Dynamics of social ties and local public good provision

Frans van Dijk*, Frans van Winden

CREED, Faculty of Economics and Econometrics, University of Amsterdam, 1018 WB Amsterdam
The Netherlands

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Abstract

A model is presented in which social ties between individuals and private contributions to a local public good are interrelated. Ties are formalized by means of utility interdependence, and depend on the history of social interaction, in this case the joint provision of the public good. The resulting dynamic model generates equilibrium values of the intensity of ties and the private provision level. The impact of public provision on these variables is analyzed. Our results are very different from those obtained with the standard model, where individuals are only interested in the utility from own consumption.

Keywords: Local public goods; Social ties; Voluntary provision

JEL classification: A13; D64; H41; H70

1. Introduction

In many economic issues social ties between individuals are important. This has been argued by social-psychologists and sociologists over the years and, more recently, by a number of economists, among which is Becker (1974). Positive social ties provide individuals with additional resources in times of need, help to control externalities, form a basis for a cooperative outcome of a prisoner’s dilemma, and ensure fair business dealings (Coleman, 1990; Granovetter, 1985). Negative ties may bring about the reverse. Where these ties are relevant, the

*Corresponding author.
factors determining their formation become of interest. In this article we investigate the development and impact of such ties in the context of the private provision of public goods in small communities where they are expected to play a significant role.

Studies of the private provision of a public good usually assume that individuals determine their contributions non-cooperatively in a one-shot game. Contributions of other individuals are taken as given, and only the utility derived from own consumption is considered. We refer to this set-up as the standard model (for a rigorous treatment, see Bergstrom et al. (1986)). Equilibrium contributions fall below the social optimum in this model, and this shortfall increases with the number of players. Disregarding corner solutions, the total availability of the public good is invariant to the income distribution as well as public provision. When the interaction is repeated a known finite number of times, the outcome is the same as that of the one-shot game. With an infinite or uncertain timespan any outcome can be sustained (see, e.g., Fudenberg and Tirole (1991)). However, if individuals are myopic, the results of the standard model are still valid. We will use this model as a bench-mark. Many variants of the standard model and alternative models have been developed (e.g., Sugden (1984); Cornes and Sandler (1985); Andreoni (1989)), but also these models do not explicitly allow for social ties. Interestingly, experimental results show that ‘group identity’ affects contributions, indicating that there is more to the provision of public goods than is currently captured by economic models (see Dawes and Thaler (1988)).

Our model focuses on the affective component of social ties, its development, and its impact on private provision. Ties are formalized by means of weighted interdependent utility functions. Weights attached to the utility of other individuals can be positive, zero or negative. An important difference with the existing literature is that these weights are not constant over time, but depend on the history of interaction between the individuals. The formation of ties is determined by the contributions of individuals, resulting in a simultaneous, dynamic model of social ties and public good contributions. We also investigate the impact of public provision and taxation. It should be emphasized immediately, however, that we do not claim to fully capture the very complex social process that is at stake here. The incorporation of the weights in the model, and the use made of psychological notions and findings is still rather ad hoc. In the absence of a fundamental theory of interpersonal relations that duly takes account of the

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1For convenience, we refer to these goods as local public goods, in contrast with the public finance literature, where the term generally refers to public goods provided by municipalities.

2In the economic literature utility interdependence is frequently assumed (ad hoc) to tackle phenomena that are hard to explain by pure individualism, such as behavior within families (including intergenerational transfers), voluntary redistribution, charity and crime (e.g., Barro (1974); Becker (1981); Hochman and Nitzan (1985)). In these models utility interdependence is used, but not explained. An exception is Guttman et al. (1992) where a uniform level of altruism is determined by education.
affective component (sentiments and emotions), this is unavoidable. Hopefully, the analysis presented below will motivate such fundamental research.

Our main results are in stark contrast with those of the standard model. First, in many cases private provision is higher. It may even approach the provision level that follows from the maximization of a social welfare function. A lower provision level can also occur, however. The outcome is determined by the combined effect of the spread of incomes and preferences, and the so-called tolerance level of individuals, which refers to the appreciation by an individual of the contributions of others. When any contribution is valued positively, and incomes are equal, a high provision level is reached. If people have a negative appreciation of any contribution smaller than their own, high provision levels cannot be attained. When in addition incomes differ the provision level falls below that of the standard model. Thus, in our model private provision is not invariant to the income distribution. Second, public provision leads to a decrease of total provision, by impeding the development of social ties. Again invariance is not found. Total provision reaches a minimum when, in the two-person case presented in detail, at least one of the private contributions has just been reduced to nil. It is only, when public provision is further raised that total provision starts to increase. Furthermore, as the development of ties takes time, a reduction in public provision is not immediately taken over by private initiative, if at all. Consequently, public programs are more difficult to end than to initiate.

The organization of the article is as follows. Social ties are formalized in Section 2. In addition, the impact of ties on public good provision is briefly examined. Section 3 goes into the dynamics of social ties. In Section 4 the complete model of private provision in the absence of government intervention is analyzed, while public provision is added in Section 5. Section 6 concludes.

2. Social ties and their impact on public good provision

Among the variables used by sociologists and social psychologists to characterize social ties—emotional intensity, reciprocal services, mutual confidence and others—the affective component is considered to be the key element of a tie; see Granovetter (1973) and the review of the empirical literature on personal attachments by Baumeister and Leary (1995). This enables a one-dimensional conceptualization as a first approach, and it provides a natural way of defining positive and negative ties, as well as the intensity of ties (which may be different for the subjects involved; see Wellman (1988)). A social tie between two individuals \(i\) and \(j\) is assumed to consist of \(i\)'s (\(j\)'s) sentiments about \(j\) (\(i\)), where sentiments or feelings are defined as the extent to which \(i\) (\(j\)) cares about \(j\)'s (\(i\)'s) welfare and derives satisfaction from it. This is formalized by using the standard concept of utility interdependence. A tie between two individuals \(i\) and \(j\) is described by the pair \((\alpha_{ij}, \alpha_{ji})\) in the following expression:
where \( V_h \) stands for the total utility of individual \( h \), and \( U_h \) for the utility \( h \) derives from his own consumption. If individuals maintain ties with other persons as well, their weighted utility enters Eq. (1) in the same way. Alternatively, one could substitute total utility \( V_h \) for \( U_h \), as in Bernheim and Stark (1988). In the case of two individuals, that will be focused on below, results are not affected by the choice of specification. The domain of \( \alpha_{hk} \) is assumed to be restricted to the interval \((-1, 1)\), which implies that people neither ‘love’ nor ‘hate’ others more than they love themselves. This seems to be a reasonable restriction for which substantial experimental evidence exists (e.g., Sawyer (1966); Liebrand (1984)).

