



## Isolation, Assurance and the Social Rate of Discount

Amartya K. Sen

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## ISOLATION, ASSURANCE AND THE SOCIAL RATE OF DISCOUNT \*

AMARTYA K. SEN

I. The isolation paradox and the assurance problem, 112.—II. Optimum savings, 116.—III. The rate of discount, 120.—IV. Rich they, poor us, 121.—V. Conclusions, 122.

Some of the recent discussions on the relationship between private and social rates of discount have been concerned with a special instance of a very general problem, being an extension of the two-person non-zero-sum game known as the "prisoners' dilemma."<sup>1</sup> In the first section of this paper the general nature of this problem will be studied, and it will also be shown that there is another problem close to this one with which it is sometimes confused, but which has a very different logical structure and involves different policy implications. In the remaining three sections the application of this general framework to the question of optimum savings and the social rate of discount will be examined, particularly in the light of some recent controversies.<sup>2</sup>

### I. THE ISOLATION PARADOX AND THE ASSURANCE PROBLEM

Consider a community of  $N$  individuals, each of whom must do one and only one of two alternatives,  $A$  and  $B$ . The payoff to each individual is a function of the actions of all individuals. Let the preference ordering of each individual satisfy the two following

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1. Due to A. W. Tucker; see R. D. Luce and H. Raiffa. *Games and Decisions* (New York: Wiley, 1958), sec. 5.4 and 5.5. See also M. Shubik (ed.), *Game Theory and Related Approaches to Social Behavior* (New York: Wiley, 1964), Chaps. 20 and 23.

2. W. J. Baumol, *Welfare Economics and the Theory of the State* (Cambridge: Harvard University Press, 1952); A.K. Sen, "On Optimizing the Rate of Saving," *Economic Journal*, LXXI (Sept. 1961); S. A. Marglin, "The Social Rate of Discount and the Optimal Rate of Investment," this *Journal*, LXXVII (Feb. 1963); G. Tullock, "The Social Rate of Discount and the Optimal Rate of Investment: Comment," this *Journal*, LXXVIII (May 1964); Robert C. Lind, "Further Comment," this *Journal*, LXXVIII (May 1964); D. Usher, "Comment," this *Journal*, LXXVIII (Nov. 1964); M. S. Feldstein, "The Social Time Preference Discount Rate in Cost-Benefit Analysis," *Economic Journal*, LXXIV (June 1964); A. C. Harberger, "Techniques of Project Appraisal," Universities-National Bureau of Economic Research, Conference on Economic Planning, Nov. 27 and 28, 1964 (mimeo.); E. S. Phelps, *Fiscal Neutrality toward Economic Growth* (New York: McGraw-Hill, 1965), Chap. 4.

conditions: (1) given the set of actions of the others (no matter what they are), the individual is better off doing  $A$  rather than  $B$ ; and (2) given the choice between everyone doing  $A$  and everyone doing  $B$ , each individual prefers the latter to the former.<sup>3</sup>

Given the two features noted of the preference pattern, certain results follow immediately. In particular the following three.

1. *Pareto-inferior outcome*: In the absence of collusion, each individual will prefer to do  $A$  rather than  $B$ , for no matter what the others do each is himself better off doing  $A$ . However, the outcome, viz.,  $A$  by all, will be regarded as strictly worse by each than the alternative  $B$  by all. Thus the outcome is Pareto-inferior, and will be rejected by everyone in a referendum.<sup>4</sup>

2. *Strict dominance of individual strategy*: The atomistic result is completely independent of the individuals' expectations of other people's action. Irrespective of each person's expectations of the others' actions, each prefers to do  $A$ , i.e., the strategy of doing  $A$  strictly dominates over the alternative. Thus we do not have to make any assumption about the individual's behavior when faced with uncertainty and conflict, with which much of game theory is concerned.

3. *Need for enforcement*: Even if the policy of everyone doing  $B$  was adopted by resolution, this would not come about (assuming self-seeking) except through compulsory enforcement. Everyone would like the others to do  $B$ , while he himself does  $A$ , so that even if a contract is arrived at, it will be in the interest of each to break it.

