Social insurance and the optimum piecewise linear income tax

Michel Strawczynski
Bank of Israel, Research Department, P.O.B. 780, Jerusalem 91007, Israel

Received 30 December 1996; received in revised form 30 July 1997; accepted 8 December 1997

Abstract

This paper calculates optimal linear income taxes when differences in income are caused by random factors ("luck") rather than by unobserved individual skills, as assumed in the classical theory of income taxation. As first shown by Varian (1980) [Varian, H.R., 1980. Redistributional taxation as social insurance, Journal of Public Economics 14, 49–68] in the former case income taxation acts as social insurance. By introducing life uncertainty and precautionary behavior, we find higher optimal marginal tax rates than those found by Varian. We also find that—in the context of a piecewise two-bracket linear tax schedule—the second marginal tax is higher than the first, a finding that contrasts with results recently obtained in the framework of classical income taxation theory, which show a lower second marginal tax. © 1998 Elsevier Science S.A.

Keywords: Linear income tax; Precautionary behavior; Life uncertainty

JEL classification: H21

1. Introduction

Following Mirrlees (1971), the main stream of the literature on optimum income taxation ascribes differences in income to unobserved differences in individual abilities. In his seminal paper, Mirrlees showed that using plausible parameters for income distribution in the economy and assuming a Benthamite social welfare function,1 the optimum income tax is approximately linear, and the

1The Benthamite ("utilitarian") social welfare function is the sum of individual utilities. Mirrlees (1971) shows results for the transformation $W=e^{-U}$ as well, where $U$ represents individual utility.
marginal tax rate is low (approximately 20%). These findings led a number of authors to check the sensitivity of the results to different assumptions underlying in Mirrlees’ analysis. Atkinson (1973) found higher optimal marginal tax rates using alternative specifications of the social welfare utility function. Stern (1976) showed that the optimum marginal tax rate is substantially increased when making realistic assumptions about the elasticity of substitution between leisure and consumption. Sheshinski (1989) found slightly higher marginal tax rates for a linear system, while in the framework of a two-bracket system the range of marginal rates was similar to the one found by Mirrlees.

More recently, Slemrod et al. (1994) found a wide range of optimal marginal tax rates (for both linear and two-bracket systems) as a function of the elasticity of substitution, the degree of inequality aversion of the social planner, and the revenue requirements of the system. A remarkable result of their paper is that assuming the same income distribution as Mirrlees, their simulations show that in a two-bracket system the optimal second marginal tax rate is lower than the first. Consequently, the usual progressivity feature of rising marginal tax rates is replaced by a softer one, which considers rising average tax rates.

An alternative way of checking the sensitivity of these results to the underlying assumptions of the analysis is to compute optimal marginal rates according to alternative theories of income formation. One of the main candidates for this kind of examination is the approach of Varian (1980) to the income formation process, which states that differences in income are mainly due to unobservable random factors, which may be called ‘luck’. According to this view, the desirability of the income tax rests on its social insurance characteristics; these are obtained, for example, by providing an equal demogrant to all individuals, as in the case of the linear income tax. If we perform this examination by looking at Varian’s results on optimal linear marginal tax rates (Varian, 1980, p. 57, table 1) we find that his

---

2 Tax rates are sensitive to the assumption on the required revenue of the government. In his simulations Mirrlees assumed a low revenue requirement of up to 9% (i.e., his figures are not far from a purely redistributive system)

3 Declining marginal tax rates at high income levels were obtained also in simulations by Kanbur and Tuomala (1994), for a non-linear income tax. Note that these results are not general. Sadka (1976) shows that it is optimal to set a zero marginal tax rate for the individual with the highest ability. Since a reduction of the marginal tax from a positive rate to nearly zero increases the economic activity of this individual, his utility is clearly enhanced (otherwise he would have not chosen to work more) and we obtain a Pareto improvement; moreover, an infinitesimal marginal tax rate would increase tax revenue, allowing for further resources to be distributed among low-income agents. However, it is not clear whether marginal taxes should start to fall before reaching the top of the income distribution. Diamond (1996) shows an example where optimal marginal taxes rise until the top of the distribution of skills, and only at the top itself should it be reduced to zero.

4 Tuomala (1984), (1990) provides an interesting extension by adding luck to the classical model of abilities (his results for the linear case are shown in Appendix B). However, he leaves open the question of the sensitivity of optimal tax rates when income is formed by the interaction of savings (investment) and luck.
simulations support *low* tax rates that are quantitatively close to those found by Mirrlees. If we consider a coefficient of variation of 0.6 as empirically plausible, the optimum tax rate computed using Varian’s model is 25%; i.e., not far from the 20% tax rate found by Mirrlees. This finding raises the question of the sensitivity of Varian’s results to the underlying assumptions of his own analysis, most notably two assumptions that seem to affect the results of his simulations: (i) complete certainty about the length of life, and (ii) the use of a quadratic utility function, which implies increasing absolute risk-aversion. The latter assumption has been seriously challenged by the modern theory of insurance, since it implies that an individual’s aversion to a fixed bet increases with income. Moreover, since marginal utility is linear under a quadratic utility function, the consumer does not exhibit precautionary behavior, an element that would clearly affect the desirability of income taxation.