We now turn to the impact of social ties on public good provision. To bring out the interesting aspects of the problem, it suffices to focus on a stylized local community with two inhabitants. The results can easily be generalized to larger communities (see Section 6). Let \( U_h = U_h(x_h, g_h) (h = i, j) \), where \( x \) denotes the consumption of a private good, and \( g \) the consumption of a public good. Preferences are assumed to be strictly convex, and the prices of both goods are taken to be one. With \( y_h \) and \( g_h \), respectively, denoting \( h \)’s income and contribution to the public good, \( y_h = x_h + g_h \) and \( g = g_i + g_j \). Individuals take each other’s contribution to the public good as given. When both \( \alpha_{hk} (h, k = i, j, h \neq k) \) are zero, the standard model is obtained. The first-order condition for the maximization of Eq. (1) is given by:

\[
\frac{\partial U_h}{\partial g} + \alpha_{hk} \frac{\partial U_k}{\partial g} + \frac{U_h}{U_k} = \frac{\partial U_h}{\partial x_h}.
\]

Eq. (2) presupposes that the left-hand side is positive, which is not necessarily so. When non-positive, \( g_h \) is zero and \( x_h \) equals \( y_h \). This case is covered by the requirement that contributions are non-negative. It follows for \( h \)’s reaction function \( g_h = \max \{ g_h(y_h, y_k, g_k, \alpha_{hk}), 0 \} \). In general, \( \partial g_h / \partial \alpha_{hk} > 0 \). Assuming that \( x \) and \( g \) are normal goods, \( 0 < \partial g_h / \partial y_k < 1 \) and \( -1 < \partial g_h / \partial g_k < 0 \). Finally, \( \partial g_h / \partial y_k \geq 0 \), as the direction of change resulting from an increase in \( y_k \) (given \( g_k \)) is not determined. It can be shown that the preferences represented by the extended utility functions are strictly convex, as long as the individuals derive positive utility from the public good (the problem is trivial if this is not the case). As a result the structure of the problem is the same as that of the standard model, and a unique equilibrium exists (see Bergstrom et al. (1986)). Assuming an interior solution,

The resulting total (or extended) utility functions are formally identical to the individual social welfare functions used by Arrow (1981). However, the underlying concepts are completely different. In Arrow’s study a social welfare function reflects an individual’s ‘social conscience’ concerning the distribution of income (a normative problem), whereas total utility \( V_h \) represents the actual concern for and satisfaction derived from the welfare of specific others.
It follows from the properties of the reaction functions that \( \frac{\partial g_h}{\partial \alpha_{hk}}>0 \), \( \frac{\partial g_k}{\partial \alpha_{hk}}<0 \) and \( \frac{\partial g}{\partial \alpha_{hk}}>0 \). Thus, the impact of an increase in \( \alpha_{hk} \) is unambiguous.  

Three special cases with regard to the \( \alpha \)'s are of interest. First, the standard model is obtained when \( \alpha_{ij} = \alpha_{ji} = 0 \). A second interesting case is \( \alpha_{ij} = \alpha_{ji} \to 1 \), where \( V_i = V_j \), which means that \( i \) and \( j \) maximize the same function under separate budget constraints. Contributions reach a maximum (\( g_{\text{max}} \)), and it can be easily shown that this leads to the same result as the maximization of a social welfare function \( S = V_i V_j \) or \( S = U_i U_j \), under an aggregate budget constraint and the constraint that the consumption of the private good by an individual does not exceed her or his income. Consequently, full cooperation is achieved. A third special case holds when \( \alpha_{ij} = \alpha_{ji} \to -1 \). In this case \( i \) and \( j \) become maximally uncooperative and \( g \) reaches a minimum (\( g_{\text{min}} \)). When preferences and incomes of \( i \) and \( j \) are equal, \( g_{\text{min}} \) is zero. Otherwise, \( g_i \) or \( g_j \) is zero. The standard model is an intermediate case, where the private provision level falls short of the maximum, but is above the potential minimum. We conclude that the sign and intensity of social ties among the inhabitants of a community influence private provision. The development of these ties is discussed in the next section.

In concluding this section we consider the consequences of government intervention, assuming that public provision is financed through (local) taxation. The budget constraint of individual \( h \) (\( h = i, j \)) now reads \( y_h = x_h + g_h + \tau_h g \), where \( g_h \) denotes the quantity of the public good provided by the government, and \( \tau_h \) is \( h \)'s tax share. Furthermore, \( g = g_i + g_j + g \). In case of an interior solution, the equilibrium is given by:

\[
g_h = g_h(y_h, y_k, \alpha_{hk}, \alpha_{kh}) - \tau_h g \quad (h, k = i, j, h \neq k).
\]

This is the familiar invariance result obtained by Warr (1982) for the standard case. For given social ties, public provision crowds out private contributions one to one.

### 3. Formation of social ties

Evidently, a uniform level of sentiments towards other individuals does not exist. Sentiments not only differ between individuals, they also vary over time. Whether some base level of affection or concern occurs among individuals who have never interacted before, is open to debate, and we abstract from it here. Regarding the dynamics of social ties, we take into account that sentiments

\footnote{Due to the secondary, undetermined cross-effect of \( y_i \) on \( g_i \) in the reaction function, the signs of the derivatives with respect to \( y_i \) are not as clear-cut. In general, \( \frac{\partial g_i}{\partial y_i} > 0 \), \( \frac{\partial g_j}{\partial y_i} < 0 \) and \( \frac{\partial g}{\partial y_i} > 0 \), and this will be assumed throughout.}
develop through prolonged interaction (Baumeister and Leary, 1995). Furthermore, as argued by Coleman (1990), ties are subject to decay over time, when not actively maintained. As to the development of ties we use the hypothesis that positively valued interaction is likely to generate positive sentiments, whereas the reverse holds for negatively valued interaction. This correspondence between interpersonal experiences and sentiments has been argued and substantiated by many social scientists over the years (see Homans (1950); Simon (1952); Fararo (1989); Frijda (1986)). Whether in our case the interaction will be valued positively or negatively depends on the contributions. Two aspects are likely to play an important role. First, as any contribution by \( j \) is helpful in producing the public good which is positively valued by \( i \), this is expected to lead to a positive valuation by \( i \) of the interaction (with similar reasoning applying to \( j \)). Second, it stands to reason that the valuation of the interaction will be positively (negatively) affected if the contribution by the other exceeds (falls short of) \( i \)'s or \( j \)'s own contribution. This comparison of contributions may be elicited by a notion of fairness.\(^5\) Weighing the one aspect against the other leads to the following criterion for positive (negative) sentiments: 

\[
G_{hk} = g_k - g_h > (>) 0 \quad (h, k = i, j, h \neq k),
\]

where the weight \( \epsilon_h \) (\( 0 \leq \epsilon_h \leq 1 \)) reflects the individual’s tolerance towards smaller contributions by the other individual. The following differential equations describe the development of the social tie (\( \alpha_{ij} \), \( \alpha_{ji} \)) over time:

\[
d\alpha_{hk}/dt = f_h(G_{hk}, \alpha_{hk}) \quad (h, k = i, j, h \neq k).
\]

