GROWTH AND EQUALITY EFFECTS OF PENSION PLANS

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Abstract

We investigate the balanced growth effects of pension plans on the rate of growth and on
the equality in a closed economy where individual decisions about education are the engine of
growth. We distinguish between two pension systems, a Beveridgean one and a Bismarckian
one, where the latter may depend on one’s entire earning history or not, and show that an
economy with a pay-as-you-go Beveridgean system has a lower growth rate than with a Bis-
marckian one, yet is also “fairer” in terms of intergenerational equality. We also show that
the rate of growth in a FF Bismarckian scheme may be increasing in the rate of contributions.

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1 Introduction

Expenditure on public pension plans and expenditure on the provision of education represent
a large part of government resources nowadays. The purpose of this paper is to examine the
implications of public pension plans on long-run growth as well as equality in a setup of endoge-
nous growth where human capital is the engine of growth. So far, apart from a few exceptions,
the literature either focuses on investigating effects on public pension plans or on investigating
effects of expenditure on the provision of education. Few papers aim at introducing both aspects
into a single model.

Regarding pension systems, in most countries these regimes are based on the pay-as-you-go prin-
ciple (PAYG) where retirement benefits are financed through payroll taxes on labour income.
Several implications of pension policies have been examined in the literature (e.g. induced distor-
tions on the labour markets, effects on saving rates and capital accumulation, and consequences

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for the provision of public goods, see World Bank, 1994). Endogenous growth implications of pension plans from a normative point of view have also been investigated. Saint-Paul (1992) investigates the case for public debt in an endogenous growth model à la Arrow-Romer and shows that the introduction of an unfunded social security system reduces capital accumulation and hence the growth rate, such that it cannot be Pareto-improving. Marchand et al. (1996) show in an endogenous growth setting à la Arrow-Romer that the case for ascending net transfers (like a PAYG pension plan) is rather weak on the balanced growth path.

Regarding expenditure on the provision of education, various studies have looked at the role of public expenditure on inequality and growth. Kaganovich & Zilcha (1999) show that there is a double benefit of public funding of education which consists in reducing income inequality and yielding a positive growth effect. Bearse, Glomm & Ravikumar (1998) investigate how the degree of centralisation of education funding influences income distribution, to name but two papers.

As pointed out, few are the papers that look both at education and pension systems. In an endogenous growth framework Docquier & Michel (1997) examine a similar problem to Marchand et al. (1996), however in a setting à la Uzawa-Lucas where education is the engine of growth, and reach quite a similar result to Marchand et al. (1996). Glomm & Kaganovich (1999) investigate the growth and inequality effects of increased spending on public provision of education that comes at a cost of decreased pension outlays and show that higher spending on public education leads to higher income inequality.

A majority of papers on pension systems ignore the fact that agents are heterogeneous. Thus, investigating pension schemes loses one interesting feature, i.e. that pensions can be used to redistribute: one can oppose a universal flat regime with identical pension benefits regardless of income and work history (the so-called Beveridgean scheme), which hence induces an explicit redistribution within a generation, to an earnings-related benefit regime with higher pension benefits for those who have earned and contributed more (a Bismarckian scheme). Clearly, this distinction makes little sense if agents are considered to be homogenous. Although Glomm & Kaganovich (1999) work in a framework of heterogeneous agents, they do not look at the effect of different pension plans as they only consider a Beveridgean PAYG scheme, their aim being to see what happens if both sectors compete for funds.

In our model we consider an endogenous growth model where human capital is the engine of growth and introduce heterogeneity by considering that individual ability to transform time invested in education into human capital differs among agents. We compare two regimes: a Beveridgean and a Bismarckian one, both if pensions are funded according to the PAYG principle or the fully-funded (FF) principle (contributions are invested and paid out to the contributors when they retire). In terms of economic growth, two opposite effects may be attached to the two regimes in our model. On the one hand, earnings-related benefit plans may create an additional
inducement to acquire human capital when people internalise the link between their pension and their education-related wage. On the other hand, flat-rate plans stimulate savings and capital accumulation since the income smoothing function is not satisfied for the richer part of the population. This too will be shown to affect the rate of human capital accumulation as changes in savings will influence the equilibrium rate of return on capital.

The structure of the model is discussed in section 2. Section 3 presents the comparison analytically as far as possible. We present the calibration of the model in section 4 to study the long-run effects for schemes where no analytical comparison is possible and compare growth and equality effects of a FF scheme as opposed to a PAYG scheme. Finally section 5 concludes.

2 The Model

Our model depicts a closed economy with three types of agents: consumers, firms and the government. Each consumer lives for three periods (youth, adulthood and retirement), the length of each being normalised to one, each generation is of identical size (there is no population growth).