It is easy to check that in the special case when there are only two individuals, the above corresponds exactly to the game of prisoners' dilemma. In fact with  $N = 2$ , the conditions (1) and (2) on the preference pattern give a complete strict ordering of the individuals over the entire field of possible outcomes, which consists

3. If  $i(x^i)$  stands for individual  $i$  pursuing strategy  $x^i$ , when  $x^i$  can be  $A$  or  $B$ , and if  $\phi^i$  stands for the payoff to individual  $i$  in terms of his own welfare units, then:

$$(1) \quad \begin{aligned} &\phi^i[1(x^1), 2(x^2), \dots, i(A), \dots, N(x^n)] > \\ &\phi^i[1(x^1), 2(x^2), \dots, i(B), \dots, N(x^n)] \end{aligned}$$

and

$$(2) \quad \begin{aligned} &\phi^i[1(B), 2(B), \dots, i(B), \dots, N(B)] > \\ &\phi^i[1(A), 2(A), \dots, i(A), \dots, N(A)]. \end{aligned}$$

4. It is possible to argue that the difference between the noncooperative equilibrium point and the collusive solution in this game corresponds precisely to Rousseau's distinction between "will of all" and the "general wills." See Marglin, *op. cit.*, p. 104, fn. 2. See also W. G. Runciman and A. K. Sen, "Games, Justice and the General Will," *Mind*, LXXIV (Oct. 1965); and R. R. Farquharson, "An Approach to the Pure Theory of Voting Procedure," Ph.D. thesis, Oxford University, 1957-58.

of four alternatives.<sup>5</sup> We shall, however, stick to the  $N$ -person version, and call it the "isolation paradox."<sup>6</sup>

The savings problem is only a special application of this. Suppose  $B$  stands for the policy of saving one more unit for the sake of the future of the community, and  $A$  for not doing it. Given the action of all others, each individual is better off not doing the additional unit of saving himself. Hence nobody will, but everyone would have preferred one more unit of saving by each than by none. This is the essence of the problem discussed by Marglin and myself.<sup>7</sup>

Consider now a somewhat different preference pattern. Let the individuals continue to hold (2), but let (1) be modified. In the special case when everyone else does  $B$ , the individual now prefers to do  $B$  himself. Excepting this special case, the individual continues to prefer doing  $A$  to  $B$  no matter what the others do, *given* their action.<sup>8</sup>

This is a near-cousin of the isolation paradox, but differs from it in some of the main results. Result (II), i.e., strict dominance no longer holds. Expectations about other people's behavior must be brought in. If it is expected that the others will all do  $B$ , then this one would prefer to do  $B$  also; otherwise he may do  $A$ . Result (I) needs some modification also. If everyone has implicit faith in everyone else doing the "right" thing, viz.,  $B$ , then it will be in everyone's interest to do the right thing also. Then the outcome need not be Pareto-inferior.<sup>9</sup> However, if each individual feels that the others are going to let him down, that is not do  $B$ , then he too may do  $A$  rather than  $B$ , and the outcome will be Pareto-inferior.

Result (III) does not hold any longer. Given that each individual has complete assurance that the other will do  $B$ , there is no problem of compulsory enforcement. Unlike in the case of the isolation paradox, it is not in the individual's interest to break the contract of everyone doing  $B$ . In this case assurance is sufficient and

5. A. K. Sen, *Choice of Techniques* (2d ed.; Oxford: Basil Blackwell, 1962), Appendix to Chap. VIII. We have in this case:

(3)  $\phi^i[i(A), j(B)] > \phi^i[i(B), j(B)] > \phi^i[i(A), j(A)] > \phi^i[i(B), j(A)]$ .

6. See Sen, "On Optimizing the Rate of Saving," *op. cit.*, Sec. II.

7. See Phelps, *op. cit.*, for an illuminating discussion of this problem in the context of others involving growth and public savings.

8. Formally, we impose the restriction on (1) that

(1\*) Not  $[x^1 = x^2 = x^3 = \dots = x^n = B]$ .

And we supplement (1), thus restricted, by:

(1.1)  $\phi^i[1(B), 2(B), \dots, i(B), \dots, N(B)] >$   
 $\phi^i[1(B), 2(B), \dots, i(A), \dots, N(B)]$ .

9. Note, however, that there is no guarantee that this will definitely not be Pareto-inferior. From (1), (1.1) and (2), we get an incomplete ordering for each individual, if  $N > 2$ .

enforcement is unnecessary, and we shall refer to this case as that of the "assurance problem."