The paper is organized as follows: Section 2 introduces the setup for the optimum linear income tax schedule. Sections 3 and 4 characterize the effects of life uncertainty and precautionary behavior on computed optimal linear tax rates. Section 5 characterizes the optimum piecewise linear system using a two-bracket schedule. Section 6 summarizes and concludes the paper.

2. An optimum linear income tax with life uncertainty

Each agent solves the following problem:

$$\text{MAX}_{x_i} u(w_i - x_i) + p E u[c(x_i + \epsilon)]$$

where \(u\) is a utility function (\(u' > 0, u'' < 0\)), \(w_i\) is person \(i\)’s income in the active period of life, \(p_i\) (0 < \(p_i\) ≤ 1) is the probability of his survival to the second and last period of life, \(E\) is the expectation operator, defined on uncertain second-period income \(x_i + \epsilon\) (where \(x_i\) are the savings at the end of the first period and \(\epsilon\) is a non-observable random shock), and \(c\) represents consumption.

Since one of the main aims of this paper is to compare its results with those obtained by Varian (1980), we will start by assuming that individuals are identical ex-ante, i.e., that \(w_i = w, \epsilon_i = \epsilon\) and \(p_i = p\) for all \(i\). Extensions to the general case

\(^5\)Varian’s table 1 shows the sensitivity of the results to different levels of the random component of income. Calculating the coefficients of variation for each case (as a percentage of the expected second-period income, \(x\)) we obtain that the empirically plausible range is between \(n = 0.2\) and \(n = 0.3\) (where the coefficients of variation are 0.44 and 0.73, respectively). Assuming that a coefficient of variation of 0.6 represents a benchmark for empirical purposes (see, e.g., Barsky, Mankiw and Zeldes, 1986), we calculated the optimum tax using Varian’s methodology for \(n = 0.25\), which corresponds to a coefficient of variation of 0.58.

\(^6\)Precautionary behavior has become a standard element of consumption analysis both on the micro and macro level; see Hubbard, Skinner and Zeldes (1994).
(where \( w_i \) is different for each \( i \)) and some of the implications for the results are presented in Section 5 and in Appendix A.

If we could actually observe luck (\( e \)) occurring, the optimal policy of the government ex-ante would be a 100% tax rate on luck. But we cannot observe luck – we can only observe actual second-period income, \( x + e \). Note also that the definition of the problem implies that \( Ec(x + e) = x \), and since individuals are egoistic they consume all the available income in the second period. Moreover, the consumption function \( c \) is linear in the proposed model, since we consider only the case of a linear income tax. The F.O.C. for an individual is:

\[
u'(w - x) = pE[u(c(x + e))c'(x + e)],
\]

It is assumed that the government collects a linear income tax from all individuals, and its revenues are distributed only to individuals who are alive in the second period:

\[
T = Et; \quad S = pD; \quad S = T
\]

where \( T \) represents taxes collected, \( S \) is the benefit distributed to consumers, \( t \) is the tax rate, \( y \) is the income subject to taxation (in our case, savings, \( x \)) and \( D \) is the demogrant given to consumers. As shown in Eq. (3), the government budget must be balanced, and therefore the demogrant equals \( tx/p \). By substituting the government budget equation in the individual maximization problem, the government’s problem is how to choose the optimum tax rate \( t \) such that:

\[
\text{MAX} \, v(t) = u[w - x^*(t)] + pE[u(c[x^*(t) + e] + tx^*(t)/p)],
\]

where \( x^*(t) \) is the optimum amount of savings chosen by the individual [by using Eq. (2)]. The F.O.C. for the government’s problem is:

\[
v'(t) = -u'(w - x)x'(t) + pE[u(c(x(t) + e) - (1 - t)x(t)(1 - p)](1 - t)x'(t) + x + e + tx'/p - x = 0.
\]

By substituting the first term according to the F.O.C. at the individual level and re-arranging terms we can re-write Eq. (5) as follows:

\[\text{In a sense, there is an annuity component in government intervention, since the government returns all the collected taxes back to the consumers, including taxes collected from individuals who died in the first period. But the annuity component does not have the effect of annuities in the usual sense, since it is used to increase the demogrant (i.e., it does not imply an increase in the available rate of return of savings as in the usual case considered for annuities, which are a function of the amount of savings, \( x \)). For the effect of annuities in the usual sense see Abel (1986).}

