TAX EVASION AND STRATEGIC BEHAVIOUR OF THE FIRMS

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The purpose of this paper is to analyze the optimal tax evasion decision in the context of an oligopolistic market with quantity setting firms. It is shown that the optimal amount of tax evasion for each firm depends not only on the degree of collusion in the market but also on the relative market shares of the firms; increasing collusion, however, leads to a larger amount of tax evasion in the market. It is proved that, with constant probability of detection, separability between shifting and evasion decisions holds, whereas it might fail to hold if the probability of detection is a function of the declared tax base. In this case, a probability function which decreases with the declared tax base will lead to lower evasion and lower shifting.

0. Introduction

A relevant part of the theory of tax incidence deals with the price and quantity effects of taxes in competitive and non-competitive markets; many of the results obtained depend crucially on the premise of an all-knowing and all-powerful State. Recent literature on tax evasion has pointed out the need for re-examining these results but, surprisingly enough, little work has been done in this field since the seminal paper by Allingham and Sandmo in 1972.¹

The purpose of this paper is to analyze the optimal tax evasion decision in the context of an oligopolistic market with quantity setting firms;² the relevance of this problem, we feel, lies in the fact that such an approach suggests some tentative conclusions on the relation between the quantity behaviour of the firms and their respective tax compliance behaviour. Indeed, there does not seem to be any a priori reason for concluding that quantity decision interactions play no role in the tax evasion decision; quite the

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²Much of the literature on tax evasion has concerned itself with a price-taker tax evader, whether he is a worker facing a given wage rate or a saver facing a given interest rate. An excellent survey of the theoretical literature can be found in Cowell (1985).

³For the sake of analytical simplicity we consider the case of a simple duopoly, although the model and the results can easily be extended to the case of more than two firms. See Dixit (1986).
contrary, one might expect some kind of correlation between the two, but we have no a priori argument for holding that a more collusive market (as opposed to a more competitive one) should lead to a higher tax declaration, or vice versa. Furthermore, we might expect some kind of interdependence between tax compliance decisions, but, again, we have no a priori argument to predict the sign of this relation. Finally, it is worth emphasizing that tax evasion is not a phenomenon just limited to the self-employed and to various forms of moonlighting: firms, especially small and middle-sized ones, do also evade tax.3

The paper is organized as follows. In section 1 the model is presented and the properties of equilibrium are examined; in section 2 some comparative statics results are derived for the unit, ad valorem and profit tax cases: in particular, the effects of firms' different strategic behaviour on the quantity and tax compliance decisions are examined.

Section 3 examines the effects of some State controlled parameters, such as tax and penalty rates and the probability of detection, on the optimum shifting and tax declaration of firms; in section 4 we modify the model to analyze the case of variable probability of detection, and, finally, section 5 presents our conclusions.

1. The model

We consider a simple duopoly market where profit maximizing firms produce a homogeneous good and simultaneously announce their quantity decisions; exchange takes place at market clearing prices. Firms face a tax function \( T(q_i) \) of the following possible forms:

\[
T^i(q_i) = t_c(R^i - C^i), \quad \text{profit tax},
\]

\[
T^i(q_i) = t_v R^i, \quad \text{ad valorem tax},
\]

\[
T^i(q_i) = t_s q_i, \quad \text{unit tax},
\]

where \( R^i \) is firm \( i \)'s total revenue \( p(Q)q_i \), \( p(Q) \) the inverse demand function, \( Q = q_i + q_j \) the total quantity, \( C^i = C^i(q_i) \) firm \( i \)'s total cost, and \( q_i \) is the quantity produced by the \( i \)th firm. In what follows, subscripts in the function symbols will denote derivatives.

If the firms decide to evade a sum \( D^i \) (with \( 0 \leq D^i \leq T^i \)), and evasion is detected, with a given probability \( \mu \), they will incur a fine (including the tax) \( \tau \) times the evaded tax \( D^i \), with the penalty tax rate \( \tau > 1 \). Alternatively, the

3See, for example, Rey (1965), Clotfelter (1983), and International Bureau of Fiscal Documentation (1979).
evasion decision could be modelled as a fraction $\alpha_i$ of the total tax, in which case the evaded tax will be a function of the tax base. The choice of which of these two hypotheses is more plausible is left to the discretion of the reader: the working of the two models is essentially the same, although the modelling of the evasion decision as $D_i$ provides more clearcut results and allows a more straightforward interpretation of them. The reason for this is the additive nature of risk in the former case as opposed to the multiplicative risk of the latter. In what follows, therefore, we will model the evasion decision as $D_i$, but we shall briefly mention the results for the other case whenever they differ.