These two problems have often been confused with each other. Marglin's and my argument for the inoptimality of market savings is based on the assumption of a situation of the type of the isolation paradox. This problem however, has been identified with Vickrey's analysis<sup>1</sup> of "the interdependence of the transfers of different donors," where "an individual might be willing to make a gift to one of his fellows *if he knew* that others were doing so even if he would not make the gift on his own."<sup>2</sup> This last is, however, a case of the assurance problem.<sup>3</sup> In our case an individual will not do the saving *even if* "he knew that others were doing so," and this makes the inoptimality of the market result certain, which it is not in Vickrey's case.

The difference is a simple one when viewed in the context of game theory. In the assurance problem with which Vickrey is concerned,<sup>4</sup> everyone doing the "right" thing, i.e., *B*, is an "equilibrium point,"<sup>5</sup> whereas in Marglin's case and in mine, this is definitely not so. Baumol's discussion of the optimum savings problem also fits in with the assurance problem, rather than with the isolation paradox, and it rests on an interdependence due to the indivisibility of public projects. The effect of trying to save alone for the sake of the future generation is "negligible," so that the individual, though endowed with altruism, does not do this, "except if he has grounds for assurance that others, too, will act in a manner designed to promote the future welfare of the community."<sup>6</sup> In the case of the isolation paradox, however, the individual will not do the saving *even with* the assurance.<sup>7</sup>

1 W. S. Vickrey, "One Economist's View of Philanthropy," in F. G. Dickinson (ed.), *Philanthropy and Public Policy* (New York: National Bureau of Economic Research, 1962).

2. Tullock, *op. cit.*, p. 331; emphasis added.

3. The preference pattern corresponds to (1) subject to (1\*) and (1.1).

4. Vickrey goes into more complex cases also. He introduces the possibility that an individual donating a sum might induce others to do the same. This restricts condition (1) even more than in the assurance problem, with the individual preference for *A* over *B* being not only not applicable when *all* others do *B*, but also when, say, a certain suitably large fraction of the total group does *B*.

5. See Luce and Raiffa, *op. cit.*, sec. 7. 8. For the classic analysis of the meaning and the existence of equilibrium points, see J. F. Nash, "Equilibrium Points in *N*-person Games," *Proceedings of the National Academy of Sciences*, Vol. 36 (1950).

6. Baumol, *op. cit.*, p. 92.

7. "This possibility of the apparent paradox is present whenever his relative evaluation of others' consumption is such that he would prefer them to sacrifice some consumption for the future generations; and sacrificing something himself might, because of the indivisibility of the political decision, be the means of achieving this to a sufficient extent to over-compensate the loss

## II. OPTIMUM SAVINGS

Consider now the following ordering of individual  $i$ . He attaches a weight of unity to his consumption today (simply a normalization assumption),  $\beta$  per unit to the consumption of his contemporaries,  $\gamma$  per unit to the consumption of his own heirs in the future, and  $\alpha$  per unit to the consumption of the others in the future generation.<sup>8</sup> For each person, only one marginal choice is considered, viz., whether to increase the saving by one unit ( $B$ ), or not to do it ( $A$ ). Also, the future consumption is taken to be a one-shot affair, though this assumption can be easily relaxed without losing anything essential. The marginal rate of return on one unit of saving (i.e., one unit less of consumption) today is an increase in the consumption in the future by  $k$  units, where  $k > 1$ . The individual expects that a proportion  $\lambda$  of the fruits of his saving will accrue to his heirs, and the rest  $(1 - \lambda)$  to others in the future generation, when  $0 \leq \lambda \leq 1$ ; we shall discuss presently the proper assumption about the value of  $\lambda$ . The individual figures that *given* the actions of other people today, the net gain  $G(i)$  from one unit more of personal saving is the following:

$$(4) \quad G(i) = [\lambda \cdot \gamma + (1 - \lambda) \alpha] \cdot k - 1.$$

If  $G(i) > 0$ , the individual  $i$  will clearly save the extra unit, i.e., do  $B$ . However, when we start with the amount of savings on which each has already made a decision (based on their atomistic calculation), and then consider the extra unit to be a tiny bit more,  $G$  clearly cannot be positive or they would not have been in atomistic equilibrium. Making the usual assumptions about well-behaved and continuously differentiable functions, we shall indeed find that in the atomistic equilibrium,  $G = 0$  for every individual, which amounts to:

$$(5) \quad [\lambda \cdot \gamma + (1 - \lambda) \alpha] k = 1.$$

Now, consider the possibility of a social contract of everyone ( $N$  in all) saving one more tiny unit, so that as evaluated by any individual the immediate loss is  $[1 + (N - 1)\beta]$ , attaching the appropriate weights to the consumption of oneself and of one's contemporaries. This has to be set against the gain in the future. The total

that he would incur from his own act of sacrifice." (Sen, *op. cit.*, pp. 488-89.) "I am willing to abide by [the no-fix system], too, if this is the price I must pay for your adherence, although my first choice is that all of you abide by the no-fix system and I buy off the police force." (Marglin, *op. cit.*, p. 100.)