The change of sentiments depends on the weighted relative contributions (\( G_{hk} \)) and the state of the sentiments (\( \alpha_{hk} \)). A diminishing marginal (positive) effect of \( G_{hk} \) is assumed. The effect of \( \alpha_{hk} \) is twofold. First, \( \alpha_{hk} \) is taken to influence the impact of \( G_{hk} \) such that sentiments remain within their boundaries (\(-1, 1\)). Second, sentiments decay over time. Consequently, \( f_h \) is taken to be an increasing, S-shaped, function of \( G_{hk} \) (the ‘impulse’), such that \( f_h \to 1 - \alpha_{hk} \) when \( G_{hk} \to \infty \) (\(-\infty\)). When \( G_{hk} = 0 \), \( f_h = 0 \) if ties do not decay over time, whereas \( f_h < 0 \) if decay occurs and \( \alpha_{hk} \neq 0 \). The function \( f_h \) is continuous and twice differentiable. In the numerical example of the appendix (Appendix A) a function exhibiting these properties is presented. Fig. 1 gives the phase-diagram of Eq. (5) for \( \epsilon_h > 0 \).

For clarification, suppose that ties would not decay over time. In that case the stationary points, where \( d\alpha_{hk}/dt = f_h = 0 \), are represented by the broken line b. When the impulse \( G_{hk} \) is zero, stationarity is obtained for any \( \alpha_{hk} \). For all positive values of \( G_{hk} \), \( \alpha_{hk} \) grows to its upperbound 1, and for all negative values to its lower bound \(-1\). For any \( \alpha_{hk} \neq 0 \) to qualify as a stationary point the countereffect of the decay of the tie requires \( G_{hk} \neq 0 \). This is shown by the fat line in the figure.

\(^5\)The relevance of fairness has been shown particularly in the context of ultimatum-bargaining games (Thaler, 1988). Its role in public good experiments is still subject of research (Ledyard, 1993). See also Rabin (1993) on fairness in general.
In that case \( \alpha_{hk} \) only approaches its boundary \( 1(-1) \) for infinite positive(negative) values of the impulse \( G_{hk} \). Thus, in the presence of attrition there exists an upward sloping differentiable function through the origin \( f^w_h \) describing the stationary points of Eq. (5):

\[
\alpha_{hk} = f^w_h(G_{hk}).
\]

(6)

4. Dynamics of social ties and private provision

4.1. Model and equilibrium outcomes

Combining the results of Section 2 and Section 3, we can now examine the development of the social tie \((\alpha, \alpha)\) and the corresponding contributions \((g, g)\). Following Coleman (1990) we assume that social ties are generally the unconscious byproduct of social interaction. Individuals are supposed to be myopic, which implies that people do not take into account the potential impact of their contributions on their own feelings and the feelings of others towards them.\(^6\) The decision structure can then be modeled as a sequence of contribution decisions

\(^6\)Strategic behavior is discussed in van Dijk and van Winden (1995).
represented by Eq. (3), which are only connected by the development of the tie over time. Finally, we ignore the possibility that social ties are affected by mortality or migration (see Section 6). Using Eqs. (3) and (5) the development of the social tie is described by the following differential equations:

$$\frac{d\alpha_{hk}}{dt} = f_i(G_{hk}(\alpha_{hk}, \alpha_{kh}), \alpha_{hk}) \quad (h, k = i, j, h \neq k) \quad (7)$$

where the income variables have been deleted, for convenience. The stationary points of these equations, determined by $f_i = 0$, can be mapped in $(\alpha_{ij}, \alpha_{ji})$-space. Neglecting extreme situations where $g_i$ or $g_j$ is zero, the curves representing these stationary points are upward sloping. Using Eq. (6), the slopes are given by:

$$\left(\frac{d\alpha_{hk}}{d\alpha_{jh}}\right)_h = \frac{\frac{\partial f_i}{\partial G_{hk}} \left( \frac{\partial g_k}{\partial \alpha_{jh}} - e_h \frac{\partial g_j}{\partial \alpha_{jh}} \right)}{1 - \frac{\partial f_i}{\partial G_{hk}} \left( \frac{\partial g_k}{\partial \alpha_{jh}} - e_h \frac{\partial g_j}{\partial \alpha_{jh}} \right)} > 0.$$ 

As regards the position of the curves in the $(\alpha_{ij}, \alpha_{ji})$-plane, the situation where individuals are identical is considered first. Fig. 2 gives the phase-diagram for the general case where $0 < e_i = e_j < 1$. For negative values of $\alpha_{hk}$, the decay of the tie over time causes an increase of $\alpha_{hk}$. For stationarity $(f_i = 0)$, this increase needs to be offset by a negative value of $G_{hk}$, and $h$’s contribution has to be larger than $k$’s. This implies that $\alpha_{hk}$ would have to be more negative than $\alpha_{hk}$. For $\alpha_{hk} = 0$, stationarity requires that $G_{hk} = 0$, and thus $\frac{g_k}{g_j} = e_h g_h$, which again implies that $\alpha_{jh}$ would have to be smaller than $\alpha_{hk}$, and negative. When $\alpha_{hk}$ becomes positive, $G_{hk}$ must also be positive to compensate the decline of sentiments due to attrition. Consequently, $\alpha_{kh}$ increases and switches sign. Given the properties of $f_i$, the
marginal impact of $G_{hk}$ then decreases with the increases of $\alpha_{hk}$ and $G_{hk}$. But the rate of decline of $\alpha_{hk}$ due to decay does not similarly decrease. Thus, as $\alpha_{hk}$ becomes more and more positive, $\alpha_{hk}$ has to increase at an ever faster rate, and at some point $\alpha_{hk}$ would have to become necessarily larger than $\alpha_{hk}$. Since individuals are assumed to be identical, the curves for $i$ and $j$ are symmetric, and intersect where $\alpha_{ij} = \alpha_{ji}$.