\[\text{Note that under a linear one-bracket income tax, } c[x(t) + e] = (1 - t)(x + e) + D, \text{ where } D = tx(t)/p.\]
Before interpreting government’s first-order condition Eq. (6), note that Eq. (11) in Varian (1980) is a particular case of this equation, for \( p = 1 \). The first term represents the ‘efficiency’ effect (i.e., the effect on savings), while the second term represents the ‘insurance’ effect (i.e., changes in the exposure of the consumer to risk). Using the same procedure as the one used by Varian, we interpret this F.O.C. by assuming a small decrease in the tax rate, \( \Delta t \). The first term increases by \( \Delta x(t) \left[ x(t) + tx(t) \right] \left[ 1 - \left( \frac{1 - p}{p} \right) \right] \epsilon / \left( 1 - t \right) \), an increase caused by the effect of the tax-cut on savings, multiplied by the marginal utility. Life uncertainty affects the demogrant, which is one of the elements of the argument of marginal utility; this argument includes savings, \( x \), the ‘net’ (after tax) income shock, \( (1 - t)\epsilon \), and the increase in wealth through the demogrant, \( tx(t)(1 - p)/p \), which takes place since the tax payment of the \( (1 - p) \) non-survivors is delivered to the \( p \) survivors. Assuming that marginal utility is decreasing, as \( p \) tends to zero, the importance of the efficiency effect diminishes since the tax-insurance system is providing the survivors with a high demogrant.

The insurance effect (second term) is related to the fact that instead of facing a risk of \( (1 - t)\epsilon \) the consumer now faces an increased risk of \( [1 - (t + \Delta t)]\epsilon \). The insurance effect is therefore:

\[
p[\epsilon - [(1 - p)/p][x(t)(1 - t) + x(t)]] [\epsilon - [(1 - p)/p][x(t)(1 - t) + x(t)]] Eu'[x(t) + [(1 - p)/p]tx(t) + (1 - t)\epsilon] \Delta t.
\]

First, note that the effect of life uncertainty on the argument of marginal utility is the same as the one described above for the efficiency term. The new element is given by the first expression, \( p[\epsilon - [(1 - p)/p][x(t)(1 - t) + x(t)]] \), which equals \( \epsilon \) when \( p = 1 \) (as in Varian, 1980). The second term in curly brackets reflects an efficiency effect through the demogrant, and thus it is weighted by the ratio of non-survivors to survivors, \( (1 - p)/p \). As \( p \) tends to zero, the whole expression tends to zero, since in this case the consumer does not survive and hence does not enjoy from insurance. Clearly, the interesting case is obtained for the range of values \( 0 < p < 1 \). Within this range, it is not clear what is the effect of income uncertainty on the results. On one hand, the lower is \( p \), the higher is the demogrant received by survivors; but on the other hand, the lower is \( p \), the lower is the probability of enjoying the insurance. The conclusion is that the introduction of life uncertainty does not imply a clear-cut conclusion on the optimal level of the system, and whether life uncertainty increases or decreases optimal taxes must be tested by running calibrated simulations, with realistic parameters. Note also from the F.O.C. shown in Eq. (6), that the strength of the insurance effect is a function of the third derivative of the utility function. This is so, because the impact of the
change $\Delta t$ on the insurance effect depends on the magnitude of the change in marginal utility (i.e., it depends on the convexity of the marginal utility). As well known, convexity of the marginal utility implies a positive third derivative of the utility function (as in the logarithmic utility function, and unlike the quadratic utility function, where marginal utility is linear).

In order to isolate the importance of life uncertainty and precautionary behavior we proceed in the following two sections by comparing the results of calibrated simulations with Varian’s benchmark: (i) we calculate optimal taxes for a quadratic utility function (as in Varian, 1980) under life uncertainty, and (ii) we calculate optimal taxes with life certainty (as in Varian) but with a logarithmic utility function.

3. The effect of life uncertainty on the optimum linear marginal tax

Assume a quadratic utility function:
\[ u = -\frac{c^2}{2} + bc, \]  
(7)
where $c$ is consumption and $b$ is a parameter to be calibrated in the simulation. The F.O.C. is:
\[ -[b - (w - x)] + \frac{p(1 - t)}{2} \left[ b - \left( 1 - t \right)(x + n) + \frac{tx}{p} \right] + b - \left( 1 - t \right)(x - n) + \frac{tx}{p} = 0, \]  
(8)
where $n$ is the income shock which takes a positive value $(+n)$ with probability 0.5 and a negative value $(-n)$ with probability 0.5. After some manipulation the following formulae for $x(t)$ and $x'(t)$ are obtained:
\[ x(t) = \frac{b[p(1 - t) - 1] + w}{1 + p(1 - t) - t(1 - p)/p}, \]  
\[ x'(t) = \frac{(2b - w)p + b[2(1 - p)t - (1 - p)^2/p] + w(1 - p)/p}{[1 + p(1 - t) - t(1 - p)/p]^2}. \]  
(9)

Using these results and the first-order condition Eq. (5) (see Appendix B.2), the formula for the optimal tax $t$ is a function of the following parameters:
\[ t = t(p, n, w, b). \]  
(10)

*For a graphical explanation of this point, see Strawczynski (1994, p. 490).
As in Varian, increases in \( n \) (risk) and \( w \) (income) and a decrease in \( b \) (which corresponds to the case of an increase in risk aversion) result in an increase of the optimum tax rate. The new element is related to \( p \); the higher \( p \) (i.e., the higher the probability of survival), two opposite effects are at work: on the one hand, the greater is the adverse effect of the tax, since marginal utility of savings is weighted by a higher probability of survival; on the other hand, if he survives, the tax-transfer system provides insurance. Since it is not clear which of these effects dominates, it is important to choose parameters that are in line with existing evidence in order to assess the importance of these effects by means of a calibrated simulation.

