is arguable that this is precisely what has happened in the UK over the last twenty years, as reflected in the much lower values for our current estimates than those obtained by Guerrero (1987) using the same methodology. Admittedly one would not want to put too much weight on year-to-year fluctuations and one might debate on whether it is the income tax narrowly defined, or income tax plus NIC, or some other fiscal instrument, that is the appropriate artistic by which to infer these values. However it seems that this provides at the very least a useful source of information additional to the indirect evidence obtained by ingenious assumptions from models of consumer behaviour.

5.3 Caveats

We began our summary with some remarks about the pervasive role of fairly abstract economic theory in any sensible treatment of this topic. There are some tiresome caveats in application too. For example it is likely that major changes in relative prices will themselves alter the welfare weights. (Christiansen 1983). As we have noted, it is well known that attempts to measure inequality or risk aversion directly appear to show a strong influence of dependence upon the personal characteristics of respondents: whether they are relatively young students or the relatively old who are about to retire, for example. Furthermore indirect estimates from savings behaviour indicate that the relevant parameter value is likely to be a function also of the individual's resources – income or wealth.

It might be reasonable to form the judgment that these interpersonal differences in the *parameter estimates* (as opposed to interpersonal differences in resources) are simply irrelevant to policy making, in which case either a compromise value from the behavioural evidence, or the "revealed social values" from the tax structure might be considered to be appropriate.

5.4 Consequence for policy formulation

In the introduction we remarked that estimating parameters of a welfare function or a utility function is not like estimating the parameters of the demand function for chocolate or cigarettes. The reason for this is that conventional consumer analysis focuses upon the structure of consumer preferences rather than intensity or level of consumer satisfaction. Although there are some analytical tricks which would enable one to transform one problem into the other, it is not self-evident that these tricks are legitimate, nor that they should be used to inform social judgments.

There are thus three separate main alternative foundations on which one can rest the case for welfare weights using some interpretation of "marginal utility":

1. Marginal satisfaction from income or consumption.
2. Explicit social valuation of inequality.
3. Attitude to risk

Although some of the literature complicates matters by blurring the distinctions between these quite different concepts it is important to bear in mind that each theoretical basis for welfare weights will require a different type of evidence to make it operational. Indirect evidence on 2 and 3 is certainly available, and possibly some direct evidence as well. The status of evidence on 1 is dubious. Given the diversity of estimates obtained for the "elasticity of marginal utility" just on interpretation 3 it is probably wise to use more than one value (more than one set of weights) in making distributional judgments. A reasonable range seems to be from 0.5 (corresponding to the indirect evidence on risk aversion from models of lifetime consumption) to 4.0 (corresponding to direct experimental evidence). This would bracket the values of 1.2 - 1.4 that are implied by a social-valuation approach that uses evidence drawn from the personal tax system.

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A Derivations

A.1 Constant Absolute Risk/Inequality Aversion

Consider the utility function

$$\phi(y) = 1 - e^{-\alpha y} \tag{32}$$

Differentiating (32) we have

$$\begin{aligned} \phi'(y) &= \alpha e^{-\alpha y} \\ \phi''(y) &= -\alpha^2 e^{-\alpha y} \end{aligned}$$

Hence, using (9) and (10) we find

$$\begin{aligned} A(y) &= -\frac{\phi''(y)}{\phi'(y)} = \alpha \\ R(y) &= -y\frac{\phi''(y)}{\phi'(y)} = \alpha y. \end{aligned}$$

Clearly the same concept can be applied to SWFs of the form (12): if one does so one obtains the "leftist" social welfare function of Kolm (1976). For this SWF the elasticity of marginal utility evaluated at mean income $\bar{y}$ is evidently $\alpha\bar{y}$.

A.2 Constant Absolute Sacrifice Model

For the derivation of (30) note that, by substitution of (29) in (28), we obtain the following condition for the absolute sacrifice model given a constant elasticity of marginal (social) utility

$$y^{1-\varepsilon} - [y - T(y)]^{1-\varepsilon} = \text{constant} \tag{33}$$

Differentiating (33) we get

$$y^{-\varepsilon} = [1 - T'(y)]\,[y - T(y)]^{-\varepsilon}$$

which implies

$$\frac{1}{1 - T'(y)} = \left[\frac{y}{y - T(y)}\right]^{\varepsilon}$$

from which (30) follows by taking logarithms.

The estimates in Table 2 are then obtained by a curve fit using ordinary least squares on (30), constraining the intercept to be zero.

A.3 Intertemporal Substitution Elasticity and Risk Aversion

As noted in the main text (see section 4.2.2), under certain assumptions about the structure of preferences in an intertemporal utility function one may connect the concept of (atemporal) risk aversion with the intertemporal elasticity of substitution.