Differences between $i$ and $j$ may exist with regard to income, preferences for the private and public goods, the tolerance level, and the speed at which ties develop and decay. For expositional reasons, only differences in income or preferences are considered. With regard to income we assume that $y_i < y_j$, and as to preferences that $\eta_i > \eta_j$, and $\eta_j < \eta_i$ for all $x_i = x_j$ and $g$, where $\eta_i$ denotes the elasticity of utility of $h$ with regard to $z$ ($z = x_i$, $g$ and $h = i$, $j$). This implies that $(\partial U_i / \partial g) / (\partial U_j / \partial g) < (\partial U_i / \partial x_i) / (\partial U_j / \partial x_j)$. In both cases $f_i = 0$ and $f_j = 0$ are no longer symmetric. Symmetry requires $G_{ij}$ for $(\alpha_{ij}, \alpha_{ji}) = (a, b)$ to be equal to $G_{ji}$ for $(\alpha_{ij}, \alpha_{ji}) = (b, a)$, whereas in the cases considered $G_{ij} < G_{ji}$. Both curves shift to the right, and at the point of intersection $\alpha_{ij}$ must be larger than $\alpha_{ji}$. When incomes or preferences differ sufficiently, $f_i = 0$ shifts to such an extent that $\alpha_{ij}$ is positive for all $\alpha_{ji}$. The following proposition presents the equilibrium outcomes.

**Proposition 1.** (a) A unique social-tie equilibrium, denoted by $(\alpha^e_{ij}, \alpha^e_{ji})$, exists; (b) with equal preferences and incomes, the tie is symmetric and positive in equilibrium $(\alpha^e_{ij} = \alpha^e_{ji} = \alpha^e > 0)$; (c) with differing preferences or income levels, the tie is asymmetric in equilibrium (in the examined case $\alpha^e_{ij} > \alpha^e_{ji}$), and the sentiment with the lower intensity may be negative $(\alpha^e_{ji} = < 0)$; (d) the equilibrium is locally stable.

Parts (a) and (b) follow directly from the geometry of Fig. 2. Note that the functions cannot intersect in the negative orthant, since $f_i = 0$ then requires that $g_i > g_j$ ($G_{ij} < 0$) and $f_j = 0$ that $g_j < g_i$ ($G_{ij} > 0$). Part (c) is based on the impact of these differences on the phase-diagram, as discussed above. Part (d) can simply be verified by examining the Jacobian of the system at its equilibrium.

These results are consistent with sociological insights emphasizing the dependence of friendship ties (symmetric positive relations) on similarity of attitudes, attributes and social positions (Feld, 1981). It is noted that in the present model a situation where both individuals have negative feelings towards each other cannot occur. In case of several public goods and differing preferences, such a situation may arise, however.

A few remarks about the extreme cases $\varepsilon = 0$ and $\varepsilon = 1$ are in order for later reference. When $\varepsilon = 0$, $G_{hk} = g_k = 0$. Consequently, $f_h$ cannot be stationary for negative values of $\alpha_{hk}$. Both curves intersect once in the positive orthant, irrespective of differences in preferences or income. The equilibrium $\alpha$'s are larger
than in the general case. When $\varepsilon = 1$, $G_{ijk} = g_k - g_i$. With equal preferences and incomes, both $f_i = 0$ and $f_j = 0$ pass through the origin. Hence, $(0,0)$ is the equilibrium. With differing preferences or incomes, both curves shift to the right. In that case, the equilibrium $\alpha$'s are of opposite sign, but equal in absolute value (since $G_{ij} = -G_{ji}$). Moreover, they are smaller than in the general case.

4.2. Provision levels: comparison with the standard model

As a benchmark we use the standard model, with $\alpha_i = \alpha_j = 0$. Comparing the benchmark provision level ($g^b$) with the equilibrium levels generated by our model ($g^s$), the following proposition holds for the general case, where $0 < \varepsilon < 1$:

Proposition 2. (a) With equal preferences and incomes: $g^s > g^b$; (b) with different preferences and equal incomes: $g^s > g^b$; (c) with different incomes and equal preferences: $g^s \leq g^b$.

Part (a) of the proposition is evident from proposition 1(b) and the properties of Eq. (3). To prove parts (b) and (c), we proceed by examining the extreme cases $\varepsilon = 0$ and $\varepsilon = 1$ first. If $\varepsilon = 0$, $g^s > g^b$, because the equilibrium $\alpha$'s are always positive irrespective of differences in preferences or income. Now, consider $\varepsilon = 1$. When preferences and incomes, and, therefore, contributions are equal, the $\alpha$'s are zero in equilibrium, and $g^s = g^b$. Allowing for differences in preferences or income gives in this case: (I) with different preferences regarding $x$ and $g$, but equal income: $g^s > g^b$; and (II) with differences in income, but equal preferences: $g^s < g^b$. If (I) holds, individual $i$ who attaches a lower weight to the public good increases her or his contribution as a result of the interaction, whereas the contribution of $j$ decreases, but to a lesser extent. The intuition is that individual $i$ takes the utility of $j$ with a relatively high valuation of the public good positively into account, while $j$ takes to the same extent the utility of $i$, with a relatively low valuation, negatively into account. However, when incomes differ (II), the increase of the contribution by the poor person as a result of interaction does not compensate for the decrease of the rich individual’s contribution. A proof is given in Appendix B. As the general case, $0 < \varepsilon < 1$, lies between these extreme cases, the proposition follows.