If the $i$th firm evades the tax and is not detected, its pay-off is given by $Y_i = R_i - C_i - T_i + D_i$ If the evasion is detected, its pay-off is $Z_i = Y_i - \tau D_i$.

The problem firms 1 and 2 face is to decide the quantity produced (and therefore tax shifting) and the amount of tax evasion which maximize the expected utility of the pay-offs. Therefore:

$$\max_{D_i, q_i} EU_i = (1 - \mu) U_i(Y_i) + \mu U_i(Z_i)$$

s.t.

$$q_i \geq 0,$$
$$T_i(q_i) \geq D_i \geq 0.$$  

First-order conditions for interior solutions are:

$$R_i - C_i - T_i = 0,$$  

$$\frac{U'_y}{U'_z} = \mu(\tau - 1) \frac{1}{(1 - \mu)}.$$  

with

$$R_i = p + p_q q_i \left(1 + \left[\frac{\delta q_i}{\delta q_i}\right] c\right),$$

where the superscript $c$ stands for conjectured and $T_i = t_c[R_i - C_i]$ for a profit tax; $T_i = t, R_i$ for an ad valorem tax and $T_i = t_s$ for a unit tax.

From the above conditions, it can be observed that, for $\tau \neq 0$, a profit tax is not shifted on to consumers, while indirect taxes are; it can be noted, furthermore, that production and tax declaration decisions display a one-
sided separation: the production decisions are not affected by tax declaration decisions but the converse is not true.4

The solution of the system of simultaneous equations (1), (i = 1, 2), determines the equilibrium values \( q^*_i, q^*_j \), whereas eqs. (2) determine, respectively, \( D^*_i \) and \( D^*_j \) as a function of both \( q^*_i \) and \( q^*_j \). Indeed, \( U^i_1/U^i_2 \) is a function of \( D_i, q_i \) and \( q_j \), say \( F^i(D_i, q_i, q_j) \), but not of \( D_j \).

In the plane \( F_i(D_i, q_i, q_j) \) and \( D_i \), for a given pair of outputs, \( q_i, q_j \), this function displays the following characteristics:

\[
F^i(0, q_i, q_j) = \frac{U^i_j(R^i - C^i - T^i)}{U^i_2(R^i - C^i - T^i)} = 1,
\]

\[0 < F^i(T^i, q_i, q_j) = \frac{U^i_2(R^i - C^i)}{U^i_2(R^i - C^i - T^i)} < 1.
\]

\[F^i < 0.
\]

The above conditions guarantee that, for a given output equilibrium pair \( q^*_i, q^*_j \), there exist a unique equilibrium pair \( D^*_i, D^*_j \).

The relative magnitudes of \( D^*_i, D^*_j \) obviously depend on the utility functions of firms \( i \) and \( j \) and on the equilibrium values of \( q_i \) and \( q_j \).

An illustrative example is given in fig. 1.5

In order to ensure the existence and uniqueness of equilibrium for \( D_i \), we need to ensure existence and uniqueness of equilibrium in the output space; to do so we assume that each firm’s profit is a concave function on its own output and that

\[p^i_Q \left\{ 1 + \left[ \frac{\delta q^*_j}{\delta q^*_i} \right]^c \right\} - C^i_{ii} \leq 0
\]

and

\[p^Q_Q \left\{ 1 + \left[ \frac{\delta q^*_j}{\delta q^*_i} \right]^c \right\} + p^Q_Q q^*_i \leq 0
\]

with strict inequality for at least one of these two expressions. For analytical simplicity we assume \( p^Q_Q = 0 \) and \( C^i_{ii} = 0 \). It is worth emphasizing that, due to

4 This confirms a result already obtained for the monopoly case [cf. Marrelli (1984)]. The above result holds, however the tax evasion decision is modelled, if the probability of detection is given (independently of the penalty scheme). However, if the choice variable is \( z \), then, for the separability to hold, we must confine ourselves to interior solutions for \( z \).