8. These values can be interpreted as the relevant marginal utilities in the utility function of individual  $i$ , and can be taken as constant for small changes (see Marglin, p. 101).

gain in physical terms is  $(N \cdot k)$  for the future generation as a whole, and let the proportion of that enjoyed by one's own heirs be  $h$ ; the appropriate assumptions for  $h$  will be discussed presently. However, the net gain  $G(s)$  from the social contract, as viewed by any individual, will be of the general form:

$$(6) \quad G(s) = N \cdot k \cdot h \cdot \gamma + N \cdot k(1 - h)\alpha - 1 \\ - (N - 1)\beta.$$

We can now examine what conditions have to be satisfied for the isolation paradox to hold. Note that (5) indicates the weak form of the preference relation (1). Since we are starting from an atomistic market equilibrium, people, in isolation from the others, do not want to save more than they are doing already. Strictly speaking they should be indifferent between  $A$  and  $B$ , since the net gain from the change is exactly nil, but we can assume that they prefer not to save when there is no net gain. For condition (2) to hold, we need  $G(s) > 0$ , i.e., everyone prefers  $B$  (saving) by each rather than  $A$  (not saving) by each.

$$(7) \quad N \cdot k[h \cdot \gamma + (1 - h)\alpha] > 1 + (N - 1)\beta.$$

When (7) is consistent with (5), we have the isolation paradox holding, and people are willing to join in the contract to save but not do so individually. To get this result, the assumption that was made by Marglin and myself has been found to be unacceptable by many, viz., that individuals do not discriminate between their own heirs and the rest of the future generation, i.e.,  $\gamma = \alpha$ . As can be readily checked, exactly the same formula holds if  $\lambda = h = 0$ , i.e., if the fruits of my saving (both in individualistic saving and social contract) accrue to the future generation in general and not to my own heirs. Neither, I agree, is a good assumption, and it will be shown presently that neither is necessary. But before that, the consequence of this assumption, however bad, can be checked immediately. Then from

(5),  $k = \frac{1}{\alpha}$  and condition (7) reduces to:

$$(7.1) \quad 1 > \beta.$$

The result is independent of the value of  $N$ , provided, of course,  $N > 1$ . It is quite reasonable to assume that  $\beta < 1$ , i.e., I value my consumption more than that of my other contemporaries.

In a closely-reasoned note on this problem, commenting on Marglin's paper, Lind has suggested an alternative set of assumptions. In effect, he assumes that the individual can pass on (if he chooses) all the fruits of his own saving to his own heir without any part of it going to others in the future generation. Then:

$\lambda = 1$ ,  $h = \frac{1}{N}$ , and  $k = \frac{1}{\gamma}$ , so that (7) reduces to:

$$(7.2) \quad \frac{1}{\gamma} > \frac{\beta}{a}.$$

We cannot be so sure that (7.2) will hold as (7.1). If  $(\frac{1}{\gamma}) > (\frac{\beta}{a})$ , the individuals will be willing to join the contract; and if  $(\frac{1}{\gamma}) < (\frac{\beta}{a})$ , the individuals will not be willing to do so. Indeed, if the latter condition held, it would be easy to show that they will be prepared to join a contract to *reduce* savings; this too fits the isolation paradox, except it now runs the other way, with *B* standing for reducing the saving by one unit each, and *A* as before. In between these values lies the case where everything is fine with atomistic allocation, and the case of  $(\frac{1}{\gamma}) = (\frac{\beta}{a})$  can be seen to be one where my relative evaluation of your heir's consumption ( $a$ ) and your consumption ( $\beta$ ), exactly corresponds to your relative evaluation of your heir's consumption ( $\gamma$ ) vis-a-vis your own (1). Lind finds this a "reasonable" assumption.<sup>9</sup>