To summarize, taking the dynamics of social ties into account widens the range of possible outcomes. Depending on the homogeneity of communities and tolerance with respect to contributions, provision may be above or below the benchmark level of the standard model. In addition, it is important to note that the provision level increases with tolerance and the equality of the income distribution (this follows from the proof in Appendix B). In the extreme case of $\varepsilon = 0$ and equal incomes, the maximum provision level $g^{max}$ is approached. Appendix A provides a numerical example.
5. Private provision and public provision

We now examine the impact of public provision. Treating public provision \((g_g)\) as exogenous and using Eq. (4), the development of \(\alpha_{hk}\) is described by:

\[
d\alpha_{hk}/dt = f_h(g_h(\alpha_{hk}, \alpha_{ih}), g_h(\alpha_{hk}, \alpha_{ik}) - (\tau_h - \epsilon_h \tau_h)g_h, \alpha_{hk})
\]

(8)

\((h, k = i, j, h \neq k)\). Assuming that the tax burden is equally split \((\tau_h = 0.5)\), and concentrating again on the general case where \(0 < \epsilon < 1\), the impact of public provision on the position of the phase-lines in Fig. 2 is readily seen. Public provision causes a decline of \(G\) if both private contributions are positive: \(g_h(\alpha_{hk}, \alpha_{ik}) - 0.5(1 - \epsilon)g_h < g_h(\alpha_{hk}, \alpha_{ih}) - 0.5(1 - \epsilon)g_h(\alpha_{ih}, \alpha_{ik})\). When \(g_g\) is increased from zero, \(f_i = 0\) shifts to the left, and \(f_j = 0\) to the right. With differing preferences or incomes the phase-diagram is again asymmetrical. The following two propositions present the equilibrium results for the social tie and the total provision level of the public good.

Proposition 3. If \(g_g\) increases from zero, then: (a) with equal preferences and income, the equilibrium \(\alpha\)'s decline uniformly until they become zero; (b) with differing preferences or incomes, the equilibrium \(\alpha\)'s decline, and the lower \((\alpha_{ji}^*)\) becomes negative, or, when already negative, becomes more negative; when \(g_g\) increases further and the lower contribution \((g_i)\) becomes zero, both equilibrium \(\alpha\)'s go to zero, i.e., the positive one \((\alpha_{ji}^*)\) decreases further, while the negative one \((\alpha_{ji}^*)\) increases.

Proposition 4. If \(g_g\) increases from zero, then: (a) the total provision level of the public good \(g^e\) (which includes \(g_g\)) decreases until, in case of identical individuals, both contributions are zero or, else, the lower contribution \((g_i)\) is zero; when \(g_g\) increases further, \(g^e\) increases; (b) when incomes differ, total provision starts to increase at a lower value of \(g_g\) than in case of identical individuals (keeping total income constant); with equidistant differences in preferences generally the same result is found\(^9\).

\(^7\)Note that we focus on observable effort, i.e., the voluntary contributions to the public good. We assume that contributing to the public good by paying taxes does not generate affective ties. Political processes seem to constitute a weaker mechanism for the formation of social ties than voluntary provision. In case of voting in elections the link between decisions and contributions is much less direct. Also, the secrecy typical for voting results in limited information about the willingness of individuals to contribute.

\(^8\)If \(\tau \neq 0.5\) the tax rate can be split in an average tax rate and an equally sized redistributive tax and subsidy rate. In that case the analysis of the effect of differences in income of Section 4.2 applies.

\(^9\)Equidistant means that the marginal rate of substitution \(d\alpha/dg\) for \(i\) is \((1 - \gamma)\) and for \(j\) \((1 + \gamma)\) times that rate in case of uniform preferences. As to the generality of the result, an exception occurs in the extreme case where \(\epsilon\) is close to 1 and tie attrition approaches zero for both individuals (see Appendix C).
Part (a) of proposition 3 follows directly from the discussed inward shift of $f_i = 0$ and $f_j = 0$. As to part (b), it is easily seen that increasing $g_x$ in a case of differences in preferences or incomes causes one equilibrium $\alpha$ to become negative. In the case where $i$ attaches lower importance to the public good, or has a lower income than $j$, $i$’s contribution becomes zero before $j$’s. In that event the impulse on $\alpha_{ji}$ becomes negative, since $G_{ji} = -e\{g_x(\alpha_{ji}, \alpha_{ji}) - 0.5g_{ji}\} < 0$. It follows that $\alpha_{ji}$ must be negative for $f_j = 0$. The intersection of $f_i = 0$ and $f_j = 0$ occurs at a negative $\alpha_{ji}$ and a positive $\alpha_{ij}$. A further increase of $g_x$ results in a decline of the negative impulse, which causes $\alpha_{ji}$ to become less negative. Eventually both equilibrium $\alpha$’s are reduced to zero. Part (a) of proposition 4 follows from the decline of the equilibrium $\alpha$’s. The increase of $g_x$ is overcompensated by the decrease of the contributions of $i$ and $j$. Total provision starts to increase only when at least one of the voluntary contributions has been reduced to zero. Part (b) reflects that with differences in income the lower contribution is always, and with differences in preferences in nearly all cases, smaller in equilibrium than the contribution in case of equal preferences and incomes. The proof of proposition 4 is straightforward, but tedious (see Appendix C). The numerical example of the appendix (Appendix A) illustrates the results.

Together, the propositions lead to the following conclusions. Public provision impedes the development of positive and negative sentiments, where the latter occurs if the provision level is sufficiently high. People become more neutral/indifferent towards each other. For identical individuals, ties and, therefore, the total provision level decline uniformly as public provision increases. When preferences or incomes differ, the impact is more complex. At low levels of public provision, sentiments become even (more) negative, and again total provision decreases. Thus, in general, increasing public provision from zero first causes total provision to decline. This constitutes a major departure from the standard model. It is only when at least one of the contributions is reduced to zero that total provision starts to increase. The larger the differences between individuals the lower the level of public provision at which this reversal occurs, and the smaller the decline of total provision.

The consequences of decreasing public provision, to cut back government expenditure for instance, can also be examined. Suppose that for a long time public provision completely crowded out private contributions, and is now totally abolished. At first, total provision declines sharply to the level $g_x(0,0) + g_y(0,0)$. When tolerance is low and large differences in income exist, private provision subsequently declines further. If tolerance is sufficiently high, private provision starts to increase and reaches a substantial level, but only after some time. The community would face a difficult transition period. Because the electorate may not accept the temporary decline of provision, the government might feel urged to maintain expenditures. Also note that, from a long term, social welfare perspective limited expenditure reductions only make sense as long as the reduced public provision level remains above the level that would be privately attained in the
absence of public provision. Otherwise, total provision is reduced below the level that can be privately achieved. The model suggests that there is a clear choice to be made: either rely on private or on public provision.