Table 1 shows the results\(^{11}\) of a simulation for different values of \( n \), the income risk. The range of \( n \) values was chosen so as to correspond to empirically accepted coefficients of variation. The choice of \( p \) is related to the limits of economic decisions concerning active and passive periods of life. Assuming that retirement age is 65 and that life expectancy is 75,\(^{12}\) and using an actuarial table to calculate the probability of survival,\(^{13}\) we estimate that \( p = 0.8 \). For the purpose of comparison, we also present the optimum tax computed using Varian’s methodology, which corresponds to the case \( p = 1 \). As in Varian, we assume that \( w = b = 1 \), implying that when \( t = 1 \), the optimum amount of savings \( x \), equals zero.

According to Table 1, the optimum tax under life uncertainty is higher than the

<table>
<thead>
<tr>
<th>( n )</th>
<th>( x )</th>
<th>Coefficient of variation</th>
<th>( p = 0.8 )</th>
<th>( p = 1 )</th>
<th>( t^* - t^{**} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p = 0.8 )</td>
<td>( p = 1 )</td>
<td>( )</td>
<td>( t^* )</td>
<td>( D^*/x )</td>
<td>( t^{**} )</td>
</tr>
<tr>
<td>0.1</td>
<td>0.39</td>
<td>0.48</td>
<td>0.26</td>
<td>0.21</td>
<td>0.26</td>
</tr>
<tr>
<td>0.2</td>
<td>0.38</td>
<td>0.45</td>
<td>0.53</td>
<td>0.44</td>
<td>0.30</td>
</tr>
<tr>
<td>0.22</td>
<td>0.37</td>
<td>0.44</td>
<td>0.59</td>
<td>0.50</td>
<td>0.31</td>
</tr>
<tr>
<td>0.25</td>
<td>0.37</td>
<td>0.43</td>
<td>0.67</td>
<td>0.58</td>
<td>0.32</td>
</tr>
<tr>
<td>0.275</td>
<td>0.37</td>
<td>0.42</td>
<td>0.75</td>
<td>0.66</td>
<td>0.34</td>
</tr>
<tr>
<td>0.3</td>
<td>0.36</td>
<td>0.41</td>
<td>0.83</td>
<td>0.73</td>
<td>0.35</td>
</tr>
</tbody>
</table>

\(^{10}\)This result is specific to the quadratic utility function, which implies increasing absolute risk aversion. Thus, the higher the wage, the more averse is the consumer to a given bet and consequently he prefers a higher tax (demogrant).

\(^{11}\)When not explicitly stated, optimal marginal tax rates in all the simulations were found on the basis of a grid search between zero and one in intervals of 0.01.

\(^{12}\)This figure corresponds to the category with a high life expectancy, which is also the relevant category for developed countries (The World Bank Atlas, 1992).

\(^{13}\)For this purpose we used a table for 1980, which appeared in Hubbard, Skinner and Zeldes (1994).
tax under life certainty by a percentage that declines as the income risk increases. The reason for this result is that under increasing absolute risk aversion the relative desirability of income taxation rises as a function of the size of the risk. When \( n = 0.25 \) (which implies coefficients of variations of 0.58 and 0.67 for \( p = 0.8 \) and \( p = 1 \), respectively) is taken as the benchmark, we find that life uncertainty raises the optimum tax rate and the share of the demogrant in savings by 7 and 15 percentage points, respectively.

4. The effect of precautionary behavior on the optimum linear marginal tax

Assume now that the utility function is \( u = \ln c \). The first-order condition in this case is:

\[
\frac{1}{w} = \frac{p(1-t)}{2} \left[ \frac{1}{x + (1-t)n} + \frac{1}{x - (1-t)n} \right].
\] (11)

Eq. (12) shows the formula for \( x \), which is obtained after solving a second-order equation:

\[
x = \frac{w(1-t)p + \sqrt{p^2 w^2 (1-t)^2 + 4[1 + (1-t)p](1-t)^2 n^2}}{2[1 + (1-t)p]}.
\] (12)

Note that when \( t = 1 \) (a 100% tax), the individual chooses not to save. Note, too, that \( x \) rises as \( p \) rises,\(^{15}\) showing that the individual is willing to save for second-period consumption.

From Eq. (11) we can calculate \( x'(t) \):

\[
x'(t) = \frac{wp + 2p^2 w^2 (1-t)^2 - 12p(1-t)^2 n^2 - 8(1-t)n^2}{\sqrt{(1-t)^2[p^2 w^2 4n^2 - 4p(1-t)n^2]}}
\]

\[\times \left[ 1 + (1-t)p \right] - 2p[w(1-t)p
\]

\[+ \sqrt{p^2 w^2 (1-t)^2 + 4[1 + (1-t)p](1-t)^2 n^2}.\]

Finally we use first-order condition Eq. (6) to compute an equation for the optimum tax rate:

\(^{14}\)For simplicity we explicitly neglect in this section the annuity effect of the demogrant (in the simulations we will consider only the case \( p = 1 \)).