Here we follow the approach of Blundell et al. (1994). Let $\mathbf{p}_t$ denote prices at time t, and x_t expenditure at t; then we may express consumption at t thus

$$c_t = \frac{x_t}{a(\mathbf{p}_t)}, \tag{34}$$

where $a(\cdot)$ plays the role of a normalisation function. Blundell et al. (1994) model within-period indirect utility in an "isoelastic" form thus:

$$V(\mathbf{p}_t, x_t) = \frac{c_t^{\{-\theta\}}}{b(\mathbf{p}_t)} \tag{35}$$

where θ is the elasticity parameter and the "{}" notation in the superscript is used to denote the function

$$c_t^{\{-\theta\}} = \begin{cases} c_t^{-\theta} & \text{if} \quad \theta \neq 0 \\ \ln c_t & \text{if} \quad \theta = 0. \end{cases}$$

Once again $b(\cdot)$ plays the role of a normalisation function in (35).

Then the intertemporal elasticity of substitution can be expressed as:

$$\frac{\partial \ln c_t}{\partial \ln [1 + i_t]} = \frac{\phi_t'}{x_t \phi_t''} = \frac{c_t^{\{\theta\}}}{\rho_t - [1+\theta]\, c_t^{\{\theta\}}} \tag{36}$$

where i_t is the interest rate at t and ρ_t is a parameter to be discussed below. Note the following about (36):

- It must be negative, given the standard assumptions about the utility function ϕ.
- The intertemporal elasticity of substitution is allowed to vary with consumption.

The parameter ρ_t in (36) plays a crucial role: it gives the transformation of within-period utility used in the intertemporal utility function which is represented as:

$$F(V_t) = \frac{\phi_t}{[1+\delta]^t} V_t^{\{1+\rho_t\}} \tag{37}$$

Note that if $\rho_t = 0$ then (36) becomes

$$-\frac{1}{1+\theta}$$

which is (minus) the coefficient of relative risk tolerance. Under this special assumption it is clear that relative risk aversion takes the form $1+\theta$.

B Risk Aversion and Marginal Utility

The formulation (8) is the representation of a person's preferences over lotteries where each lottery is represented by a collection of S (probability,income)-pairs $[(\pi_1, y_1), ..., (\pi_S, y_S)]$ corresponding to the S states of the world. If one accepts

1. the existence of utility interpreted as satisfaction or well-being and
2. that utility is consistent with preferences,

then the *utility* associated with the preference structure (8) is given by

$$\Phi\left(\sum_{s=1}^{S} \pi_s \phi(y_s)\right) \tag{38}$$

where Φ is an increasing function of one argument. If we want we can be specific that (38) is associated with individual i rather than j by introducing a personal subscript:

$$\Phi_i\left(\sum_{s=1}^{S} \pi_s \phi_i(y_{is})\right) \tag{39}$$

(we are assuming here – though we do not need to do so – that the probabilities are the same for all persons). Now suppose that y_{is} is generated by a mixture of personal choice (portfolio investment for example) and chance (random rates of return on assets): each y_{is} is then dependent on y_i the amount of resources – income or wealth – that the person has before the investment choice is made and before the state-of-the-world is realised. Typically this is given by a linear constraint

$$\sum_{s=1}^{S} \theta_s y_{is} \leq y_i$$

where the terms θ_s are determined by rates of return, asset prices, gambling odds. The (ex ante) utility of income y_i is then:

$$v_i(y_i) = \max_{\sum_{s=1}^{S} \theta_s y_{is} \leq y_i} \Phi_i\left(\sum_{s=1}^{S} \pi_s \phi_i(y_{is})\right) \tag{40}$$

The assumption of risk aversion is enough to ensure that the function $\phi_i(\cdot)$ (right-hand side of 40) is concave, but it does not say anything about whether $v_i(\cdot)$ (left-hand side of 40) is concave, i.e. it says nothing about whether we have decreasing marginal utility of income, interpreted in the sense of satisfaction.

C Principal Notation

a_i	personal attributes of household i
A	index of absolute risk aversion
c_t	consumption in period t
C	cost or expenditure function
E_i	evaluation of externality by i
h_i	size of household i
m	number of commodities
n	number of individuals
p_k	price of good k
P	index of partial risk aversion
q_k	quantity consumed of good k
R	index of relative risk aversion
S	number of states of the world
$T(y)$	tax payable on income y
u	individual utility level
U	"social-utility" or evaluation function
v_i	utility function of person i
V	indirect utility function
w	wealth level
W	social-welfare function
x_i	equivalised income of person i
y_i	income of person i

α	coefficient of absolute risk/inequality aversion
δ	personal discount rate
ε	coefficient of relative risk/inequality aversion
μ	parameter of lognormal distribution
ν	number of equivalent adults
π_i	subjective probability of event i
σ	parameter of lognormal distribution
ϕ	"felicity" or cardinal utility function
Φ	utility transformation
χ	equivalisation function
ω_i	welfare weight