6. Concluding remarks

Our analysis points at conditions under which individuals succeed in providing local public goods without government intervention. They will fail to do so in cases of low tolerance with respect to the contributions of others, in particular when combined with an unequal income distribution. In that case there is a rationale for government intervention. Moreover, crowding out private contributions would also forestall negative sentiments. From a wider perspective, this is important in itself as such sentiments may cause social tensions that endanger public order and result in costs to society. Another rationale for government intervention suggested by the model concerns geographical mobility. In contrast with the standard model, migration affects the provision level of the public good in our model, because it disrupts existing social ties and necessitates the build-up of new ties. van Dijk (1997) shows that generally the average intensity of social ties decreases and that the provision level gets closer to the prediction of the standard model. Community size could provide a third rationale for government intervention. In the standard model the provision level, relative to the level that is found maximizing a social welfare function, becomes rapidly inconsequential when the number of inhabitants increases. Extending our model to larger communities, it can be shown that the relative decline of the provision level proceeds much more gradually as long as the inhabitants are informed about the contributions of a substantial number of their fellow-inhabitants. If this condition is not met, the results approach those of the standard model.

In other instances private provision can reach a high level, and public provision is unnecessary. When it does not crowd out private provision completely, public provision is counterproductive, as it reduces the total provision level. At any rate, it hampers the development of positive social ties that would develop through the private provision process, and may thereby induce further public intervention. A case in point is voluntary assistance to people in need. Following our model, there is no general willingness to lend such support; only individuals who have positive sentiments towards a person in need would be willing to help. When the development of such sentiments is blocked, the need for costly welfare and social security arrangements increases.

While staying close to standard economic theory, our model underscores the

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10It may also shed a new light on the occurrence of cooperation in public good experiments. The model suggests that account should be taken of the possibility that the motivation of subjects changes under the social interaction that takes place within the experiment.
important role played by social ties in the provision of public goods. Ignoring the effects of public policy on these ties, may, on the one hand, unwittingly result in the decline of social conditions that stimulate individuals to take care of their own needs, and, on the other hand, in social tensions and insufficient public amenities. In the first case, government intervention is too strong, in the second too weak. In both cases public policy is not attuned to the social environment.

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Appendix A

Numerical example

To illustrate the social-ties mechanism, Table 1 presents some numerical examples. The (own consumption) utility functions are specified as: $U_{hi} = x_{hi}^\rho g^{1-\rho}$ ($h = i, j$). For Eq. (5) a discrete-time version of the following specification is used:

$$\frac{d\alpha_{hk}}{dt} = \frac{e^{\alpha_{hi}^*(g_{hi} - x_{hi})} - (1 - \delta h \alpha_{hi})/(1 + \delta h \alpha_{hi})}{e^{\alpha_{hi}^*(g_{hi} - x_{hi})} + (1 - \delta h \alpha_{hi})/(1 + \delta h \alpha_{hi})} - \alpha_{hk}.$$

Table 1

<table>
<thead>
<tr>
<th>$g_s$</th>
<th>$\varepsilon$</th>
<th>$y_i = y_j = 100$</th>
<th>$y_i = 70, y_j = 130$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$g^0$</td>
<td>$g^*$</td>
<td>$\alpha^*_i$</td>
</tr>
<tr>
<td>0</td>
<td>67</td>
<td>95</td>
<td>0.81</td>
</tr>
<tr>
<td>0.5</td>
<td>67</td>
<td>89</td>
<td>0.59</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>67</td>
<td>67</td>
</tr>
<tr>
<td>20</td>
<td>0.5</td>
<td>67</td>
<td>85</td>
</tr>
<tr>
<td>50</td>
<td>0.5</td>
<td>67</td>
<td>76</td>
</tr>
<tr>
<td>67</td>
<td>0.5</td>
<td>67</td>
<td>67</td>
</tr>
<tr>
<td>80</td>
<td>0.5</td>
<td>67</td>
<td>80</td>
</tr>
</tbody>
</table>

Note: $\beta = 0.5, \delta = 0.8$ and $\sigma = 0.015$ for $h = i, j$. The minimum provision level is reached at $g_s = 50$ for $y_i = 70, y_j = 130$, and at $g_s = 67$ for $y_i = y_j = 100$. In case of the maximization of a social welfare function $S = \max[V_j]$ for $\alpha^*_i = \alpha^*_j = 1$, or $S = U_i/U_j$, the optimum is 100.
As defined in Section 4.1, with differences in preferences: 

For a wide range of utility functions (e.g., CES functions with elasticity of substitution $\gamma$, and differences in preferences are with regard to, respectively, $x_i$ and $x_j$. When $\alpha_{ij}=0$, in equilibrium individuals $i$ and $j$ choose points denoted by (1) and (2) in the $(x, g)$-plane, such that: 

$$(\alpha U_i/\alpha g)/(\alpha U_i/\alpha x_i)_{(1)} = (\alpha U_i/\alpha g)/(\alpha U_i/\alpha x_i)_{(2)}.$$  

We first consider differences in preferences. Of course, $g$ is equal for both individuals and $x_i > x_j$. By setting $\alpha_{ij} = \alpha_i = \alpha$, where $\alpha$ is marginally larger than zero, we can examine the change of the slopes of $i$ and $j$’s indifference curves through the points (1) and (2), when $\alpha_{ij}$ and $\alpha_j$ increase from zero. For positive $\alpha$ the slopes are with regard to, respectively, $i$ and $j$ in absolute value:

$$s_i = [(\alpha U_i/\alpha g)/(\alpha U_i/\alpha x_i)]_{(1)} + \alpha [(\alpha U_i/\alpha g)/(\alpha U_i/\alpha x_i)]_{(2)},$$  

$$s_j = [(\alpha U_i/\alpha g)/(\alpha U_i/\alpha x_j)]_{(2)} + \alpha [(\alpha U_i/\alpha g)/(\alpha U_i/\alpha x_j)]_{(1)}.$$  

For a wide range of utility functions (e.g., CES functions with elasticity of substitution $\gamma = -1$, and thus also a Cobb–Douglas function): 

$$(\alpha U_i/\alpha g)/(\alpha U_i/\alpha x_i)_k = (\alpha U_i/\alpha g)/(\alpha U_i/\alpha x_i)_k$$  

This implies: 

$$s_i \leq s_i(1) = [(\alpha U_i/\alpha g)/(\alpha U_i/\alpha x_i)]_{(1)} + \alpha [(\alpha U_i/\alpha g)/(\alpha U_i/\alpha x_i)]_{(2)}$$  

$$s_j \leq s_j(2) = [(\alpha U_i/\alpha g)/(\alpha U_i/\alpha x_j)]_{(2)} + \alpha [(\alpha U_i/\alpha g)/(\alpha U_i/\alpha x_j)]_{(1)}.$$  

As defined in Section 4.1, with differences in preferences: 

$$(\alpha U_i/\alpha g)/(\alpha U_i/\alpha x_i)_k < (\alpha U_i/\alpha g)/(\alpha U_i/\alpha x_j)_k.$$  