2.1 Overlapping generations structure

Agents born at time $t$ have the opportunity to allocate a proportion $e_{i,t}$ of their youth to education ($0 \leq e_{i,t} \leq 1$), the remaining time of this period is spent working. We therefore implicitly understand the term ‘young’ as being over the age of mandatory schooling. The consequence of investing in the accumulation of human capital is that it will increase one’s productivity when adult - a period that is spent in full-time work (labour is supplied inelastically). In our model agents differ in their individual ability to transform education into productivity. Thus, for a given time spent investing in education, agents that have a higher ability, $b_i$, to transform education into human capital will increase their human capital more than an agent with a lower ability. This is captured by the following equation characterising generation $t$ individuals during their active life (i.e. period $t + 1$), where $h_{i,t+1}$ denotes the level of human capital agent $i$ born at time $t$ has when he is adult (i.e. at time $t + 1$) and $\bar{h}_t$ denotes the level of human capital young agents are endowed with at time $t$.

$$h_{i,t+1} = \bar{h}_t \left[ 1 + b_i e_{i,t} \right] \tag{1}$$

We assume that the level of human capital each young generation starts with, $\bar{h}_t$, is the average level of human capital of its parent generation. This assumption is made in order to have education as the (endogenous) engine of growth and can be justified by the fact that human capital accumulation profiles are very similar during the period of mandatory schooling - a period not considered in our model - and largely dependent on the knowledge of the adult generation.\(^1\)

\(^1\)The more educated the adult generation is, the higher the average productivity of their children will be.
Thus, the level of human capital generation \( t \) inherits is:

\[
\begin{align*}
\hat{h}_t &= Z \int_b b f(b) \, db
\end{align*}
\]

Hence, while agents are identical at birth in terms of their inherited human capital, due to differing abilities individuals are heterogeneous in terms of their levels of human capital when adult. Formally, we assume that abilities to educate, \( b \), are uniformly distributed on \( b \in [0, 1] \), i.e. \( f(b) = \frac{1}{1-b} \).

The lifetime income of an agent of type \( i \) is the discounted sum of his net wage income earned during the time devoted to work during his youth, his net wage income earned during adulthood and his pension. As \( h_{i,t+1} \) is also the productivity expressed in efficiency units of labour, lifetime income is given by:

\[
W_{i,t} = \int_t^{t+1} (1 - e_{i,t}) + \frac{h_{i,t+1}}{R_{t+1}} + \frac{p_{t+2}}{R_{t+1}R_{t+2}} \]

where \( e_{i,t} = (1 - \xi_t) w_t \) denotes the net wage per efficiency unit of labour, \( w_t \) the gross wage rate, and \( \xi_t \) the proportional rate of contributions to the social security system at time \( t \). \( p_{i,t+2} \) denotes the pension an individual \( i \) of generation \( t \) will receive, \( R_t = 1 + r_t \) is one plus the interest rate between periods \( t \) and \( t+1 \).

As we are interested in the two PAYG pension regimes explained above, the Beveridgean and the Bismarckian scheme, each individual’s pension at time \( t \) consists of two parts: a Beveridgean (lump-sum) transfer, \( B_t \), and a Bismarckian transfer. The latter can be modelled in two separate ways: one can either assume that the Bismarckian pension depends on one’s complete earnings (and hence contribution) history, or that it only depends on individual wage income when middle-aged\(^2\). Both options are feasible, examples are e.g. the US for the latter case – in effect only the top 35 earnings years are taken into account when working out the PIA (primary insurance amount) and e.g. Germany for the former. For the moment we focus merely on the case where one’s entire earnings history is taken into account, as this is the ‘purer’ Bismarckian system.\(^3\)

We discuss how the set-up changes if it only depends on one’s adult earnings.

Hence, denoting the replacement ratio of the Bismarckian scheme with \( \frac{1}{\mu} \), the pension an individual receives is given by

\[
p_{i,t+2} = \frac{1}{\mu_t+2} [\mu_i(1 - e_{i,t}) h_t R_{t+1} + h_{i,t+1} + B_{t+2}] \]

Where \( \mu \) signals whether we are in a scheme where the Bismarckian pension depends only on

\(^2\)We rule out negative transfers of any sort to pensioners

\(^3\)A discussion as to why it is purer will be given later.
one's adult income ($\mu = 0$) or on one's entire earnings history ($\mu = 1$). We will later only compare polar cases of a pure Bismarckian or pure Beveridgean, thus the latter is achieved by setting $\frac{1}{2}t = 0$, the former by setting $B_t = 0$.

Each individual maximises an additively time separable utility function of the CES type$^4$ subject to his budget constraint. We denote the time discount factor with $\delta$ and the inter-temporal elasticity of substitution by $\eta$. Both are identical for all agents. Hence we have that individuals maximise

$$\max U_t = \frac{\sum_{k=1}^{\frac{3}{2}} \delta_k \left( q_{t+1} + \left( \frac{1}{1+\frac{1}{\delta_t}} \right) \right)^{\frac{1}{\eta}}}{1+\frac{1}{\delta_t}}$$

subject to

$$W_{i; t} = c_{i; t+1; 1} + \frac{q_{t+1; 1}}{R_{t+1}} + \frac{q_{t+1; 2}}{R_{t+1}R_{t+2}}$$

Agents are of course allowed to supplement their state pension with private savings. In other words, old age consumption is financed by the pension and private savings. The agent’s optimal solution to the above problem yields:

$$q_{t; 1} = \frac{q_{t+1; 2}}{(R_{t+1})^\eta} = \frac{q_{t+1; 2}}{(R_{t+1}R_{t+2})} = \frac{W_{i; t}}{1 + \left( \frac{(R_{t+1})^\eta}{R_{t+1}} + \frac{(R_{t+1}R_{t+2})^\eta}{R_{t+1}R_{t+2}} \right)}$$

Note that we do not take liquidity constraints into account but assume for reasons of simplicity and tractability that individuals have perfect access to credit markets. A discussion of the issues that arise when liquidity constraints are introduced is given in appendix 7.1.