\(^{15}\)The derivative of \( x \) with respect to \( p \) is equal to \( \frac{\left[2w(1-t)+[1+(1-t)p]\left[2pw^2(1-t)^2]\right]'}{(p^2 w^2 (1-t)^2 + 4[1 + (1-t)p](1-t)^2 n^2)^{3/2}} / \left[4[1 + (1-t)p]^2\right].\)
Table 2
Logarithmic utility function and life certainty

<table>
<thead>
<tr>
<th>n</th>
<th>x</th>
<th>Coefficient of variation</th>
<th>t* (log)</th>
<th>t** (quadr.)</th>
<th>t* - t**</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Log</td>
<td>Quadr.</td>
<td>Log</td>
</tr>
<tr>
<td>0.1</td>
<td>0.50</td>
<td>0.48</td>
<td>0.20</td>
<td>0.21</td>
<td>0.04</td>
</tr>
<tr>
<td>0.2</td>
<td>0.47</td>
<td>0.45</td>
<td>0.43</td>
<td>0.44</td>
<td>0.22</td>
</tr>
<tr>
<td>0.22</td>
<td>0.45</td>
<td>0.44</td>
<td>0.49</td>
<td>0.50</td>
<td>0.29</td>
</tr>
<tr>
<td>0.25</td>
<td>0.39</td>
<td>0.43</td>
<td>0.65</td>
<td>0.58</td>
<td>0.45</td>
</tr>
<tr>
<td>0.252</td>
<td>0.38</td>
<td>0.43</td>
<td>0.67</td>
<td>0.59</td>
<td>0.47</td>
</tr>
<tr>
<td>0.255</td>
<td>0.36</td>
<td>0.43</td>
<td>0.70</td>
<td>0.60</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Note that with life certainty the optimal D/x is equal to the optimal tax.

\[
\frac{x(t)[1-p(1-t)]x'(t)}{1-t} = n^2. \tag{13}
\]

In order to isolate the effect of assuming a function with decreasing absolute risk aversion (logarithmic) instead of increasing absolute risk aversion (quadratic), we consider only the case of \( p = 1 \). Comparing the results using these two functions allows us to estimate the effect of precautionary behavior. The results of the simulation for different levels of income risk are shown in Table 2. Again, all simulations are performed for \( w = 1 \).

The results show that the effect of insurance rapidly increases as the size of income risk rises. The optimal tax rate rises from 30 to 50% for a range of coefficients of variation between 0.5 and 0.7. Note also that the difference between the optimal taxes in the two cases increases as income risk increases, a result that may be attributed to the fact that with precautionary behavior, the importance of insurance increases as a function of the income risk. Taking the case of \( n = 0.25 \) as a benchmark (as in Section 3), we find that the optimum tax rate for the logarithmic utility function is 20% higher than the one obtained by Varian. It is important to note that a logarithmic utility function implies a coefficient of relative risk aversion of 1. Higher coefficients of relative risk aversion clearly imply even higher optimal marginal tax rates.

5. An optimum piecewise linear schedule

In this section we design the simplest stylized setup for a two-bracket linear schedule. The main purpose of the analysis is to compare the results to those obtained in the framework of classical income taxation theory, and especially to see whether optimal taxes follow a rising pattern of marginal tax rates as in actual tax systems, or a declining pattern as in Slemrod et al. (1994). For analytical
convenience we first consider the case of luck as the only source of income, and then we characterize optimal taxes when both skills and luck are sources of income.

5.1. Luck as the only source of income

Assume that the income formation process emerges in the following four possible states of nature (each with probability 0.25):

I) \( y = x - \epsilon_2 \)

II) \( y = x - \epsilon_1 \)

III) \( y = x + \epsilon_1 \)

IV) \( y = x + \epsilon_2 \)

where \( y \) represents second-period resources, \( x \) first-period savings and \( \epsilon_1 \) and \( \epsilon_2 \) are fixed random shocks (\( \epsilon_2 > \epsilon_1 \)). Further assume that first-period income \( w \) is equal for all individuals, and consequently the only difference in second-period resources is caused by luck.\(^{16}\)

For simplicity, we assume in this sub-section that although the social planner cannot observe low occurrences of luck (\( \epsilon_1 \)), high occurrences of luck (\( \epsilon_2 - \epsilon_1 \)) are observable. The case where unobservable differences in income maybe due also to different levels of skill will be studied in the next sub-section and in Appendix A. Since by assumption there are two types of shocks, the income tax schedule in this setup will be defined as follows:

- On the first \( d \) dollars, \( 0 < d < x + \epsilon_1 \), the marginal tax rate is \( t_1 \).
- On the next \( (\epsilon_2 - \epsilon_1) \) dollars, the marginal tax rate is \( t_2 \).