The difficulty with Lind's assumption of the balance of emotions is that it makes insufficient allowance for the personal nature of egoism. My egoism might not extend as much to my heirs vis-à-vis yours, as it applies to me personally vis-à-vis you. The longer the distance in time that we consider the more is this likely to be the case. So that  $(\frac{a}{\beta}) > \gamma$ , does not seem to be a particularly bad assumption. However, for the sake of argument, let us grant this balance of emotions, and assume that  $\gamma$  is exactly equal to  $(\frac{a}{\beta})$ , no more and no less. There is the further question of  $\lambda = 1$ , which is the only case Lind discusses. If it is assumed that there is a gap between the marginal productivity of capital and the rate of interest,

9. Lind, *op. cit.* pp. 341-42, 345. In commenting on my *Economic Journal* paper on the subject, the same suggestion was made in a personal communication (dated April 4, 1962) by the late Sir Dennis Robertson.

1. In much of the literature on economic development, it is conventional to assume that the market wage rate is above the social opportunity cost of labor, which will also make  $\lambda < 1$ , since the marginal benefits to the owners of capital will fall short of the marginal benefits to the community. This is a sufficient assumption for  $\lambda < 1$ , but, of course, not necessary.

$\lambda$  must be below 1.<sup>1</sup> But even when there is no such gap, in a society with taxation this will happen again. Even if we assume, in a competitive dream world, that no one is prevented from enjoying the full "return" of his own investment through taxes, this does not rule out taxes, such as estate duties, that apply to unrequitted transfers. And these too will make  $\lambda < 1$ . This is not to say that Marglin's and my assumption of  $a = \gamma$ , or alternatively, of  $\lambda = 0$ , is a good assumption, but neither is the other special case of  $\lambda = 1$ , with exactly balanced emotions. The natural question to ask is what happens when  $\lambda$  takes a value in between these extremes, i.e., when it is selected from the open interval  $]0, 1[$ . The answer is that the condition for the isolation paradox remains *exactly* the same as with  $\lambda = 0$ . Lind's case of  $\lambda = 1$  is the one exceptional value; the rest of the interval gives the same condition. This is shown below.

Let us assume first that in the case of the social contract to save more, my heir gets only  $\lambda$  part of my own savings, and nothing of other people's savings. In that case:

$$(8.1) \quad h = \frac{\lambda}{N}.$$

Then the required condition (7) reduces to:

$$(7.3) \quad [\lambda \cdot \gamma + (N - \lambda)a] \cdot k > 1 + (N - 1)\beta.$$

In view of (5) and the Lindian balanced emotions ( $\gamma = \frac{a}{\beta}$ ), this is equivalent to:

$$(7.1) \quad \beta < 1.$$

Precisely the same condition, as with  $a = \gamma$ , or with  $\lambda = 0$ .

Now, this assumption of my heir getting only  $\lambda$  part of the fruits of my savings in the social contract with other people's heirs getting the whole of the fruit of their savings *plus*  $(1 - \lambda)$  part of the result of my saving, seems a poor one. Surely my heirs will do better than this from the social contract; they can expect to get a part of the  $(1 - \lambda)$  portion of other people's savings that goes to future generations in general. But, if this is the case, (7.1) will be *a fortiori* sufficient for the isolation paradox, as can be readily

checked from the fact that  $\gamma > a$ , since  $\gamma = \frac{a}{\beta}$ , and  $\beta < 1$ . If

(7.1) is sufficient to induce me to join the contract even with the minimum share of my heirs, it is naturally sufficient if they get a higher share.

## III. THE RATE OF DISCOUNT

To derive the formula for the social rate of discount implicit in this, we have to specify the value of  $h$  more than working with its minimum magnitude. The symmetrical assumption is the following: each set of heirs get  $\lambda$  part of the results of their progenitor's savings, while  $(1 - \lambda)$  goes to a general pool of the "future generation," out of the total of which each set again gets  $(\frac{1}{N})$  portion. It is readily seen:

$$(8.2) \quad h = \frac{1}{N}.$$

And with this symmetrical sharing of the general pool, the social rate of discount ( $\rho$ ) is given by exactly the same formula as that of Lind (except in the special case of  $\lambda = 0$ ):<sup>2</sup>

$$(9) \quad \rho = \frac{1 + (N - 1)\beta}{\gamma + (N - 1)\alpha} - 1.$$

Marglin's formula coincides with this only when  $\alpha = \gamma$ , or when  $N$  is very large, which reduces this to  $[\frac{\beta}{\alpha} - 1]$ , and also reduces Marglin's to the same value. However, the difference between the private and the social rate of discount continues to hold, except when the knife-edge balance happens to occur. This is the dual to the problem of the optimum rate of saving, and (7.1) can be seen to be sufficient for the social rate to be below the private rate of discount.