2.2 Discrete education choice

We assume here a discrete choice on education investment. Individuals face a “take-it or leave-it” decision, i.e. they can decide either to devote an exogenously given time to education ($e_{i; t} = e$) or not to invest in education ($e_{i; t} = 0$). This is plausible if one regards the time spent investing in human capital when young as time attending e.g. a university or taking part in a training course: the length of such a programme is given and one ‘cannot’ decide to stop attending before it is over.$^5$

Due to their different ability to transform time spent investing in human capital into productivity, agents will be heterogeneous in terms of ability, and hence earnings, when adult. Clearly, it is preferable to select $e$ if the resulting lifetime income is higher with than without education. Agents therefore compare what their lifetime income would be if they decided not to invest in education to what they would enjoy if they decided to opt for the investment (they do not

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$^4$Any subscript after the time-subscript refers to what period the agent is in. Hence $q_{i; t+1; 1}$ refers to consumption of a young individual $i$ at time period $t$, $q_{i; t+1; 2}$ refers to consumption of an adult individual $i$ at time $t$.

$^5$’Cannot’ may seem a harsh notion, but one could capture the notion by assuming that without a certificate/diploma confirming successful completion of the programme, no productivity gains are realised.
take account of general equilibrium effects, as each agent considers his actions to be negligible upon the aggregate. As the abilities of agents are uniformly and continuously distributed, there will be an agent who is indifferent to investing in education or not: his cost of doing so, the foregone income when young, will just balance the present value of the additional gain of his higher productivity. We will label the ability this agent has “the critical value of ability”, $b_c$.

Agents with a lower ability will choose not to invest whilst agents with a higher ability will choose to do so. As perfect access to credit markets is given, $b_c$ is obtained by comparing the income one would have with education to that without education. Comparison yields that the critical cut-off of abilities is thus:

$$b_c = e_1 - \frac{R_{t+2} + \mu \delta_{t+2}}{R_t + \delta_{t+2}}$$

Thus, we can see that on the balanced growth path where $\delta_t = \delta_{t+1} = \delta$ and $w_t = w_{t+1} = w$, neither the rate of taxation nor the wage rate have a direct effect on the critical level of abilities. Figure 1 gives the distribution of labour efficiency amongst adults.

Let us point out that if liquidity constraints were introduced, the result would be that the critical level of ability would not be lower than in the liquid case. As agents deciding to invest in education all spend the same time doing so, savings are increasing in ability as lifetime income is increasing in ability. Hence, ruling out negative savings would mean that the lowest ability types that choose to invest in education in the unconstrained case could no longer afford to do so, the critical level of abilities would not decrease.\(^6\)

\section*{2.3 Endogenous growth}

Economic growth comes from the fact that the average stock of human capital of adults is transmitted to the young of the next generation (equation (2) above). This gives the growth

\(^6\)Again, the reader is referred to appendix 7.1 for a more detailed discussion.
rate of human capital, if $f(b)$ is taken as a uniform distribution, as

$$g_{t+1} = \frac{\Phi_{t+1} \Phi_t}{\Phi_t} = \frac{\sigma \beta^2 b_i b^2}{2 \beta_i \beta}$$  \hspace{1cm} (9)

### 2.4 Production and factor prices

At each period of time a composite good, $Y_t$, is produced by a representative firm using capital, $K_t$, and labour, $L_t$, measured in efficiency units. The production function is of the CES type with constant returns to scale:

$$Y_t = A^{-\theta} K_t^{\frac{1}{\theta}} + (1-\theta) L_t^{\frac{1}{1-\theta}}$$  \hspace{1cm} (10)

where $\theta$ is the share of capital income, $A$ is a scale parameter and $\theta$ measures the elasticity of substitution in production. The firm acts competitively so that it equates the gross wage and the rate of return on capital with the respective marginal productivity:

$$w_t = A^{\frac{1}{\theta}} (1-\theta) Y_t^{\frac{1}{L_t}}$$  \hspace{1cm} (11)

$$1 + r_t = 1 + A^{\frac{1}{\theta}} (1-\theta) Y_t^{\frac{1}{K_t}}$$  \hspace{1cm} (12)

Capital is assumed to depreciate completely each period.

### 2.5 Closing the model

As noted above, human capital is equivalent to productivity, which is expressed in efficiency units. Hence, labour supply during period $t$ is given in efficiency units by

$$L_t = \mu_i \beta_i \frac{b^2}{b_i} \Phi_t$$  \hspace{1cm} (13)

where the first term denotes total labour supply of the young at time $t$ and the second term labour supply of the adult at time $t$. The government levies a proportional tax (i.e. contribution to the social security scheme) on wage income. A share of these contributions may be invested to a social security reserve/fund. Let us denote the social security fund available at the start of period $t$ by $K^c_t$. The government budget constraint is given by

$$\ell_t w_t L_t + R_t K^c_t = \int_0^\infty \rho_t f(b) \, db + K^c_{t+1}$$  \hspace{1cm} (14)

We denote with $\hat{A}$ the share of the contributions to the social security system that are funded at time $t$, so that $1 - \hat{A}$ is the share distributed according to the PAYG principle. We also assume that the total fund, principal and earned return, is used after one period.

$$K^c_{t+1} = \hat{A} \ell_t w_t L_t$$  \hspace{1cm} (15)

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7The size of each generation has been normalised to 1 as there is no population growth.
A pure PAYG pension scheme is obtained when $\dot{A}_t = 0$, we have in our model a publicly (fully) funded system if $\dot{A}_t = 1$ as opposed to an individualised fully funded system ($\dot{\zeta} = 0$) which is equivalent to private savings. Given the way we have modelled pensions in (4), a publicly fully funded Bismarckian system where pensions depend on one's entire earning is equivalent to private savings. We will see later however that a different Bismarckian system (e.g. where $\mu \neq 1$) may not display this equivalence.