5 The convexity or concavity of the \( F^i \) function depends on the shape of the utility function; sufficient condition for convexity is \( U^i_{yyp}/U^i_y > (\tau - 1)^2 U^i_{zzz}/U^i_z \). The \( F^i \) functions in fig. 1 have been drawn under this assumption.
the separability of the shifting and evasion problem, our model does not show any interdependence between the two firms' evasion variables.

The implications of the one-sided separability are quite straightforward: indeed, one consequence is that greater collusion leads to greater evasion only to the extent that it leads to greater profits; to this extent, evasion is, in some sense, a 'luxury good'. However, no a priori conclusion can be reached on the relation between the degree of collusion and the amount of profits.

Obviously, separability fails either if the probability of detection is some function of the choice variable \( q_i \), or if, among the arguments of the utility function, we have some variable representing social disapproval or government expenditure (with a government budget constraint) or, finally, if firms are not profit maximizers.

The case of variable probability of detection will be considered in section 4 [for an example of the effects of public expenditure (in the form of public goods) see Cowell and Gordon (1987)].

However, in a world of expected-utility-of-profit maximizing oligopolistic firms, with a given probability of detection and absence of externalities, there is no rational room for evasion decision interactions.

2. The effects of collusive strategies

In the previous section we have shown that the system of four equations [eqs. (1) and (2)] determines the equilibrium values of the four unknowns, \( D^*_i, q^*_i \), for \( i = 1, 2 \); in this section we examine the comparative statics of the
model with respect to the strategic behaviour of the firms as indicated by their conjectural variations. In the next section, the same exercise will be carried out with respect to the parameters controlled by the State (μ, τ, t).

As is well known, the equilibrium solution of a static oligopoly model crucially depends on the conjectures held by the firms about their rivals' reactions to an infinitesimal variation in their own output. Such conjectures, when assumed to be constant, are expressed by a parameter, b. It is usually held in the literature that when the parameter takes the values of -1, 0, and +1, the well-known competitive, Cournot and collusive solutions are, respectively, achieved. In what follows we will limit the analysis to the comparative statics of conjectural equilibria, thus ignoring the problem of conjectural consistency; furthermore, we assume that conjectural variations are constant and are represented by the parameter b, and we limit the analysis to more or less collusive markets without side-payments. Finally, we will assume that b > -1. The reason for this being the need to ensure the concavity of the profit functions and therefore the existence and the determinacy of equilibrium.

We start by examining the effects of infinitesimal changes in the strategic behaviour of firms, as represented by the parameter b (equal for both firms) on the equilibrium values of qi and Di. The implicit function theorem, applied to eqs. (1) and (2) yields:

\[ J \cdot x = k, \]

where J is a 4 x 4 matrix and x and k are column vectors. In particular:

\[
J = \begin{bmatrix}
R_{11}' - T_{11}' & R_{12}' - T_{12}' & 0 & 0 \\
R_{21}' - T_{21}' & R_{22}' - T_{22}' & 0 & 0 \\
0 & a_{32} & a_{33} & 0 \\
a_{41} & 0 & 0 & a_{44}
\end{bmatrix},
\]

\[ a_{32}, a_{41} = [A_y^i - A_z^i][R_y' - T_y'], \quad \text{with } i, j = 1, 2; j \neq i, \]

\[ a_{33}, a_{44} = - A_y^i - (\tau - 1)A_z^i, \quad i = 1, 2, \text{ respectively}, \]

and A_y^i and A_z^i are the Arrow–Pratt measures of absolute risk aversion at Y and Z for individual i.

From the above definitions it is easy to notice that a_{33}, a_{44} < 0, and a_{32}, a_{41} < 0 for decreasing absolute risk aversion.

\[
x = \begin{bmatrix}
dq_1 y \\
dq_2 y \\
D_1 y \\
D_2 y
\end{bmatrix},
\]
\[ k = \begin{bmatrix} -p_0 q_1^*(1-s), & -p_0 q_2^*(1-s), & 0, & 0 \end{bmatrix}^T, \]

with

\[ s = 0 \text{ for the unit tax}, \]

\[ s = t \text{ for the profit tax and the ad-valorem tax cases}. \]

It can easily be verified that, for \( b > -1 \), \( J \) is non-singular, and its determinant \( A \) is greater than zero.