Representing the private rate of discount by  $\pi$ , it is seen from (5) and (9) that:

$$(10) \quad \begin{array}{l} < \\ \rho = \pi, \text{ according as:} \\ > \end{array} \frac{1 + (N - 1)\beta}{1 + (N - 1)\frac{\alpha}{\gamma}}$$

$$\begin{array}{l} < \\ = \\ > \end{array} \frac{1}{\lambda + (1 - \lambda)\frac{\alpha}{\gamma}}.$$

Lind gets  $\rho = \pi$ ; by assuming the balance of emotions  $(\frac{\alpha}{\gamma}) = \beta$ , and that  $\lambda = 1$ . The more general set of conditions is given by taking (10) as an equality; there are pairs of values of  $(\lambda, \gamma)$  that satisfy this equality, of which Lind's is one. However, there is nothing at

2. Lind points out that the assumption of identical individuals is quite crucial for this formulation, and indeed the existence, of a the social rate of discount *op. cit.*, (pp. 342-45).

all in the market mechanism to guarantee that we shall indeed have one of these critical pairs holding. Of course, it *can* happen, but it will be an accidental outcome.

Marglin's case can be obtained from (10) by putting  $\alpha = \gamma$ , a different balance of emotions. This reduces (10) to:

$$(10.1) \quad \rho \underset{>}{\overset{<}{=}} \pi; \text{ according as } \beta \underset{>}{\overset{<}{=}} 1.$$

We would get the same condition if  $\lambda = 0$ , and  $h = 0$ , when the social rate of discount is as given in Marglin.<sup>3</sup> Suppose, however, we grant (as in our discussion of the dual problem to this) that Lind's emotional balance holds; i.e.,  $(\frac{\alpha}{\beta}) = \gamma$ ; then (10) becomes:

$$(10.2) \quad \rho \underset{>}{\overset{<}{=}} \pi, \text{ according as } \beta(1 - \lambda) \underset{>}{\overset{<}{=}} (1 - \lambda).$$

The equality holds only in Lind's special case of  $\lambda = 1$ . For any other value of  $\lambda$ , (10.2) reduces into (10.1). Thus, while  $\lambda = 1$ , and  $\lambda = 0$ , look like two extreme cases, they are not symmetrical in the generality of their respective results.

#### IV. RICH THEY, POOR US

Finally, the point has been raised<sup>4</sup> whether the isolation paradox in savings is at all likely to arise when it is borne in mind that the future generation is going to be a great deal more wealthy than the present generation. In the determination of the social rate of discount, this will undoubtedly be an important consideration. This can be checked readily by looking at (9), where the wealth of the future generation will tend to make the values of  $\gamma$  and  $\alpha$  relatively lower. However, will this affect the condition for the isolation paradox to hold? Not at all! It is seen from (7.1) and (7.2) that a proportionate fall in  $\gamma$  and  $\alpha$  (the values attached to the consumption of the future generation) vis-à-vis the values attached to the consumption of members of the present generation, will not make any difference to the fulfillment of the conditions.

The explanation is simple; the fact that the future generation will be wealthier has already been taken into account in the atomistic allocation of resources, and the average wealth of the future generation vis-à-vis that of the present generation does not affect the *relative* profitability of atomistic allocation and the social contract.

3. Equation (13), Marglin, *op. cit.*, p. 106.

4. Tullock, *op. cit.*, pp. 333-35; Harberger, *op. cit.*, Sec. 4.

Indeed substituting the value of  $k$  from (5) into (7), we find that (7) is equivalent to:

$$(7^*) \quad \frac{N[h \cdot \gamma + (1 - h)a]}{[\lambda \cdot \gamma + (1 - \lambda)a]} > 1 + (N - 1)\beta.$$

In this general condition a proportionate change in the values of  $\gamma$  and  $a$  would leave the fulfillment of the inequality entirely unchanged.

Exactly the same is true, naturally, in the dual to this problem, i.e., in the difference between the private and the social rate of discounts. A proportionate change in  $\gamma$  and  $a$  will leave condition (10) exactly the same, as is obvious from its form.