We also want to point out here that a fully funded system in our model differs from the usual definition of a fully funded scheme: fully funded schemes in the standard literature do not usually allow for any intergenerational and intragenerational redistribution. As most models are two-period OLG models, each young generations' contributions are invested into a fund and paid out one period later when the young have retired. In our model however, a fully funded scheme does imply intergenerational redistribution: as we work in a three-period OLG set-up, the contributions of the young and adult generation are invested in a fund for one period. Hence, retirees benefit not only from their own contributions to the fund made when they were in full-time occupation, they also benefit from the contributions of the young last period (compare $L_t$ in (13) and (15)). Intragenerational redistribution enters in our setup of pensions only if pensions are not purely Bismarckian and depend on one's entire earning history, as we will later see.

We have to close the model by introducing the capital stock dynamics. As the capital stock in period $t + 1$ is given by the total savings of period $t$, we have:

$$K_{t+1} = \bar{e}_{t:1} + \bar{e}_{t:2} + K^c_{t+1}$$

(17)

where $\bar{e}_{t:1} = \frac{R}{\beta} \bar{s}_{t:1} f (b) db$ and $\bar{e}_{t:2} = \frac{R}{\beta} \bar{s}_{t:2} f (b) db$ are the average savings of the young and of adults at time $t$.

2.6 Balanced Growth Path

We want to examine the long run growth and equality effects of PAYG pension plans and focus exclusively on balanced growth results. To do so, we have to show that a balanced growth path exists, i.e. that all variables grow at the same rate.

Rewriting (3) using (4), we see that lifetime income is proportional to the average level of human capital in the economy:

$$W_{i:t} = \frac{1}{\beta} t (1 + \bar{e}_t) + \frac{1}{R_{t+1}} \left( 1 + e^{\gamma}_{it} \right) + \frac{\mu}{R_{t+2}} t (1 + e_{it}) + \frac{B_{t+2}}{R_{t+1}} + \frac{1}{R_{t+2}}$$

(18)
Given the CES utility function we selected in (5), we saw above that an individual’s consumption in all periods of his life is proportional to his lifetime income. However, as the average level of human capital grows over time at the rate $g$, so does wealth, hence do consumption and utility. We therefore have to defate the latter three and all other intensive variables (stock of capital, savings) by the average level of human capital.

3 Comparison of Pension Plans

3.1 Inequality

To examine the long run growth and equality effects of pension plans, we focus exclusively on balanced growth results. To compare pension schemes in terms of inequality we first have to define a measure of inequality. No doubt, selecting a measure of inequality is not an easy task. We choose here a positive measure defined by the coefficient of variation of lifetime income - a measure that attaches equal weight/importance to proportional transfers at different levels of incomes and is measured relative to the mean level of lifetime income (the measure is therefore not affected if all incomes were to increase proportionally).

The measure of inequality is given by the term:

$$r = \frac{\text{E}(W_{i,t})^2}{\text{E}(W_{i,t})} = \frac{\text{E}(W_{i,t})^2 - \text{E}((W_{i,t}))^2}{\text{E}(W_{i,t})}$$

where $W_{i,t}$ is defined in (3). Inequality is lowest if all have the same lifetime income.

We expect that inequality will be a decreasing function of the rate of contributions to a pure Beveridgean pension system irrespective of the financing method, i.e. FF or PAYG: a higher rate of contributions implies a higher Beveridgean transfer, which implies progressive redistribution. Also, the need for private savings decreases. An implication of this is that there is less capital available on the capital markets, which pushes the rate of return on capital up. This will decrease the proportion of agents deciding to invest in education. Furthermore, as the uneducated save a (relatively) larger proportion of their labour income for old-age consumption, their savings earn more interest, which increases their lifetime income relative to the educated. Thus, overall inequality will be lower the higher the rate of contributions is.

3.2 What to compare

Focussing on the balanced growth path, we want to

1. compare a pure Bismarckian PAYG system to a pure PAYG Beveridgean one
2. compare a pure FF Beveridgean system to a pure FF Bismarckian one.

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8See Sen (1973) for a discussion of positive and normative measures of inequality.
In addition, we will compare how the growth rate and inequality vary as the rate of contributions changes. The comparisons will be made by asking how the pure schemes differ in terms of inequality and growth. They can be indicated in terms of Figure 2: for 1: we compare economies along the trajectory $x_1$ to $x_2$ to economies along the trajectory $x_3$ to $x_4$. Likewise, for 2: compare we will compare economies along the trajectory $x_5$ to $x_6$ to economies along the trajectory $x_7$ to $x_8$.