Proofs of parts (b) and (c) of proposition 2

The conclusions as to $\varepsilon=1$ remain to be proven: when preferences or incomes differ such that $i$ has a lower demand for the public good, then: $g_i^b < g_j^b$, and $\alpha_{ij} = -\alpha_{ji} > 0$. $g_j^b > g_j^b$, $g_j^b > g_j^b$, and $g_j^b > g_j^b$. We will examine whether $g_i^b < g_j^b$. In the benchmark case $\alpha_{ij}$ and $\alpha_{ji}$ are both zero. Change of the $\alpha$’s is always symmetrical. Thus, it must be proven that: $\partial g_i/\partial x_i - \partial g_j/\partial x_j \geq 0$ for $\alpha_{ij} = \alpha_{ji} = 0$ and for $\alpha_{ij} = \alpha_{ji} = -\delta$, $0 < \delta < \delta_i$, where $\delta_i$ follows from the requirement $g_j > g_i$. It is assumed that an interior solution exists. When $\alpha_{ij} = \alpha_{ji} = 0$, in equilibrium individuals $i$ and $j$ choose points denoted by (1) and (2) in the $(x, g)$-plane, such that: 

$$(\alpha U_i/\alpha g)/(\alpha U_i/\alpha x_i)_{(1)} = (\alpha U_i/\alpha g)/(\alpha U_i/\alpha x_i)_{(2)}.$$  

We will examine whether

Thus: 

$s_i(1) > (1 + \alpha) [(\alpha U_i/\alpha g)/(\alpha U_i/\alpha x_i)]_{(1)}$.
and $s_j(2) < (1 + \alpha)[(\partial U_i/\partial g)/(\partial U_i/\partial x_j)]_{(2)}$.

As the r.h.s. of both inequalities are equal, $s_j(1) > s_j(2)$ and thus also $s_i > s_j$.

Consequently, the slope $dx/dg$ of $i$'s new indifference curve through (1) is steeper than the slope of $j$'s new indifference curve through (2). As also $i$'s effective income $y_i + g_i$ is larger than $j$'s effective income $y_j + g_j$, it must necessarily hold that $\partial g_i/\partial \alpha_i > \partial g_j/\partial \alpha_j$. And, using the results of Section 2, $\partial g_i/\partial \alpha_i > \partial g_j/\partial \alpha_j$, as it can be shown that the difference between $\partial g_i/\partial g_i$ and $\partial g_j/\partial g_j$ works in the same direction. For $\alpha_i = \delta$ and $\alpha_j = -\delta$ with $\delta$ marginally different from zero, we can repeat this analysis. This is left to the reader (see, however, below for the case of differences in income).

With differences in incomes $(y_i < y_j)$, the indifference curves of $i$ and $j$ are identical, when $\alpha_j = \alpha_i = 0$ and also when $\alpha_j = \alpha_i = \alpha$. In the case $\alpha_j = \alpha_i = 0$, in equilibrium $x_i = x_j$ and thus $y_i + g_i = y_j + g_j$. This implies that at $\alpha = 0$ $\partial g_i/\partial \alpha = \partial g_j/\partial \alpha$. When $\alpha_i = \delta$ and $\alpha_j = -\delta$, $i$ and $j$ choose points in the $(x, g)$-plane denoted again as (1) and (2), such that:

$$s_i' = s_j'$$

where:

$$s_i' = [(\partial U_i/\partial g)/(\partial U_i/\partial x_i)]_{(1)} + \delta[(\partial U_i/\partial g)/U_i]_{(2)}/[(\partial U_i/\partial x_i)/U_i]_{(1)}$$

$$s_j' = [(\partial U_i/\partial g)/(\partial U_i/\partial x_j)]_{(2)} - \delta[(\partial U_i/\partial g)/U_i]_{(1)}/[(\partial U_i/\partial x_j)/U_i]_{(2)}$$

Now, in equilibrium $x_i = x_j$. It may also be noted that $[(\partial U_i/\partial g)/(\partial U_i/\partial x_i)]_{(1)} < [(\partial U_i/\partial g)/(\partial U_i/\partial x_j)]_{(2)}$. Increasing $\alpha_j$ and $\alpha_i$ marginally with $\epsilon$, the slopes of the indifference curves become:

$$s_i = s_i' + \epsilon[(\partial U_i/\partial g)/U_i]_{(2)}/[(\partial U_i/\partial x_i)/U_i]_{(1)}$$

$$s_j = s_j' + \epsilon[(\partial U_i/\partial g)/U_i]_{(1)}/[(\partial U_i/\partial x_j)/U_i]_{(2)}$$

As $x_i = x_j$ and (direct) preferences are equal,

$$[(\partial U_i/\partial g)/U_i]_{(2)}/[(\partial U_i/\partial x_i)/U_i]_{(1)} \leq [(\partial U_i/\partial g)/U_i]_{(1)}/[(\partial U_i/\partial x_i)/U_i]_{(1)}$$

$$= [(\partial U_i/\partial g)/U_i]_{(1)}/[(\partial U_i/\partial x_i)/U_i]_{(1)}$$

$$[(\partial U_i/\partial g)/U_i]_{(1)}/[(\partial U_i/\partial x_i)/U_i]_{(2)} \geq [(\partial U_i/\partial g)/U_i]_{(2)}/[(\partial U_i/\partial x_i)/U_i]_{(2)}$$

And thus $s_i < s_j$, as the extreme r.h.s. for $i$ is smaller than that for $j$. Consequently, $dx/dg$ rotates less than $dx_j/dg$, and, as $x_i < x_j$ implies that also $y_i + g_i < y_j + g_j$, $\partial g_i/\partial \alpha_i < \partial g_j/\partial \alpha_j$. Again it follows that also $\partial g_i/\partial \alpha_i < \partial g_j/\partial \alpha_j$. This procedure can again be repeated.
Appendix C