Recalling the law of large numbers, we sort the individuals in the economy into four groups, according to the different idiosyncratic shocks. We further impose a restriction on the social planner, according to which he must set a common demogrant for all individuals, i.e., \( ET = S = 4D \) (where \( T \) are taxes collected, \( S \) are transfers, and \( D \) is the equal demogrant). The following is the expression for the demogrant:

\[
D = t_1 x + \frac{t_2 (\epsilon_2 - \epsilon_1)}{4}.
\]

Using this expression we can write after-tax second-period resources \( y_i^a \) for each state of nature \( i = 1, 2, 3, 4 \):

\(^{16}\)For simplicity we do not take into account the annuity effect on the demogrant (as in Section 4, we consider in the simulations only the case where \( p = 1 \)).
The problem of a utilitarian government is to set optimal tax rates \( t_i \) and \( t_{i+1} \) such that the representative individual ex-ante utility is maximized:

\[
\text{\text{MAX}}_{i=1,2} u(w - x) + \frac{p}{4}.
\]

Note that \( x \) is equal for all individuals, since we assume that ex-ante all individuals have the same first-period income, \( w \).

\textbf{Proposition:} In this setup, the second optimum marginal tax rate, \( t_2 \), for a utilitarian social planner is higher than \( t_1 \) and is equal to 100%.

\textbf{Proof:} The individual's first-order condition is:

\[
u'(w - x) = \frac{p}{4} \sum_{i=1}^{4} u'(y_i^\#),
\]

where \( u' \) is the derivative with respect to \( x \), and tax rates in Eq. (16) are adjusted to their optimum level. Note that \( y_i^\# = y_j^\# \) for all \( t_i \) and \( t_{i+1} \) (0 \( \leq t_i \leq 1, i = 1, 2)):

\[
y_i^\# = x + \epsilon_2 - \epsilon_i t_i - t_{i+1} \frac{3(\epsilon_2 - \epsilon_1)}{4} \geq y_i^\# = x + \epsilon_1 - t_i \epsilon_i + t_{i+1} \frac{3(\epsilon_2 - \epsilon_1)}{4}
\]

By contradiction, assume that the optimum \( t_2 \) equals \( k, 0 < k < 1 \). Clearly, an increase in \( k \) implies that \( y_2^\# \) goes down while \( y_1^\# \) and \( y_3^\# \) increase. Thus, an increase in \( k \) is welfare-improving, and consequently \( k \neq 1 \) is not an optimum, contradicting the initial hypothesis.

We now show that \( t_1 < t_2 \) (\( = 1 \)). By contradiction, assume that for the optimum \( x > 0 \), \( t_1 \) equals 1. Since in the present case \( x \) is a function of \( t_i \), by using Eq. (6) we show that the efficiency effect on savings is a function of \( t_1 \). It follows

\text{In Appendix C we analyze the case of a Rawlsian Social Planner.}
immediately that when $t_1 = 1$ optimal savings equal zero, which contradicts the initial hypothesis of $x > 0$.

This result contrasts with the findings of Slemrod et al. (1994), since optimal marginal tax rates rise with income. The intuition of the result runs as follows: if the only way to become rich, as in the present framework (i.e., achieving $y_n$), is to have good luck, government intervention through a well-defined two-bracket system succeeds in transferring resources from good states of nature to bad ones through the second marginal tax. Efficiency effects are confined to the first marginal tax rate, which is the only relevant marginal tax for saving decisions in the present context.

The upper part of Table 3 shows the results of a simulation for this case; optimal two-bracket tax rates are calculated for $n = 0.25$ and $n = 0.3$, assuming a quadratic utility function.\(^1\) Note that for $n_1 = n$ (see for example $n_1 = n = 0.2$) the figures for optimal savings are the same as those shown in Section 3. The reason is related to optimum behavior with a quadratic utility function: although the second marginal rate reduces the variance of second-period after-tax income, provided no precautionary behavior exists, the optimum amount of savings $x$ remains unchanged.\(^1\) The share of the demogrant in savings, is higher than in Section 3, because the existence of a second marginal tax allows for an improved insurance

<table>
<thead>
<tr>
<th>$n$</th>
<th>$x$</th>
<th>Coefficient of variation</th>
<th>$t_1^*$</th>
<th>$t_2^*$</th>
<th>$D^*/x$</th>
<th>Average marginal tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.47</td>
<td>0.53</td>
<td>0.15</td>
<td>1</td>
<td>0.26</td>
<td>0.36</td>
</tr>
<tr>
<td>0.3</td>
<td>0.45</td>
<td>0.67</td>
<td>0.22</td>
<td>1</td>
<td>0.33</td>
<td>0.42</td>
</tr>
</tbody>
</table>

\(^a\) In all simulations $p = 1$.

\(^b\) For $n = 0.25$ it was assumed that $n_1 = 0.15$ and $n_2 = 0.35$; for $n = 0.3$: $n_1 = 0.2$ and $n_2 = 0.4$.