The reason why we focus solely on pure schemes and not on a mix of schemes is that when altering the rate of contributions, we wish to be able to single out what effects are due to which scheme. In the case of a mix of both schemes this would obviously be cumbersome. Furthermore, it should be clear that a comparison moving ‘diagonally’ makes little sense, e.g. comparing $x_1$ to $x_4$ will not give much information, as one has a different rate of contributions as well as a different kind of redistribution mechanism (in this case a Bismarckian scheme as opposed to Beveridgean one). A ‘diagonal’ comparison of e.g. $x_2$ to $x_5$ makes even less sense as one learns little by changing the rate of contributions, the redistribution mechanism and the financing scheme.

To be able to perform such comparisons, we need to know the sign of the derivative of growth with respect to the contribution rate and thus need to turn to the critical cut-off level of abilities, $b^\phi$, and the growth rate, $g$.

\footnote{Note that in Figure b the height of the intersection of the lines $X_{1i} > X_2$ and $X_{7i} > X_8$ with the vertical (1/2) axis respectively is at the point that balances the social security budget when $B = 0$. Hence, the height of $X_1$ compared to that of $X_7$ need not be identical.}
Rewriting (8) and (9) for the balanced growth path, we have:

\[
bt^* = e^b \Theta(R + \mu \frac{\Theta'}{R + \frac{1}{2} R})
\]

\[g = \frac{e^g b_i^2 b^c}{2 b_i b^c} \tag{20}\]

\[g = \frac{e^g b_i^2 b^c}{2 b_i b^c} \tag{21}\]

### 3.2.1 Altering the rate of contributions

**Growth** Taking the derivative of (21) with respect to \(\dot{t}\), we obtain:

if \(\mu = 1\):

\[
\frac{\partial g}{\partial \dot{t}} = i \frac{e^g b_i^2 b^c}{b_i b^c} \tag{22}\]

if \(\mu = 0\):

\[
\frac{\partial g}{\partial \dot{t}} = i \frac{e^g b_i^2 b^c}{b_i b^c} + \frac{\mu}{b_i b^c} \frac{R e^g b_i^2 b^c}{(R + \frac{1}{2})} \tag{23}\]

Note that (22) holds both for a pure Bismarckian if the pension depends on one's complete earnings history as well as any pure Beveridgean scheme, irrespective of what earnings it depends on, for any type of funding (fully funded or pay-as-you-go). (23) holds for a pure Bismarckian scheme that depends only on one's adult earnings.

**Conclusion 1**: In an economy where pensions depend on one's entire earnings (i.e. contributing) history, the balanced growth rate is decreasing in the rate of contributions, no matter whether one has a PAYG or a FF scheme.

This can be seen upon closer inspection of (22): the sign of \(\frac{\partial g}{\partial \dot{t}}\) is positive as increasing contributions can have two possible effects. On the one hand, it reduces in a partial equilibrium framework the need for savings as pensions increase. In terms of a welfare effect, agents have less disposable income and hence save less. Both of these effects lower the balanced growth level of capital per efficiency unit of labour, one will observe a higher rate of return on capital on the balanced growth path.

It is also obvious that the term \(\frac{\partial g}{\partial \dot{t}}\) in (23) is non-negative: increasing the rate of contributions in a pure Bismarckian plan allows to increase the pension replacement rate. However, when investigating the effect of an increase in the contribution rate on the balanced growth rate of a Bismarckian scheme that only depends on one's adult income, no analytical conclusion can be made as it is impossible in our set-up to obtain a closed form for \(R\) and hence for \(\frac{\partial g}{\partial \dot{t}}\). Hence, for the case where the Bismarckian pension only depends on one's adult income, we have to turn to numerical simulations. To do so, we ...rst have to calibrate our model, which is done below in section 4.
Inequality: An intragenerationally redistributive Bismarckian PAYG scheme where $\mu = 0$ redistributes regressively\(^{10}\). Thus inequality in a pure Bismarckian scheme is lower if pensions depend on complete earnings compared to what would be the case if regressive intragenerational redistribution takes place, irrespectively of whether one has a FF or a PAYG scheme.

Growth: In a Bismarckian scheme all individuals receive a higher pension when $\mu = 1$ than when $\mu = 0$. Thus, everybody saves less if $\mu = 1$ as compared to $\mu = 0$: In a PAYG scheme, this implies a higher rate of return on capital on the balanced growth path as less is saved, which hence makes education more costly and thus crowds low ability types out of the market for investment in human capital, thus growth is lower than in the case with PAYG pensions depending only on adult income.

In a FF scheme the decrease in private savings is compensated by the increase in the capital stock of the pension fund, $K_c$. Hence, a priori, no conclusion can be made whether growth is lower or higher if the FF Bismarckian pension depends on one's entire contributing history or not. We will turn to this issue in the next section.

Conclusion 2: Inequality is not significantly affected by the contribution rate if one's pension in a Bismarckian scheme depends on one's complete earnings history or only on one's adult income. However, the rate of growth as well as how the steady state level of growth is affected by the contribution rate is influenced by what the Bismarckian pension is based upon (complete or incomplete earnings history).

4 Calibration

We now focus on pension schemes where the Bismarckian transfer only depends on the earnings one had when adult. Implicit in this is hence some intragenerational redistribution: the agents who do not spend part of their youth investing in education contribute more (in terms of time) to the social security benefit (contributions that are not taken into account when calculating their pension).