Solving in (3) for \( dq_1^*/db \), by Cramer's rule, and remembering that \( R_{ii} = p + p_0 q_i (1 + b) \) we get:

\[
\frac{dq_1^*}{db} = \frac{q_2^* - (2 + b)q_1^*}{(2 + b)^2 - 1}, \tag{4}
\]

\[
\frac{dq_2^*}{db} = \frac{q_1^* - (2 + b)q_2^*}{(2 + b)^2 - 1}. \tag{5}
\]

Eqs. (4) and (5) reveal an important characteristic of this model, which has perhaps been overlooked in the literature: take for example the Cournot-Nash equilibrium \((b = 0)\), a notional movement of the strategic behaviour of the firms toward a more collusive one \((db > 0)\) would not necessarily give rise to a reduction in the quantity produced by each firm. This would be the case only if the two equilibrium quantities were not 'too far apart'. To see this, consider the graphical representation of (4) and (5) on the \( q_1, q_2 \) plane, shown in fig. 2.

It is easy to see that, for \( dq_1^*/db < 0 \), we must have \( q_2^* < (2 + b)q_1^* \) and we will be looking at the plane with an upper bound of \( m \); on the other hand, for \( dq_2^*/db < 0 \), we must have \( q_2^* > q_1^*/(2 + b) \) and we will be considering the set with \( l \) as a lower bound.

Therefore, for \( dq_1^*/db < 0 \) the equilibrium pair \( q_1^*, q_2^* \) must be an element of the open set \( \Omega = \{ q_1^*, q_2^* | q_2^* < (2 + b)q_1^* \} \), which, in fig. 2, is represented by the convex cone bounded by the \( m \) and \( l \) rays. It is easily verified that, if this condition does not hold, \( dq_1^*/dq_2^* < 0 \).

As can easily be recognized, the asymmetry given by the different cost conditions of the firms is critical for this result. An increase in the degree of collusion, at the original Nash equilibrium, reduces the marginal revenue of both firms, thus creating an incentive to cut output. However, the reduction in output of the largest firm causes an increase in the marginal revenue of the smaller one, thus eliciting an increase in the output of the latter [cf. \( \text{J.F.E. C} \).]

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6 If the tax evasion is modelled as a fraction \( a \) of the tax liability then the Jacobian will show \( a_{11}, a_{22} < 0, a_{33}, a_{44} < 0, \) and \( a_{32}, a_{41} < 0 \) for decreasing relative risk aversion.
Levin (1985)]. However, it is worth emphasizing that for $b > -1$ total output declines as the degree of collusion increases. Indeed, let $Q^* = q_1^* + q_2^*$, then

$$\frac{dQ^*}{db} = -\frac{q_1^* + q_2^*}{(3+b)} < 0.$$  

Solving the system for $dD_i/db$ we get:

$$\frac{dD_1^*}{db} = \frac{a_3[(2+b)q_2^* - q_1^*]}{a_3[(2+b)^2 - 1](1-s)} \geq 0, \quad (6)$$

iff $q_1^* - q_2^*(2+b) \leq 0$, and

$$\frac{dD_2^*}{db} = \frac{a_{41}[q_1^*(2+b) - q_2^*]}{a_{44}[(2+b)^2 - 1](1-s)} \geq 0, \quad (7)$$

iff $q_1^* - q_2^*(2+b) \leq 0$. Therefore, a more collusive market gives rise to higher evasion if the output equilibrium pair lies within the convex cone bounded by the rays $m$ and $l$.

On the contrary, if the pair $q_1^*, q_2^*$ lies outside the above-mentioned cone,
then an increase in collusion will give rise to an increase in the degree of tax compliance for the firm with the higher market share and to a decrease for the one with the lower share. In this case, no clear-cut conclusion can be reached on the effects of a change in the degree of collusion on the total amount of tax evasion. Indeed, let \( D = D_1 + D_2 \), then

\[
\frac{dD}{db} = a_{44}a_{32}\left[q_2^*(2+b) - q_2^*\right] + a_{33}a_{44}\left[q_1^*(2+b) - q_2^*\right]
\]

A sufficient condition for \( \frac{dD}{db} > 0 \) is

\[
q_T < q_r(2 + b)
\]

and

\[
q_T(2 + b) > q_r.
\]

If the equilibrium pair \( q_i^*, q_T^* \) is such that the above condition is not met, then the sign of the derivative will depend, among other things, on the relative magnitudes of absolute risk aversion of the entrepreneurs (as indicated by the coefficients \( a_{32}, a_{33}, a_{44}, a_{44} \)).