\(^c\) $H$ represents high skill, $L$ low skill and $A$ average; $n$ is the conditional standard deviation. The basic case was calibrated according to a coefficient of variation of 0.53 ($w_1 = 0.5$, $w_2 = 1.8$, $n_1 = 0.2$ and $n_2 = 0.34$). The increase of the coefficient of variation from 0.53 to 0.67 was obtained by a proportional increase in risk for both skill levels. The grid search interval for this case is 0.05.

\(^1\) Clearly we would like to run a simulation with a function that exhibits precautionary behavior. Unfortunately the solution for the logarithmic case implies a fourth-order equation for $x(t)$, which is analytically untractable.

\(^2\) See, e.g. (Abel, 1988, p. 49).
mechanism. Note that when the case of $n = 0.25$ is taken as a benchmark – as in previous sections – the optimum average marginal tax rate is 36%, which is quite high in view of the fact that precautionary behavior is absent. These findings strengthen the ones obtained under the linear system, i.e., optimal marginal tax rates are higher than 25%, as computed by Varian.

However, the result of a 100% second tax rate derives straightforward from the assumption of observable luck as the only source of being millionaire. As Eaton and Rosen (1980) p. 358 point out, “if labour supply were completely exogenous, the individual would desire a 100% earnings tax, with the expected value of earnings returned as a lump sum”. Consequently, it is important to test whether the result of a higher second marginal tax rate holds also in a model where income is formed by a combination of both luck and skills. Eaton and Rosen (1980) found that the optimal structure of the tax system is affected in this case by the assumptions about inequality of skills and uncertainty. In the next sub-section we extend their results to the case of a two-bracket income tax.

5.2. Skills and luck as sources of income

As noted by Eaton and Rosen (1980); Slemrod et al. (1994) the results on optimal taxation depend on the given set of assumptions, for example, the utility function, the revenue requirement, the nature of the social planner, the extent of risk aversion and the extent of uncertainty and inequality.

Although we do not aim to perform a complete characterization of the optimal system by testing these assumptions, it is interesting to examine whether the result of a higher second marginal tax rate is valid in a framework where both luck and abilities are sources of income formation. Since it is not possible to obtain general results, we proceed by performing a calibrated simulation using a similar set of assumptions as in the previous sections.

For simplicity we still assume a four individuals economy with two realizations of luck ($+\epsilon$ and $-\epsilon$) which are applied respectively to two levels of skill, $w_1$ and $w_2$ ($w_1 < w_2$); i.e., the richest person in the economy will be high ability and lucky, while the poorest low ability and unlucky. In order to achieve simplicity in the design of the tax system, we assume that a low skill and lucky individual is poorer than a high skill and unlucky individual. This assumption allows for a separation of the savings decision as a function of skills, where the first individual faces the first marginal tax rate, $t_1$, and the second faces the second marginal tax rate, $t_2$.\footnote{This assumption does not mean that luck is an unimportant source of income. In fact, an additive component in the income shock may imply that the low level skill faces considerable uncertainty and luck may represent a substantial component of his ex-post income.}

The results are shown in the lower part of Table 3. The simulations were
calibrated so as to match the coefficients of variation of 0.53 and 0.67, as in the previous sub-section. The most interesting result is that in both cases the second marginal tax rate is significantly higher than the first one. Although the efficiency effect reduces marginal tax rates, both the insurance effect and the existence of inequality call for high average marginal tax rates, above 40%. Note also that the increase of uncertainty affects differentially the first and second tax rate; while the first tax rate increases and thus provides insurance against the negative shock to poorer individuals, the optimal second marginal tax rate goes down and thus allows for a higher level of savings by high skill individuals.

6. Summary and conclusions

This paper considers the sensitivity of the optimal marginal linear income tax when differences in income are caused by random factors rather than by unobserved skills, as in the classical theory of optimum income tax. We found that the optimum tax is higher than 25%, which is the figure obtained by Varian in the linear tax framework. For the benchmark parameters used in the simulation, we found that both life uncertainty and precautionary behavior substantially raise the optimum linear income tax. The effect of precautionary behavior would probably have been greater had we used more realistic assumptions about relative risk aversion.

A remarkable finding, which contrasts with the one recently obtained by Slemrod et al. in the context of the classical model of income taxation, is that for a utilitarian social planner and a piecewise two-bracket linear income schedule, the second marginal rate is higher than the first.

Acknowledgements

I am thankful to Efraim Sadka and two referees of the journal for very helpful comments. I also thank Eytan Sheshinski, Shlomo Yitzhaki and Oved Yosha for their remarks, and Irina Blits for excellent research assistance. The views expressed and remaining errors are my own.