The result that we cannot show analytically whether the balanced growth rate is either increasing or decreasing when the Bismarckian pension depends only on one's adult earnings is not satisfactory. We therefore turn to numerical simulations of our model in order to be able to infer how the pension system is likely to affect the economy. This requires careful calibration of the parameters of the model in order to obtain solutions in accordance with long-run observations.

Our parameters are the marginal return of education in the human capital accumulation function (\(\beta\)), the share of capital in the production function of consumption goods (\(\bar{\alpha}\)), the scaling factor of the production function (\(A\)), the inter-temporal elasticity of substitution in consumption (\(\gamma\),

\(^{10}\)i.e. from uneducated to educated, as the former do not benefit from their longer contribution history.
the rate of time preference (°), the proportion of their youth individuals deciding to invest in acquiring education devote to human capital formation (e), the upper and lower bounds of ability type (b and B), the rate of population growth (n), the elasticity of substitution in production (e) and the replacement rate of the earnings-related scheme (½). There is some empirical agreement on the share of capital income (¯) and on the inter-temporal elasticity of substitution (¾). Unfortunately, the parameter of the human capital accumulation technology is little known.

A pure pay-as-you-go earnings-related scheme, i.e. where no Beveridgean transfer to pensioners exists, is drawn upon as a baseline case for the comparison. The calibrated values of the parameters are given in the table below:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>A</th>
<th>B</th>
<th>e</th>
<th>°</th>
<th>b</th>
<th>B</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline case</td>
<td>.2</td>
<td>:27</td>
<td>16</td>
<td>.8</td>
<td>:75</td>
<td>:45</td>
<td>15</td>
</tr>
</tbody>
</table>

Table 1

In particular, for the baseline case we choose an inter-temporal elasticity of substitution lower than unity (¾ = :80) which is a common assumption in overlapping generations models. The time-preference rate per generation is chosen to be 75%, which represents an annual discount factor of 1.3% over 22 years (the length of a generation). The elasticity of substitution in production is chosen to be one, which means that we have a Cobb-Douglas production function. The share of capital income is set at 27%, a value which is compatible with long-run data.

The proportion of time young individuals spend acquiring education is a value that is difficult to quantify. In our model we assume that the length of one generation in our model represents approximately 22 years. Furthermore, agents are considered to be 'young' in our model once they reach 18 (hence "adult" aged between 41 and 63 where they work full-time, and "retirees" thereafter). The age span birth-18th birthday is neglected in our model for the simple reason that not only are education, i.e. human capital accumulation, profiles very similar during this span (due to mandatory schooling), but also as we do not consider individuals younger than 18 as being an active part of the labour force. We choose a value for e of 45%, i.e. individuals who decide to increase their productivity must spend 45% of their first period doing so. Although this may seem a high value, empirically it is not so as it implies that one must spend 9.9 years acquiring human capital (full-time) - a number that is not too different to the number of average schooling years (8.88) in the total population above 15 in Belgium, 1990.

Furthermore, the influence of how the time spent acquiring human capital affects future productivity, i.e. ® is likely to be a crucial character and was somewhat arbitrarily chosen to be 20%. This was done so that our baseline economy exhibits feasible values of growth, moreover, what is likely to be more of importance is the relative magnitude of ® to the share of capital in income, ̅, rather than the absolute magnitude ®.

The parameters of our baseline case therefore give an annual growth rate (g) of 3% per annum.
(which is equivalent to 76% growth per generation) and a 6% p.a. return on capital \( r \). 18% of the young invest in education \( \text{edu} \). As we have a positive contribution rate to the social security system and have ruled out Beveridgean transfers for the baseline case, one can solve the government's social security constraint and can conclude that retirees receive a pension which represents 60% \( (r) \) of their labour income received when adult. The young agents who decide not to invest in education save a proportion of 28.5% of their wage income \( (s_{1,u}) \). These values are plausible from an empirical point of view.

\[
\begin{array}{cccccc}
 r(p,a) & g(p,a) & \frac{1}{2} & \text{edu} & \hat{\iota} & s_{1,u} \\
 6\% & 3\% & 60\% & 46\% & 15\% & 28.5\% \\
\end{array}
\]

We will see that although the effect of contributions on the growth rate is likely to be negative for realistic parameters if we have a PAYG scheme, one cannot rule out that the growth rate will increase if the rate of contributions is increased. Surprisingly, in a FF scheme the effect of an increase in the rate of contributions is to increase the growth rate.

4.1 PAYG

Figures 3 and 4 give the growth and equality effects of a PAYG pension scheme when the contribution rate \( (\hat{\iota}) \) moves from 0.0 to 0.45. Two cases are considered: a pure Bismarckian scheme and a pure Beveridgean scheme. It appears that

1. both growth and inequality are lower the higher the rate of contributions is

2. in a Beveridgean scheme the rate of growth is lower than in a Bismarckian one, yet equality is higher.

\[
\begin{array}{cccccc}
 1.20 & 1.30 & 1.40 & 1.50 & 1.60 & 1.70 & 1.80 & 1.90 & 2.00 & 2.10 \\
 0.00 & 0.05 & 0.10 & 0.15 & 0.20 & 0.25 & 0.30 & 0.35 & 0.40 & 0.45 \\
\end{array}
\]

\text{Contribution Rate}

\begin{itemize}
  \item \text{Bis}
  \item \text{Bev}
\end{itemize}

Figure 3: Growth effects in PAYG scheme \((\mu = 0)\)

The fact that growth in a Bismarckian system is higher than in a Beveridgean one was to be expected: for a given positive contribution rate, individuals have a higher incentive to invest
in human capital under a Bismarckian scheme as individuals then benefit from their increased productivity when old as well as when adult under such a pension system.