From the above results, we can observe that the effects of a more collusive market on the tax compliance of the two firms is the same, and, in particular, there is a lower tax declaration, irrespective of the tax in existence, if market shares are such that:

\[
1/(3+b) < q_i/Q < (2+b)/(3+b).
\]

In the Cournot case, for example, for the above result to hold:

\[
1/3 < q_i/Q < 2/3.
\]

As already said, due to the separability property of the evasion and shifting decisions, evasion is, in some sense, a 'luxury' good; greater collusion leads to a greater evasion only to the extent that it leads to greater profits.

It is not always true, however, that greater profits arise from a higher degree of collusion. Indeed:

\[
\frac{d\Pi^i}{db} = \Pi^i \frac{dq_i}{db} + \Pi^j \frac{dq_j}{db} = p_Q q_i - (2+b)q_j
\]

since \( \Pi^i = 0 \) from the first-order condition.

Therefore, \( d\Pi^j/db \leq 0 \) iff \( q_i - (2+b)q_j \geq 0 \) since \( p_Q < 0 \).

Then, \( d\Pi/db \leq 0 \) iff \( q_i^2 + q_j^2 - 2q_i q_j (2+b) \geq 0 \).

It is worth pointing out that if the equilibrium output pair lies inside the convex cone bounded by the \( m-l \) rays, then \( d\Pi/db > 0 \) (\( b > -1 \)).

\footnote{This result obviously rests on the assumption of \( p_{QQ} = 0 \) and \( C_i = 0 \). However, relaxing this assumption will only modify quantitatively our results, but not the essence of the proposition.}
3. The effects of State policies

In this section we analyze the effects of infinitesimal changes in the State controlled parameters \((\mu, \tau, t)\) on the equilibrium values of the firms' outputs and tax declarations.

By implicit differentiation of eqs. (1) and (2) with respect to \(\mu, \tau\) and \(t\) we get, in matrix form:

\[
J \cdot x = k,
\]

where \(J\) is the Jacobian previously defined and \(x\) and \(k\) are column vectors whose values depend on the State controlled parameters with respect to which we differentiate; in particular:

\[
x = \begin{bmatrix}
\frac{dq_i}{dv_h} & \frac{dD_i}{dv_h}
\end{bmatrix}^T,
\]

where \(v_h = \tau, \mu, t\) for \(h = 1, \ldots, 3\). The vector \(k\) is the column vector of the constant terms, which is different for each State controlled parameter. In general, \(k_\tau, k_\mu, k_t\) are positive vectors for decreasing absolute risk aversion.

Solving, we get:

\[
\frac{dq_i^*}{dv_h} = 0, \quad \text{for } h = 1, 2, \tag{9}
\]

which is what we expected, given the separability of decisions between quantity and tax compliance variables.

On the other hand, it can be easily verified that

\[
\frac{dD_i^*}{dv_h} < 0, \quad \text{for } h = 1, 2. \tag{10}
\]

An increase in the probability of detection, or in the fine rate, gives rise to an increase in the amount of the declared tax base, irrespective of the type of tax in use.

The results obtained for the tax rate are definitely more interesting; indeed, it can be easily shown that, in this case:

\[
\begin{align*}
\frac{dq_i}{dt} &< 0 \quad \text{iff} \quad R_i^d(2 + b) \leq R_j^d, \quad \text{for the unit tax,}^8 \\
\frac{dq_i}{dt} &\geq 0 \quad \text{iff} \quad R_i^d(2 + b) > R_j^d, \quad \text{for the ad valorem tax,} \\
\frac{dq_i}{dt} &= 0 \quad \text{for the profit tax.} \tag{11}
\end{align*}
\]

^8In the unit tax case, the unambiguous sign depends on the assumption of \(p_{Q0}\) and \(C_{ii}^d = 0\). Cf. Levin (1985, p. 285).
The economic interpretation of the above results is straightforward. Given the one-sided property of the model, the profit tax gives rise to zero shifting, as in the no evasion problem and, therefore, \( dq_i/dt = 0 \). The different results obtained for the indirect tax cases need a more careful explanation; an increase in the unit tax creates an incentive to cut output. The reduction in output, in the case of \( \rho_0 = C'_{ij} = 0 \), has no asymmetric effects on the profits of the two firms. On the other hand, in the case of an ad valorem tax, even with linear marginal revenue and cost schedules, asymmetry is critical; the reduction in the output of the bigger firm caused by an increase in the tax rate causes an increase in marginal revenue of the smaller firm, which may more than offset the incentive to cut output due to the higher tax rate.