## V. CONCLUSIONS

In Section I two specific problems concerning individual and social actions were studied. One, the isolation paradox, is an  $N$ -person extension of the two-person non-zero-sum game of the prisoners' dilemma. Here each individual has a strictly dominant strategy, and the pursuit of this by each produces an overall result that is Pareto-inferior. Individuals can do better than this by collusion, but the collusive solution requires enforcement.

The second, the assurance problem, which is sometimes confused with the first, has a different analytical structure and imply different policy questions. Here there is no strictly dominant strategy, and one of the equilibrium points in the noncooperative game may be Pareto-optimal. Whether this will be the outcome of the noncooperative game or whether the outcome will be Pareto-inferior depends on what each individual expects about the others' action. To get out of the problem all that is necessary is that each individual is assured that the others are doing the "right" thing, and then it is in one's own interest also to do the "right" thing. No enforcement is necessary.

Marglin's and my discussion of the inoptimality of market savings corresponds to the isolation paradox, whereas that of Baumol, and of Vickrey (in the context of philanthropy), corresponds to the assurance problem. The distinction is important analytically as well as for policy decisions.

In the last three sections, Marglin's and my formulation of the problem of the inoptimality of market savings was examined as an application of the paradox of isolation. It should be conceded immediately that if one is ready to make some rather special assump-

tions, the problem can in fact be assumed away. Lind's conditions of balance ( $\gamma = \frac{\alpha}{\beta}$ , and  $\lambda = 1$ ) achieve this,<sup>5</sup> and, as we have seen, so does a family of pairs of values of  $(\gamma, \lambda)$ . However, there is nothing in the market mechanism that will ensure this achievement. This inoptimality of the market mechanism, and the possibility of a social contract by which everyone will agree to do something he would not be ready to do individually, is not a surprising result when viewed in the context of games such as the prisoners' dilemma. It is a paradox only as an apparent one, as most paradoxes are. Even from the point of view of usual theories of optimum allocation through decentralized decisions, the result need not be viewed as particularly contrary, for the basis of it lies in an *external* concern for members of the future generation vis-à-vis those of the present.

If the emotional balance of the type proposed by Lind is taken ( $\gamma = \frac{\alpha}{\beta}$ ), then the optimality of the market rate of saving and that of the market rate of discount are unlikely to hold in any economy that we know of. With this type of balanced emotions, the isolation paradox can be ruled out, as Lind does, by assuming  $\lambda = 1$ . Marglin's and my assumptions about people's concern for others ( $\gamma = \alpha$ ) differ from this, but if  $\lambda = 0$  then, of course, the condition for the existence of the isolation problem are the same as with  $\gamma = \alpha$ , no matter what emotional assumptions we make. While  $\lambda = 1$  and  $\lambda = 0$ , represent two extreme cases, their positions are not symmetrical. In fact, for any value of  $\lambda$  in between these limits, i.e., from the open interval  $]0, 1[$ , the condition for the isolation paradox, with Lindian balanced emotions, will be exactly the same as with  $\lambda = 0$ , i.e., precisely the result we get from Marglin's and my assumptions.

Finally, the question of the future generation being, on the average, much richer than the present generation was studied as grounds for an objection that has been raised against the chances of the isolation paradox.<sup>6</sup> It was shown that a change in the average

5. Lind's condition is not, of course, in conflict with the claim that "this possibility of the apparent paradox is present whenever his relative valuation of others' consumption is such that he would prefer them to sacrifice some consumption for the future generations" (Sen, *op. cit.*, p. 488). With  $\gamma = \frac{\alpha}{\beta}$ , and  $\lambda = 1$ , the condition is not met, and the consequence does not, naturally, follow. See also Marglin, *op. cit.*, pp. 100-2.

6. Tullock, *op. cit.*, pp. 333-35; Harberger, *op. cit.*, Sec. 4. Harberger has a further argument which we have not discussed. "The third argument (c), best reflected by Sen and Marglin, smacks of charity. . . . My reaction to this is simple: any individual who wants to help others, and to make sure that his

wealth of the future generation vis-à-vis that of the present generation, bringing about a change in  $a$  and  $r$  in the same proportion, leave the possibility of the isolation paradox completely unchanged.

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contribution is not dissipated, can do so by selecting one or more people of the present generation to help" (pp. 14-15). This seems a good way of working off one's irrepressible urge towards charity, but surely this need not cure the misallocation in the rate of saving.