Going back to the discussion on how inequality will be affected in a pure Bismarckian scheme, we can deduce from figure 3 that the education disincentive of a higher rate of return on capital outweighs the education incentive effect of a higher Bismarckian factor when the rate of contributions is larger, hence inequality is lower.

The attractiveness of a Bismarckian scheme in terms of higher growth stands however in contrast to the drawback that inequality is also higher under such a scheme: in a Bismarckian scheme the educated benefit from their higher productivity when adult as well as when retired. Due to benefiting from their education investment twice hence, inequality is of course larger. The adoption of a PAYG-Bismarckian instead of a PAYG-Beveridgean scheme will therefore depend on whether growth is more valued than equality.\textsuperscript{11}

4.2 Comparison with a fully funded scheme

Since part of the growth cost of the PAYG pension system stems from general equilibrium effects of the interest rate on savings $\frac{1}{\alpha} \frac{\partial \alpha}{\partial \omega}$, it is interesting to investigate the effects of changing the rate of contributions to a fully funded public pension scheme. Let us recall, that in our set-up a fully funded system only implies investing contributions for one period whilst maintaining the benefit rule (Bismarckian or Beveridgean), yet redistribution is still effective.\textsuperscript{12}

The table below gives values where the pension system is now a FF Bismarckian one ($\hat{A} = 1$)

\textsuperscript{11}A robustness analysis of our results is not presented in the paper but has nevertheless been performed and shows that the results obtained above are not sensitive to the chosen values of our parameters in table 1.

\textsuperscript{12}Here we are dealing with a compulsory fully funded scheme, where contributions are still levied. However, one should not forget that in any case, by reducing the contribution rate to the social security system, individuals still have the possibility to save privately and we therefore also have a private fully funded scheme.
yet where we use the same parameters as the baseline case given in table 1. We see that in comparison to a PAYG Bismarckian scheme, more individuals decide to invest in education scheme in a fully funded Bismarckian scheme. The rate of return on capital is lower in a FF Bismarckian scheme than in a PAYG Bismarckian one and the Bismarckian replacement rate \( \frac{1}{2} \) is lower. Furthermore, the young deciding not to invest in education save a slightly lower percentage of their wage income than in the PAYG Bismarckian scheme.

\[
\begin{align*}
\text{r} & (\text{p:a}) \quad \text{g} (\text{p:a}) \quad \frac{1}{2} \quad \text{edu} \quad \xi \quad s_{1:w} \\
5.5\% & \quad 3.2\% \quad 18\% \quad 55\% \quad 15\% \quad 26\%
\end{align*}
\]

Figures 5 and 6 show the growth and equality effects of this funded regime compared to the PAYG scheme above. A striking feature is that a fully funded Bismarckian scheme differs from the other schemes investigated in this paper in such that the balanced growth rate and inequality are actually increasing in the contribution rate. One should therefore ask what the effects are that lead to an increase in the growth rate in a fully funded Bismarckian system instead of a decrease as in a PAYG Bismarckian scheme when the contribution rate is increased.

A higher contribution rate obviously implies, ceteris paribus, a higher Bismarckian pension - both in the pure FF and pure PAYG Bismarckian scheme. Rewriting equation (16) for a pure Bismarckian scheme \( B = 0 \) and solving for \( \frac{1}{2} \) we get on the balanced growth path:

\[
\frac{1}{2} = \frac{\mu}{2} \left( \frac{b_i \cdot b f}{b_i} \right) \cdot \eta \cdot (1 + \Delta + R \Delta) \cdot \frac{i_1}{\lambda} (1 + g)
\]

(24)

Ignoring for the moment the effect of the return on capital and the critical level of abilities (and hence the growth rate), we see that \( \frac{\partial \mu}{\partial R} > 0 \). Given this, we see that the incentive to invest in human capital increases as one increases the contributions to a social security scheme and that savings of individuals will decrease. However, as the decrease in private savings is compensated
by an increase in public savings (in form of the pension fund), this offsets the disincentive effect of a higher rate of return on capital on the investment of human capital. Hence, we can conclude that a higher rate of contributions to the FF scheme increases the incentive of individuals to invest in education as the rate of return on capital falls, which outweighs the decrease in the Bismarckian replacement rate.

From (23), we thus deduce that $R \frac{\partial \tilde{g}}{\partial \tilde{\xi}} > \frac{\partial R}{\partial \tilde{\xi}} (R + 2\tilde{\xi})$ and hence $\frac{\partial \tilde{g}}{\partial \tilde{\xi}} > 0$.

Turning to a funded Beveridgean scheme, an increase in the growth rate due to an increase in the rate of contributions can be ruled out by virtue of (22).

We also see from figures (5) and (6) that compared to a PAYG Beveridgean scheme a funded Beveridgean scheme induces a higher growth rate, it unfortunately also increases inequality: as the return on contributions in a Beveridgean funded scheme is higher in our economy than in a PAYG one, the same contributions yield a higher pension. Thus, every agents decides to save less. However, again, the decline in private savings is more than compensated by the savings of the fund, hence the incentive to invest in human capital is higher in a Beveridgean funded scheme and growth is higher compared to a Beveridgean PAYG scheme, yet so is inequality.