Non-reported simulations that assume a lower inequality of skills show lower values for the second marginal tax rate; however, in these simulations the second marginal tax rate was always higher than the first one.
Appendix A

A model with heterogeneous agents

Assume that \( w_i \) is different for each \( i \), so that individuals are different ex-ante. The agent solves the following problem (assuming life certainty):

\[
\max_i w_i - x_i + Eu[c(x_i, 1 + \varepsilon)],
\]

where \( w_i \) is the initial endowment (skill) and \( \varepsilon \) is the independently distributed idiosyncratic random shock. Assuming a Benthamite social utility function, the government’s problem is to maximize:

\[
\max \sum_{i=1}^{n} \left[ u[w_i - x_i(t)] + Ec[x_i(t) + \varepsilon] + \sum_{i=1}^{n} \frac{tx_i(t)}{n} \right].
\]

Clearly, the solution depends on the distributions of both skills and luck. The existence of a demogrant implies that redistribution is both from the lucky to the unlucky and from high-skill to low-skill individuals.

Concerning government intervention, note that this general model provides an example of market failure. Assuming that skills are not observable and that there is asymmetric information (i.e., individuals know which type they belong to, but the government or the insurance companies do not), a private insurance company will fail to provide insurance based only on the ‘luck’ component, because redistribution occurs as a consequence of both luck and skill. Therefore, as a result of adverse selection, we may encounter a situation in which high-skill individuals will prefer not to buy such insurance in order to avoid redistribution to low-skill individuals. This problem could be solved by making universal participation compulsory, which is the case that calls for government intervention.

Appendix B

Optimal linear tax rates

<table>
<thead>
<tr>
<th>Source</th>
<th>( t ) (%)</th>
<th>Model, Main Issue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stern (1976), p. 161</td>
<td>54</td>
<td>Classical, elasticity of substitution</td>
</tr>
<tr>
<td>Atkinson (1973)</td>
<td>Up to 50</td>
<td>Classical, social planner</td>
</tr>
<tr>
<td>Slemrod et al. (1994), table 1</td>
<td>12.5–76</td>
<td>Classical</td>
</tr>
<tr>
<td>Tuomala (1990), p. 145</td>
<td>26</td>
<td>Classical, labor income uncertainty</td>
</tr>
<tr>
<td>Varian (1980)</td>
<td>25</td>
<td>Luck as a source of income</td>
</tr>
<tr>
<td>Present paper</td>
<td>Over 32</td>
<td>Luck, life uncertainty and precautionary behavior</td>
</tr>
</tbody>
</table>
Optimum tax formula under life uncertainty

Eq. (10) is obtained by using first-order condition Eq. (5) and the formulae obtained for \( x(t) \) and \( x'(t) \), as shown in Eq. (9):

\[
[b - (w - x)]x'(t) = \rho \frac{b}{2} \left[ x(t) + p(t) - (1 - t)x(t)(1 - p) \right] + x(t) + \frac{tx'(t) - x(t)}{p} + \rho \frac{b}{2} \left[ (1 - t)x'(t) - \frac{x(t) - p(1 - t)e - (1 - t)x(t)(1 - p)}{p} \right] + x(t) - \frac{x(t) - x'(t) - x(t)}{p}.
\]

This equation serves as the basis for the simulation in Table 1.

Appendix C

A Rawlsian social planner

In this appendix we show that when luck is the only source of income and for the parameters used in the simulation in the upper part of Table 3, a Rawlsian social planner will adopt a higher first marginal tax than will a utilitarian social planner. A Rawlsian social planner will maximize the income of an individual with the worst realization \((-\epsilon_2)\):

\[
\text{MAX}_{t_1,t_2} y^* = x(t_1) - \epsilon_2 + \epsilon_2 t_1 + \frac{t_2(\epsilon_2 - \epsilon_1)}{4}.
\]

This equation immediately shows that the optimum policy is to set \( t_2 = 1 \). With respect to \( t_1 \), let us denote by \( x^* \) and \( t \) the optimal values in the utilitarian case. First note that if \( x^* < (1 - t)\epsilon_2 \), a Rawlsian planner will prefer the policy \( t_1 = 1 \) rather than the utilitarian solution \( t_1 = t_1^* \), since in the former case \( y_1 \) is higher (note that for \( t_1 = 1 \), in the optimal solution \( x = 0 \), and the tax-transfer policy leads to equalization of after-tax income).

After noting that for the parameters used in our simulation \( x^* > (1 - t_1)\epsilon_2 \), we ask whether \( t_1^* \) is an optimum for a Rawlsian planner. The answer to this question depends on the reaction of \( x \) to changes in \( t_1 \), compared to the change in the term \( -(1 - t_1)\epsilon_2 \) (‘bad luck’). Raising \( t_1 \) lowers the effect of ‘bad luck’ because the
most unlucky individual is less exposed to the negative shock $\epsilon_r$. At the same time, raising $t_1$ discourages savings (the higher $t_1$, the higher this effect). As shown in Table 4, which is based on the same parameters as Table 3 (for $n=0.25$), the choice of the optimum $t_1$ for a Rawlsian planner depends on the parameters of the simulation.

The last column of Table 4 shows the impact of raising the tax rate on the income of the individual with the lowest income. The optimum tax is somewhere between 0.4 and 0.5, compared with 0.15 in the utilitarian case.\(^{22}\)

### References


Helpman and Sadka (1978) have shown that in the context of the classical model of income taxation, a Rawlsian planner chooses a higher linear optimal tax rate than a Benthamite planner.