Proof of proposition 4

Part (a): we define i’s equilibrium private contribution in the absence of public provision \( g_i^e = 0 \) as: \( g_i^{e*} = g_i(\alpha_{ij}^*, \alpha_{ji}^*) \), and with public provision as: \( g_i^{e*} = g_i(\alpha_{ij}^*, \alpha_{ji}^*) - 0.5g_j \geq 0 \). We first examine \( g_i^{e*} > 0 \) and \( g_j^{e*} > 0 \). As follows from proposition 3(a), \( \alpha_{ij}^* < \alpha_{ij}^e \) and \( \alpha_{ji}^* < \alpha_{ji}^e \) and thus \( g_i(\alpha_{ij}^*, \alpha_{ji}^*) < g_i(\alpha_{ij}^e, \alpha_{ji}^e) \). Consequently, \( g_i^{e*} < g_i^e - 0.5g_j \). A similar result holds for \( g_j^{e*} \), and thus: \( g_i^{e*} + g_j^{e*} < g_i^e + g_j^e - g_s^e \). Thus: \( g_i^{e*} + g_j^{e*} < g_i^e \). Thus we have proven that in the new equilibrium with public provision, when private contributions are positive, the total provision level is lower than before. We will now investigate what happens, when one or both contributions are reduced to zero. When individuals are identical with respect to preferences and income, \( g_i^{e*} \) and \( g_j^{e*} \) are reduced to zero by the same \( g_s^e \). When \( g_i^e \) is further increased, private contributions remain of course zero, and total provision increases. Thus, increasing \( g_s^e \) from zero, first results in a decline of total provision. Only, when both private contributions are reduced to zero, total provision starts to increase. Total provision is at a minimum, when both contributions are just reduced to zero.

When individuals differ (assuming again \( \partial U_i / \partial g_i / \partial U_j / \partial g_j / \partial x_i \) or \( y_i < y_j \), we know that \( g_i^e < g_j^e \). This is also necessarily the case when there is public provision and the tax burden is shared equally \( (\tau = 0.5) \): \( g_i^{e*} < g_j^{e*} \). Thus, as \( g_s^e \) is increased, \( g_i \) is reduced to zero before \( g_j \). Using Eq. (4), we find (suppressing \( y_i \) and \( y_j \)): \( g_i^{e*} = g_i(\tau g_j, \alpha_{ji}^e) - (1 - \tau)g_j \). Total provision is: \( g_i^{e*} = g_i(\tau g_j, \alpha_{ji}^e) + \tau g_j \), and: \( \partial g_i^{e*} / \partial g_j = \tau \partial g_i^e / \partial g_j + \tau \). Note that \( -1 < \partial g_i^e / \partial g_j < 0 \). Thus, an increase of \( g_j^e \) increases the total provision level. Consequently, when \( g_j^e \) is increased from zero, total provision drops until \( g_i \) is zero. When \( g_j^e \) is further increased, total provision increases.

Part (b): Using proposition 1(c) while in case of differences in income holding total income constant and in case of differences in preferences considering equidistant preferences (see footnote 9), \( \alpha_{ij}^e \) (equal) < \( \alpha_{ij}^j \) (unequal) and \( \alpha_{ji}^j \) (equal) > \( \alpha_{ji}^j \) (unequal). And \( g_i^e < g_j^e \). The inequality regarding \( \alpha_{ij}^j \) implies: \( (1 - \varepsilon)g_j^e \) (equal) < \( g_j^e \) (unequal) < \( \varepsilon g_j^e \) (unequal), and the inequality regarding \( \alpha_{ji}^j \): \( (1 - \varepsilon)g_j^e \) (equal) < \( g_j^e \) (unequal) < \( \varepsilon g_j^e \) (unequal). We will first examine the extreme cases with regard to \( \varepsilon \). When \( \varepsilon = 0 \), the inequalities reduce to: \( g_j^e \) (equal) < \( g_j^e \) (unequal) and \( g_i^e \) (equal) > \( g_i^e \) (unequal), which implies that \( g_i^e \) (unequal) is reduced to zero before \( g_j^e \) (equal) when increasing public provision. This result holds for differences in income as well as preferences.

When \( \varepsilon = 1 \), both inequalities reduce to: \( g_j^e \) (unequal) > \( g_i^e \) (unequal), which does not allow direct conclusions. We know, however, that: \( g^e \) (equal) = \( g^b \) (equal), and in case of differences in income: \( g^b \) (equal) = \( g^s \) (unequal) and from the proof in Appendix B: \( g^s \) (unequal) < \( g^s \) (unequal). Thus: \( g^e \) (equal) > \( g^s \) (unequal). And, as
\( g_i^*(\text{unequal}) > g_i^*(\text{equal}) \) and of course \( g_j^*(\text{equal}) = g_j^*(\text{equal}) > g_j^*(\text{equal}) \). This implies that in case of differences in income \( g_i^*(\text{unequal}) \) is again reduced to zero before \( g_i^*(\text{equal}) \) when increasing public provision. This result now obviously holds also for the general case \( 0<\varepsilon<1 \).

In case of equidistant preferences as defined earlier (still for \( \varepsilon = 1 \)), it can be easily shown that \( g_i^b(\text{equal}) > g_i^b(\text{unequal}) \). As \( g_i^*(\text{equal}) = g_i^b(\text{equal}) \) and from the proof in Appendix B \( g_i^*(\text{unequal}) > g_i^b(\text{unequal}) \), we can not reach immediate conclusions as to \( g_i^*(\text{equal}) < g_i^*(\text{unequal}) \). By considering again an extreme case, results can be reached. The smaller the weakening of relations, the higher \( \alpha_{ij}^* \) and \( g_i^* \) are. In the extreme case, when relations do not weaken at all over time, \( g_i^*(\text{unequal}) = g_i^*(\text{unequal}) \). This implies that \( i \) and \( j \) maximize the same extended utility function. By substituting the definitions of equidistant preferences in the optimality conditions of \( i \) and \( j \) and considering extreme cases regarding the allocation of the difference between the marginal rates of substitution of \( i \) and \( j \) over \( (\partial U_i/\partial x_i)/U_i \) and \( (\partial U_i/\partial g_i)/U_i \) for \( h = i, j \), it can be shown that:

- in all situations with regard to the allocation over \( x \) and \( g \), \( i \)'s equilibrium contribution in case of different preferences is equal or close to his equilibrium contribution in case of equal preferences;
- when both are not equal, \( g_i^*(\text{unequal}) \) may be larger or smaller than \( g_i^*(\text{equal}) \), depending on the allocation over \( x \) and \( g \).

To summarize the results for differing preferences, only when \( \varepsilon \) is close to 1 and the rate of decay is close to zero, \( i \)'s equilibrium contribution in case of different preferences may be (somewhat) larger than (or equal to) his equilibrium contribution in case of equal preferences. In all other cases \( g_i^*(\text{unequal}) \) is smaller than \( g_i^*(\text{equal}) \), and, consequently, \( g_i^*(\text{unequal}) \) is again reduced to zero before \( g_i^*(\text{equal}) \), when public provision is increased.

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