Therefore, an increase in the ad valorem tax rate does not always cause a reduction in the equilibrium outputs of both firms; it may well be the case that the output of one firm increases while the other decreases. The results depend on initial market shares and, therefore, given the structure of our model, on relative costs and conjectural variations. From the above conditions it is easy to see that if market shares are such that both equilibrium marginal revenues lie in the convex cone bounded by (4) and (5) expressed as strict equalities, then both quantities will decrease, while if market shares are such that marginal revenues are 'too far apart', then equilibrium quantities will move in opposite directions.

Turning now to the effects of a change in the marginal tax rate on the amount of tax evasion, it is easy to see that for the profit tax, \( dD_i/dt < 0 \) since, in this case, \( d^2T/dq_i dt = R_i - C_i = 0 \), from the first-order conditions, while, for indirect taxes:

\[
\frac{dD_i}{dt} \leq 0
\]  

(12)

according to whether the direct effect on tax liability, due to the change in the tax rate at constant quantity, is lower or higher than the indirect effect that such a change causes through shifting and, therefore, quantity variations of both firms. The first, direct, effect is always positive, since an increase in the marginal tax rate, at a given quantity, increases tax liabilities; the second, indirect, effect is negative since the shifting of the taxes tends to reduce tax liability. It is through this interaction of these opposite effects on the total amount of profits and on the amount of the fine that, given decreasing risk aversion, we observe changes in the tax compliance behaviour.

It is worth pointing out that, in the case of the profit tax, the absence of any tax shifting renders more transparent the result which, in this case, confirms the existence of a positive correlation between tax compliance and
tax rate if the fine is levied on the evaded tax rather than on the unreported tax base [cf. Yitzhaki (1974)].

4. The case of variable probability of detection

In the previous sections we have examined the effects of different strategic behaviour of the firms, and of the variation of some State controlled parameters on the shifting and evasion variables, with exogenously given probability of detection. It should be emphasized that on this last assumption rest the one-sided separability property of our model and, therefore, many of the results obtained.

The aim of this section is to investigate the characteristics of the model when the assumption of a fixed probability of detection is removed. By doing so, it is possible to examine a model where shifting and evasion decisions interact. Indeed, if the auditing program is structured in such a way as to make the probability of being audited positively depend on the difference between the average tax base declared and the tax base declared by the ith firm, we can write the probability perceived by firm i as:

\[ p' = h + m \left[ \frac{(T^i - D^j + T^j - D^i)}{t} - \frac{(T^i - D^i)}{t} \right], \quad i, j = 1, 2, i \neq j, \]

where \( p' \) is defined on the interval \([h, 1]\) with \( 0 \leq h \leq 1 \).

The first-order conditions for an interior solution to the maximization problem for firm i, given that \( dY_i/dq_i = dZ_i/dq_i \), can be written as:

\[ \frac{d \mu'_i}{dq_i}(U_i'Z' - U_i'Y') + \left[ (1 - \mu'_i)U'_y + \mu'_iU'_z \right] \frac{dY_i}{dq_i} = 0, \quad (13) \]

\[ \frac{d \mu'_i}{dD_i}(U'_iZ' - U'_iY') + (1 - \mu'_i)U'_y + \mu'_i(1 - \tau)U'_z = 0. \quad (14) \]

Since \( U'_iZ' - U'_iY' < 0 \) and \( (1 - \mu'_i)U'_y + \mu'_iU'_z > 0 \), for an interior solution to eq. (13) it is necessary that

\[ \text{sign} \frac{d \mu'_i}{dq_i} = \text{sign} \frac{dY'_i}{dq_i}. \quad (15) \]

First, we explore the case of profit taxes; in this case:

9The same result is achieved if utility is assumed to be a function of government expenditure of tax receipts. If a balanced budget is taken into account, then the amount of evasion by firm i's competitor will affect its expected utility. For a model which, in a different context, uses a similar approach, see Cowell and Gordon (1987).
A variation in the output of the $i$th firm will be perceived by the $i$th entrepreneur as giving rise to a change in the probability of being detected which will depend on the effects it has on both firms’ profits. Since $\Pi_j = p(q_i + q_j)q_j - C_j$, and ignoring second-order effects, the $i$th entrepreneur will perceive the effects of a change in his own output on the $j$th firm’s profits as $p_0q_j + b(p + p_0q_j - C_j)$, which explains eq. (16). Thence, it is easy to show that:

$$\frac{d\mu_i}{dq_i} = \frac{m}{2} \left[ p_0q_j(1 + b) + b(p - C_j) - \Pi_j \right].$$

Separability for both firms requires that: $dY_i/dq_i = dY_j/dq_j = 0$; the above condition implies:

$$q_j - q_i = \frac{(p - C_i) - (p - C_j)}{p_0(1 + b)}.$$

which, taking account of eq. (15), implies that in the profit tax case, separability can be achieved only by collusive firms ($b = 1$).

In the other cases ($b \neq 1$), the equilibrium output pair will depend on the utility of the two firms and on the asymmetry in the cost conditions.

Let us now turn to the unit tax. In this case $d\mu_i/dq_i = (m/2t)(b - 1)$. We can note that for collusive firms in the output space ($b = 1$), $d\mu_i/dq_i = 0$, and, given (15), $dY_i/dq_i = 0$. Again, the separability property holds.

On the other hand, for $b < 1$, $d\mu_i/dq_i < 0$, and for interior solutions, $dY_i/dq_i < 0$; therefore, $\Pi_i - \tau < 0$, which implies that each firm’s equilibrium output will now be larger than that produced in the full-shifting case.

Furthermore, by substituting for $[U'(Y_i) - U'(Z_i)]$ into eq. (14) the value found in eq. (13), we find an interior solution for $D_i$ from:

$$\frac{U_y^i}{U_z^i} = \frac{\mu}{(1 - \mu)} \left[ 1 - \frac{\tau}{1 + \frac{1}{\gamma} \frac{dY}{dq}} \right],$$

where $\gamma = (dD_i/dD_j)^C$, and where $c$ stands for conjectured.

Since $U_y^i/U_z^i > 0$, a necessary condition for an interior solution is that:
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\[
\left[1 - \frac{\tau}{1 + \frac{1-\gamma}{t(1-b)} \frac{dY}{dq}} \right] < 0,
\]

and, therefore:

\[
\frac{dY^i}{dq_i} > -\frac{t(1-b)}{1-\gamma},
\]

which sets an upper bound to the equilibrium values of \(q^*_i, q^*_j\), say \(\bar{q}_i, \bar{q}_j\). Therefore, for interior solutions in \(q_i\) and \(D_i (i=1,2)\), \(\bar{q}_i > q^*_i > \bar{q}_j; [\Pi^i - t = 0]\).

In the special case of \(b = \gamma = 0\), \(-t < \frac{dY^i}{dq_i} < 0\), and therefore the equilibrium output will lie between the full shifting quantity and the non-shifting one. Finally, the equilibrium values of \(D_i\) will be greater or smaller than those of the fixed probability case depending on the conjectural variations parameters. In particular, if \(\gamma = 1\), and separability holds, then the amount of evasion will be the same as in the fixed probability case, if and only if \(\mu^i(q^*_i, q^*_j) = \bar{\mu}_i\) (constant probability of detection).

5. Conclusions

In this paper we have examined tax evasion in the context of an oligopolistic market. In particular we have shown how the optimal amount of tax evasion depends not only on the collusiveness of the market but also on the relative market shares. If market shares are not 'too far apart', an increase in collusion gives rise unambiguously to an increase in tax evasion irrespective of the type of tax in existence. Furthermore, we analyzed the effects of State controlled parameters (such as the probability of detection, the penalty and the tax rate) on the tax compliance variable, confirming the qualitative results obtained in the literature for other types of market.

We are well aware of the very simplified nature of the model employed; however even with such a simplified model, one can gain some insights into the nature of the tax evasion phenomenon and, therefore, on the characteristics a design of efficient tax-enforcement policies should have.

References

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