**Conclusion 3** The growth rate in a Bismarckian FF scheme is increasing in the rate of contributions if the Bismarckian transfer only depends on one’s adult earnings.

**5 Conclusion**

In an endogenous growth economy with human capital accumulation we illustrate that an economy with a PAYG public pension scheme with lump-sum benefits (Beveridgean system) has a lower equilibrium growth rate than an economy with a redistributive PAYG Bismarckian scheme as the incentive to invest in human capital in the former is lower in comparison to the latter.
Comparing a PAYG scheme to a FF scheme with the same pension rule (i.e. Bismarckian on Beveridgean), we see that the incentive to invest in human capital is higher in the FF scheme. This is the case as the total amount of capital in the economy is higher with a FF system, which results in a lower rate of return on capital in comparison to a PAYG scheme. This in turn reduces the cost to invest in human capital.

Interestingly, whereas a higher contribution rate to a PAYG scheme will imply a lower rate of growth on the balanced growth path, this is not true for a FF Bismarckian system if pensions depend only on one's adult earnings as opposed to one's entire earnings history. Here the balanced growth rate is increasing in the contribution rate! This is due to the fact that an intragenerationally redistributive Bismarckian PAYG scheme (i.e. one where $\mu = 0$) redistributes regressively. Hence inequality is lower if the scheme depends on complete earnings in a Bismarckian system compared to what would be the case if intragenerational redistribution takes place, irrespectively of whether one has a FF or a PAYG scheme.

It seems that whereas inequality is not significantly affected if a Bismarckian scheme depends on one's complete earnings history or only on one's adult income\(^{13}\), it does have a significant impact on the rate of growth as well as how the steady state level of growth is affected by the contribution rate: as all individuals receive a higher pension in the case without intragenerational redistribution, everybody reduces their savings. In a PAYG scheme, this results in an increase in the return on capital on the balanced growth path, which hence makes education more costly and thus crowds low ability types out of the market for investment in human capital, thus growth is lower than in the case with pensions depending only on adult income. In a FF scheme the decrease in private savings is more than compensated by the increase in the capital of the pension fund, hence the opposite is the case: the rate of return on capital decreases, making education less costly, which crowds lower ability types in and hence increases the growth rate.

Pension schemes must however also be appreciated in terms of equality in a set-up of heterogeneous agents. We illustrated that a Beveridgean scheme induces a strong redistribution amongst individuals and that the higher the contribution rate, the more equal the economy becomes: a higher contribution rate crowds low ability individuals out of the market for investment in human capital, resulting in a reduction of differences in income among agents. Again, the opposite is true for a fully funded Bismarckian scheme (related to adult earnings): inequality increases the higher the rate of contributions to the social security system is, as a higher contribution rate actually crowds in low ability types.

\(^{13}\)Graph omitted here.
6 REFERENCES


² MARCHAND M., MICHEL, P. & PESTIEAU, P. (1990), "Optimal Intergenerational Transfers in a Model with Fertility and Productivity Changes", Core Discussion Paper N\textsuperscript{±}9059


7 Appendix

7.1 Liquidity Constraints

As stated above, we neglect liquidity constraints in the main results of the paper. The reason why we neglect these constraints is merely for practical reasons as we are not able to find a closed form solution for the level of capital in the economy. If liquidity constraints were introduced, the result would however be that the critical level of ability would not be lower than in the case where credit is readily available.

The intuition is as follows: if agents are constrained in their consumption when young, they will only invest in education if their ability is even higher than the critical level in the case of no liquidity constraints as the increase in adult and old-age consumption must make up for the lower, liquidity constrained, level of consumption when young. As a result, even fewer individuals in the economy will invest in education and the growth rate will be lower.

In what follows, we assume that the rate of return on capital in and the wage rate in the liquidity constrained and the unconstrained case are identical (i.e. we take our economy as being a small open one and hence ignore general equilibrium effects) and attempt at introducing liquidity constraints. When introducing liquidity constraints, two constraints must be taken into account when deciding whether to devote time to education or not when young:

1. lifetime wealth if devoting time to education must be at least as high as if an individual does not decide to invest in education
2. his utility when investing in education must equally at least be as large if he decided not to do so.

From above we know that for 1. to be fulfilled, the critical level of abilities must be as in (8). We now have to take the second constraint into account. This second constraint arises from the fact that is individuals are liquidity constrained when young, they consume less than they would...nd optimal. I.e. (7) no longer holds.

Instead, we get the result that a rationed educated individual will choose:

\[ c_{et;1} = (1 - e_t) ! t h_t, c_{et+1;2} = \frac{W_t e_t (1 - e_t) ! t h_t (1 + (R_{t+2})^2)}{1 + (R_{t+2})^2} R_{t+1} \text{ and } c_{et+2;3} = (R_{t+2})^2 c_{et+1;2} \]

Inserting these into the condition that \( U_t i_{et} = e_t \) , \( U_t i_{et} = 0 \) we get:

\[ c_{et;1} + c_{et+1;2} + c_{et+2;3} \text{, } c_{ut;1} + c_{ut+1;2} + c_{ut+2;3} \]

where an additional \( e \) is added to the subscript to denote the educated individual, a \( u \) is added to the uneducated.